MOVABLE BRIDGES

Volume II.—MACHINERY

MOVABLE BRIDGES

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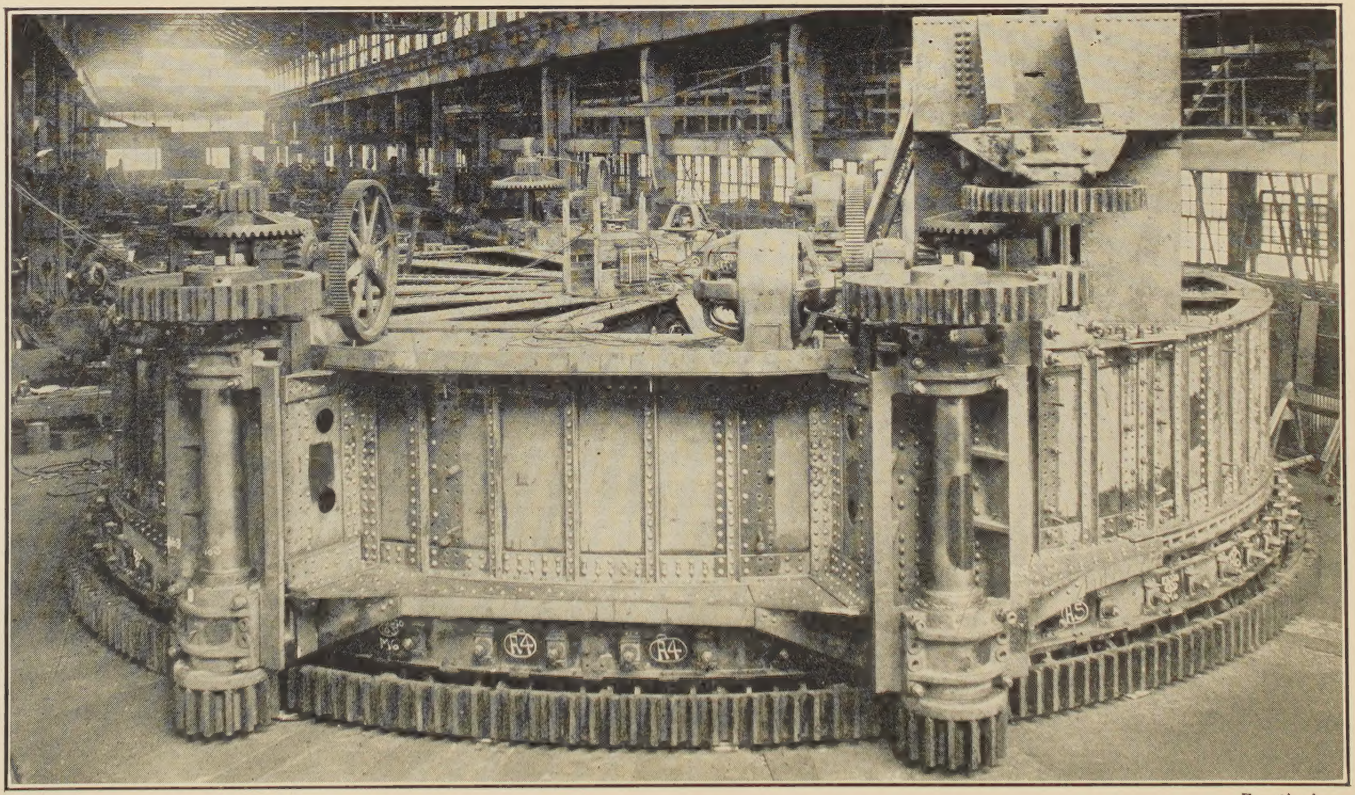

Frontispiece

ATCHISON, TOPEKA, & SANTA FE RAILWAY SYSTEM MISSISSIPPI RIVER BRIDGE

ALBERT F. ROBINSON, M. AM. SOC. C. E., BRIDGE ENGINEER, SYSTEM

Double-track, double-deck railway and highway swing span 525 ft. long.

Swinging weight, 10,000,000 lbs. Four 75-h.p. electric motors for turning.

MOVABLE BRIDGES

BY

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Member of the American Society of Civil Engineers, American Society of Mechanical Engineers, American Society for Testing Materials, American Iron and Steel Institute, and Associate Member of the American Railway Engineering Association

Assistant Chief Engineer of American Bridge Company

IN TWO VOLUMES

VOLUME II.—MACHINERY

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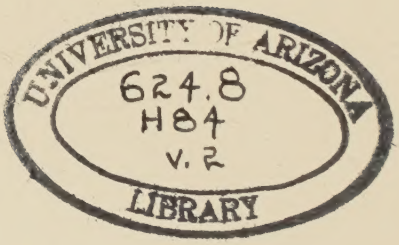

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TO MY WIFE,

WHO INSPIRED THE BEGINNING OF ITS PREPARATION,

AND WHOSE ENCOURAGEMENT

HAS MADE ITS COMPLETION POSSIBLE,

THIS BOOK

IS AFFECTIONATELY DEDICATED

72438

PREFACE

VOLUME I of this treatise was devoted mainly to a discussion of the structural portion of movable bridges. Some parts of the machinery that serve to support the structural work of swing bridges also were included.

The present volume begins with a discussion of the power required to operate movable bridges. In this chapter the determination of the minimum power required is treated as a single inclusive problem, and not divided into a series of separate parts. The theory developed has been in use for several years, and the power plants of many bridges of various types have been designed and built in accordance with this method with satisfactory results.

Another chapter treats of the general arrangements of the operating machinery, and includes many examples which were taken from bridges in successful operation.

The materials suitable for the construction of machinery, the frictional resistance of machine elements, motor and engine torques, and the appropriate unit stresses for various materials and for the various parts of the machinery are treated in following chapters.

The remaining chapters discuss the design of the machine elements that occur most frequently in movable bridges. Many formulae, tables, and charts are given. The numerical constants in the formulae have been reduced to their simplest values, and the tables and charts have been calculated and arranged for convenient and rapid use. Much of this material has been in use for several years, and it has been found to facilitate the correct and rapid design of this class of machinery. This part of the book is intended to be used as a manual of design for heavy, slow-moving machinery, whether for movable bridges or for other purposes.

Appendix A is a specification for movable bridges and their machinery, and represents the best modern practice. Appendix B gives an analysis of large sheaves loaded by ropes under tension. Appendix C is partly original and partly compiled from many sources. The group-

ing of this miscellaneous information will save the time that would otherwise be spent in searching for it elsewhere.

The author acknowledges, with thanks, the permission to use several cuts, credit having been given in connection with each cut. He also is indebted to the American Bridge Company for permission to use much of the material included in Chapter II and Chapters VII to XIII, which he first prepared for the use of this company; also for permission to include the specifications in Appendix A, which were originally written for the use of its engineers; to F. L. Dudley, M. Am. Soc. C. E., for assistance in the preparation of parts of the chapters on machine design; to C. G. E. Larsson, M. Am. Soc. C. E., for the fundamental equations in Chapter II, Art. 2; to Clement Kendall, M. Am. Soc. C. E., for assistance in preparing Chapter XIII, Arts. 6 to 9; to the Carnegie Steel Company for permission to take miscellaneous data from their "Pocket Companion"; and also to other friends for information and suggestions.

Otis Ellis Hovey.

September, 1926.

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MOVABLE BRIDGES

CHAPTER I

INTRODUCTION

1. General.—The power required to operate a movable bridge within a given time is the first problem to be considered when the design of the operating machinery is begun. The time allowed for operation is often fixed by public authority, and in such cases the engineer has little opportunity to develop an economical design. The time for the operation of a movable bridge should be determined after a thorough investigation of all conditions near the site which affect traffic in the navigable channel and on the bridge. The density of traffic in the channel and across the bridge has an important bearing on the time for operation. If the channel is straight and clear, and there is little bridge traffic, the time for operation should be made longer than if the channel is tortuous and obstructed by adjacent bridges and the traffic on the bridge is dense. The length and weight of the movable span should also be considered.

For economy in the cost of construction, operation, and maintenance, the time of operation should be made as long as feasible without inconvenience to traffic. The economic importance of allowing a reasonable time for the operation of the bridge will be appreciated if it is remembered that the power required increases almost in inverse proportion to the cube of the time for operation.

2. Power.—The most economical and reliable source of power should be determined. Where a reliable supply of electric current is available, electric motors are the most satisfactory of all types. Steam plants are the most reliable of all types, for they are independent and self-contained units. At the present time they are seldom used. While steam plants are the more reliable, the first cost and the expenses of attendance and operation are greater than those of electrically operated plants, and most engineers consider that the increased costs

are too much to pay for the greater reliability of steam plants. Gasoline engines are more economical than steam, but their starting torques are small and they are more likely to get out of order than steam engines. Where electric current is not available, and a movable bridge is operated only a few times daily, gasoline-engine plants are the most economical and desirable of all types.

3. Arrangement of machinery.—When planning the general arrangement of the machinery of a movable bridge, the first considerations should be its adaptability to the work it must perform and the strength of its parts. It is of almost equal importance that the machinery be laid out in the simplest possible way. A machine may be of ample strength and capable of doing its work, and still be of such a complex design that it is expensive to manufacture, install, and maintain. It has been noted that, in general, the cost of maintenance is about proportional to the power transmitted by the mechanism. This has a direct influence on the general design of the bridge itself, which should be of the type that is most appropriate for the site and requires the minimum power for operation. In a similar manner, the machinery for the application of the power should be laid out so that the number of parts is small and each part is of the simplest construction that will do the work. The first cost and maintenance will then be a minimum. There will be less lost work from friction when the design is simple and the application direct than when the machine is complicated, with many gears, shafts, levers, and bearings.

4. Gear trains.—The gear trains should be designed so that they can be manufactured, adjusted, and installed in compact units. This is often difficult to accomplish, on account of the necessary arrangements of the superstructure, but serious study will enable the designer to approximate the ideal conditions.

One of the most common faults of design is to adopt arrangements requiring that the shafts pass through structural members. In too many cases the couplings and gears are placed on opposite sides of these members. Erection, adjustment, maintenance, and repairs then become difficult, for the shafts must be stripped in order to be put in or taken out.

When the mechanism can be made in assembled units, the shafts and gears are permanently aligned and adjusted in the shop and it is then only necessary to locate the units in proper relation to each other on the bridge. When each shaft bearing is on a separate structural support, it is a slow and expensive operation to secure perfect alignment and correct meshing of the gears. This is particularly true of pairs of bevel gears.

5. Gears.—Bevel gears should not be used when the work can be done by spur gears. Similarly, large speed reductions should be made by properly designed worms and wheels rather than by screws and moving nuts. Worms and wheels may be the more expensive, but they can be accurately fitted in a gear box at the shop. The worm should always be placed under the wheel, and the housing made oil-tight, and continuous and adequate lubrication should be provided. Hindley worms and wheels should not be used on movable bridges, as the adjustments are so delicate that trouble is almost sure to develop. Hindley worms and wheels also need better care than movable-bridge machinery is likely to receive.

Continuous lubrication of screw gearing is difficult to secure. When the machinery is at rest, the lubricant is exposed on all parts of the screw except within the nut. It collects dirt and becomes wet, and the oil oxidizes, so that frequent attention is required.

Gears should be located between the shaft bearings. When they are on extended cantilever ends of shafts, the stresses on both shafts and bearings are increased. The deflections also are larger, with the result that there is unequal pressure on the bearings.

6. Shafts and couplings.—Short shafts, when rigidly supported, may have flanged couplings. Sellers or some other form of muff couplings will sometimes be found convenient and desirable. Long line shafts, particularly when supported on structural members subject to elastic deflections, should be connected by jaw couplings designed to center the ends of the shafts and at the same time to allow small changes in alignment, due to the deflections of the structural supports. When this arrangement is used, each alternate length of shaft should have two bearings, with couplings on short cantilevers beyond them. The other lengths are then supported by the jaw couplings. When the end lifts of swing bridges are driven from the middle of the span, this form of construction is suitable.

Flexible couplings are desirable when transmission shafts must be coupled directly to rapidly rotating elements, like the armatures of electric motors. In rare cases, when it is impossible to avoid angles in the shafting, it may become necessary to use universal-joint couplings. A bearing should then be placed near each universal joint. The joints should be in pairs, with equal angles, so that the angular velocity of the driving and driven shafts may be the same.

When the mechanism must complete an operation at the end of a definite cycle, as in the case of an end-lifting apparatus, it is often necessary to pay particular attention to the elastic torsional stiffness of the shafts as well as to their strength.

All shafts should be prevented from moving longitudinally by contacts between the gear hubs and the bearings, or by special thrust collars. The thrust of bevel gears should receive special attention. Cast-steel hubs running against cast-steel bearings will seize and cut at small pressures. Bronze thrust washers should be used between the steel surfaces.

7. Equalizers.—In cases where one motor is used to drive more than one main pinion, as in heavy swing bridges or in bascules that are driven at each truss, an effective equalizer should be introduced, preferably in the first counter shaft from the motor shaft. In a few designs, attempts have been made to avoid equalizers and to depend on precise workmanship and installation to secure equal loads on the main pinions. When the shafts were relatively short and stiff the results were unsatisfactory, and equalizers had to be added later at much greater cost than if they had been provided in the original design. The machinery may be accurately made and the gears and racks cut, but when the supporting members are of structural steel it is not possible to predict in advance just where the supports will be when erected, no matter how much care is taken in the installation. Variations of small fractions of an inch may occur anywhere and may result in large differences in the rack loads from the pinions. When equalizers are provided, the pinion and rack do not need to be cut, and the resulting economy in shop and field may more than compensate for the cost of the equalizers.

8. Trunnions.—Trunnions carrying heavy loads and rotating at small velocities are often used on bascule and vertical-lift bridges. It is important that such trunnions be designed to avoid any excessive deflections under loads. If the supports of the trunnion bearings are too rigid, the deflections of the trunnions cause unequal distribution of the pressures on the bearings and the maximum pressure may be great enough to prevent lubrication of the bearing linings.

Good lubrication of slow-moving trunnions is very important. Care should also be taken to align such trunnions accurately during erection.

9. Brakes and buffers.—Effective brakes should be provided on the main driving mechanism of all types of movable bridges that are operated by power. When the power is supplied by electric motors on swing and vertical-lift bridges, the main driving motors should have solenoid brakes on the armature shafts. For such bridges of ordinary size and weight, no other brakes are needed. Unusually large swing and vertical-lift bridges and all bascules should have effective emergency brakes in addition to the solenoid brakes. These should

be operated independently of the electric current and should be applied whenever the current fails. Bridges operated by steam or internal combustion engines should also have brakes. For bridges of ordinary size and weight, the brakes can be operated by hand or foot levers. Those for large heavy spans should be operated by power.

Bascule and vertical-lift bridges must be brought to rest when in fully open or closed position. Air buffers should be supplied to assist in stopping them without shock.

10. Specifications.—There are many other general features of machine design that should be observed. It is suggested that the entire Specification in Appendix A be carefully studied before the design of the machinery is begun.

CHAPTER II

POWER FOR MOVABLE BRIDGES

1. Introduction.—The problems involved in determining the power required to rotate a swing draw span within a given time will be considered in the following order:

Article 2 develops a rational theory of the minimum time in which a given horse-power will rotate a swing draw span through a given angle. Many power plants have been built in accordance with this theory, and experience has demonstrated the accuracy of the method. The maximum rack force, or tooth load, occurs during retardation and is equal to the maxiumum resistance of the engine, working to resist motion, plus the frictional resistances of the machinery from the engine to the rack, both reduced to their equivalent effect at the pitch circle of the rack.

This method should be used when the motive power is a reversing steam engine and when it is more economical to provide for the large rack force than to furnish the larger engine that would be required if the theory developed in Art. 3 were adopted. The rack, its pinion, and the entire gear train, back to the engine, must be stronger than would be required by the theory developed in Art. 3.

Article 3 develops a rational theory of the minimum time in which a given horse-power will rotate a swing span through a given angle, but with the rack force, or tooth load, of constant amount during both acceleration and retardation. According to this theory, a larger horse-power is needed to turn a bridge through a given angle within a given time, but the intensity of the rack force is less than would be required under the theory developed in Art. 2.

The theory developed in Art. 3 is particularly adapted for use in cases where an electric motor is used and the retardation is caused by brakes, which are designed so that the total rack force, which is the sum of the braking effect and the machinery resistances from the motor to the rack, does not exceed the tractive effort of the motor less the machinery resistances from the motor to the rack.

This theory should be used in all cases where it is more economical

to provide the larger motor required than the stronger and heavier rack and mechanism needed for the method of Art. 2.

Article 4 develops a theory of the resistances due to rotating a swing span in still air, and is preliminary to Art. 5.

Article 5 develops a theory of the resistances due to rotating a swing span, with equal arms, against a wind of known velocity, which is assumed to blow at right angles to the bridge throughout the period of acceleration. The maximum resistance occurs at the end of the last second of acceleration.

Article 6 develops a theory of the resistances due to rotating a swing span, with unequal arms, against a wind of known velocity, and is a special application of Art. 5.

Article 7 develops the relations of velocity, time, and space at the rack circle of a swing span rotating in a wind of known velocity.

Experience with many swing-span power plants, designed in accordance with the theory developed in Arts. 2 and 3, indicates that no increase of power is needed unless the time required for opening in a wind of known velocity becomes too great. The span will simply move more slowly. Should the time of opening in a 50-mile wind exceed the time required with no wind by 50 per cent. or more, the horse-power, obtained from Art. 2 or 3, should be increased to reduce the time to within 150 per cent. of the time required without wind.

Article 8 applies the theories developed in Arts. 2, 5, and 7 to an actual swing draw span.

Article 9 applies the theories developed in Arts. 3, 5, and 7 to an actual swing draw span.

Article 10 summarizes the results obtained from Arts. 8 and 9 for rotating the swing span 90° in 45 seconds and also gives comparative figures on the assumption that the time of opening was increased to 90 seconds. The widely different results obtained from the application of the two methods developed in Arts. 2 and 3 should be noted.

Article 11 is a special application of the same fundamental principles to the power required to move a vertical-lift bridge within a given time. In this case it is assumed that the motor exerts more than its normal torque during acceleration.

Article 12 applies the results obtained in Art. 11 to an actual vertical-lift bridge problem.

Important note.—The time for opening a movable bridge should be carefully chosen. When too much time is assumed for moving short and light spans, the horse-power of the motor becomes small and the speed of the armatures of such motors is large. This results in the necessity of using a large gear ratio, unduly increasing the rack force during

acceleration and retardation. While a shorter time will require a larger motor, the armature speed and gear ratio are smaller and in many cases the rack force is materially reduced.

2. Minimum time for a given horse-power. Case I.—Assume that the engine develops a constant torque during acceleration and retardation and that frictional resistances oppose the engine during acceleration and assist it during retardation. Find the minimum time in which a given horse-power will rotate a swing draw span through an angle of 90°.

Let P = number of horse-power;

W = weight of the draw span, as it turns, expressed in pounds;

W_1 = the portion of the weight of the draw span that is used in finding the mass equivalent, also in pounds;

L = the length of the draw span, in feet;

B = the width of the draw span, in feet;

I = the moment of inertia of the draw span; this may be calculated by the following approximate formula:

$$I = \frac{W_1}{g}\left(\frac{L^2 + B^2}{12}\right), \text{ in which } g = 32.2;$$

M = a mass concentrated at the rack circle having the same polar moment of inertia as the moving part of the draw span;

r = the pitch radius of the rack circle; then,

$$M = \frac{I}{r^2} = \frac{W_1}{gr^2}\left(\frac{L^2 + B^2}{12}\right);$$

$C = \frac{1}{4}$ of the circumference of the rack circle = the distance traveled by the main pinion in opening the draw 90°;

t_1 = the time of acceleration in seconds;

t_2 = the time of uniform velocity in seconds;

t_3 = the time of retardation in seconds;

t = the total time for opening 90° = $t_1 + t_2 + t_3$;

X = a tangential force, acting at a radius r, delivered by the motor, or engine, with no deduction for friction in the mechanism between the motor, or engine, and the rack. This force, X, is assumed to be constant during the time of acceleration, t_1, and to have the same constant value, but to act in the opposite direction, during the time of retardation, t_3. In the operation of a steam engine this may be effected by reversing the engine at the beginning of the period of retardation. When an electric motor or a gasoline engine is used, the power may be shut off and a brake, capable of exerting a retarding effect equal to X, applied. The force X acts during the whole of the period of acceleration, t_1,

so that the maximum velocity is attained at the end of time, t_1. During time t_2 the bridge will continue to move at the maximum velocity; but the tangential force, at radius r, will be less than X.

f = the frictional resistance of all mechanism from the motor, or engine, to the rack, expressed as a fractional part of the tangential force, X;

R_1 = the frictional resistance at the rack due to center discs in case of a center-bearing bridge; or the rolling friction and thrust-bearing friction in the case of a drum-, or rim-bearing, draw bridge.

The force at the rack, or tooth load, during acceleration is $(1-f)X$.

The force at the rack, or tooth load, during retardation is $(1+f)X$.

The velocity at the rack circle at the end of acceleration is equal to that at the beginning of retardation; therefore,

$$\frac{(1-f)X-R_1}{M}t_1=\frac{(1+f)X+R_1}{M}t_3, \quad \text{or} \quad t_3=\frac{(1-f)X-R_1}{(1+f)X+R_1}t_1. \quad (1)$$

The average velocity times the total time equals the distance traveled; therefore,

$$\frac{(1-f)X-R_1}{M}\times\frac{t_1}{2}(t_1+t_3)+\frac{(1-f)X-R_1}{M}t_1t_2=C. \quad (2)$$

From this,

$$t_2=\frac{MC}{[(1-f)X-R_1]t_1}-\frac{t_1+t_3}{2}. \quad (3)$$

The work performed by the engine in the time t_1 must equal the work of the force X during the same time; then,

$$\frac{550P}{2}t_1=X\left[\frac{(1-f)X-R_1}{M}\right]\frac{t_1}{2}t_1 \quad \text{or} \quad t_1=\frac{550MP}{X[(1-f)X-R_1]}. \quad (4)$$

Also

$$t=t_1+t_2+t_3. \quad (5)$$

Eqs. (1), (3), (4), and (5) give

$$t=\frac{CX}{550P}+\frac{550PM}{(1-f^2)X^2-2fXR_1-R_1{}^2}. \quad (6)$$

The problem is to determine the minimum time, t, in which the given horse-power will open the bridge. To evaluate Eq. (6) for a minimum time, t, place $\frac{dt}{dX}=0$ and solve,

$$\frac{(1-f^2)X^2-R_1{}^2-2fXR_1}{\sqrt{(1-f^2)X-fR_1}}=550P\sqrt{\frac{2M}{C}}. \quad (7)$$

When beginning an investigation of the power for rotating a swing span, the time for opening is usually determined by the local requirements, or it may be definitely specified by the owner of the bridge, while the horse-power is unknown. The first step toward a solution of Eq. (7) is to find an approximate horse-power which will do the work in the given time.

An analysis has been made, assuming that $t_1 = t_2 = t_3 = \frac{t}{3}$, from which the horse-power, P, for center-bearing spans, was found to be approximately:

$$P = \frac{W_{T_1}(L^2 + B^2)}{6t^3} + \frac{\sqrt{W_T{}^3}}{36.5t}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

In this expression,

P = the required number of horse-power;

W_{T_1} = the weight moved, in tons of 2000 lbs., which is used to find the mass-moment-of-inertia-equivalent at the rack;

W_T = the total weight, in tons of 2000 lbs., which rests on the center discs;

L = the total length of the span, in feet;

B = the total width of the span, in feet;

t = the total time, in seconds, for turning the span 90°.

For rim-bearing spans the expression becomes

$$P = \frac{W_{T_1}(L^2 + B^2)}{6t^3} + \frac{2.1R_T\sqrt{L^2 + B^2}}{t}. \quad . \quad . \quad . \quad (9)$$

In this formula, P, W_{T_1}, L, B, and t have the same significance as in Eq. (8). R_T = the friction, expressed in tons of 2000 lbs., from the discs, rollers, thrust bearings, etc., but excluding the friction of the transmission machinery from the engine to the rack, reduced to a tangential force acting at a radius of

$$\sqrt{\frac{L^2 + B^2}{12}}.$$

Equation (7) is most readily solved by trial after inserting the value of P from Eq. (8) or (9), and assuming a value for X. The approximate value of X, if f and R_1 are neglected, is

$$X = 10\sqrt[3]{\frac{605P^2M}{C}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The first trial in Eq. (7) may well be made with a value of X from Eq. (10) increased by about 10 per cent.

After the value of X has been found by trial, the values of t, t_1, t_2, and t_3 can be found in the following order:

t from Eq. (6).
t_1 from Eq. (4).
t_3 from Eq. (1).
t_2 from Eq. (3).

3. Minimum time for a given horse-power. Case II.—Assume that the rack force, or tooth load, is constant, and is the same during acceleration and retardation.

Find the minimum time in which a given horse-power will rotate a swing draw span through an angle of 90°.

Let P_1 = number of horse-power;

X_1 = a tangential force, acting at a radius r, delivered by the motor, with no deduction for friction in the mechanism between the motor and the rack. This force, X_1, is assumed to be constant during the time of acceleration, t_1. It is also assumed that the retarding force is constant during retardation and of the same amount as the accelerating force. This can be accomplished by making sure that the sum of the braking effect and the retarding force due to frictional resistances does not exceed the force X_1 less the frictional resistances.

Let all other quantities be the same as given in Art. 2.

The force at the rack, or the tooth load, during acceleration is $(1 - f)X_1$, and in accordance with the conditions it will be the same during retardation.

The velocity at the rack circle at the end of acceleration is equal to that at the beginning of retardation, hence

$$\frac{(1-f)X_1 - R_1}{M} t_1 = \frac{(1-f)X_1 - R_1}{M} t_3. \quad . \quad . \quad . \quad (11)$$

Therefore

$$t_1 = t_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11a)$$

The average velocity times the total time equals the distance traveled, hence

$$\frac{(1-f)X_1 - R_1}{M} t_1 \times \frac{t_1 + t_3}{2} + \frac{(1-f)X_1 - R_1}{M} t_1 t_2 = C, . \quad (12)$$

but $t_1 = t_3$; therefore

$$\frac{(1-f)X_1 - R_1}{M} {t_1}^2 + \frac{(1-f)X_1 - R_1}{M} t_1 t_2 = C. \quad . \quad . \quad (13)$$

From this,

$$t_2 = \frac{MC - [(1-f)X_1 - R_1]t_1^2}{[(1-f)X_1 - R_1]t_1}. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The work performed by the motor in the time t_1 must equal the work of the force X_1 during the same time, then

$$550P_1\frac{t_1}{2} = X_1\left[\frac{(1-f)X_1 - R_1}{M}\right]t_1\frac{t_1}{2}, \text{ or } t_1 = \frac{550P_1M}{(1-f)X_1^2 - R_1X_1}. \quad (15)$$

Substitute this value of t_1 in Eq. (14) and reduce, and

$$t_2 = \frac{CX_1}{550P_1} - \frac{550P_1M}{(1-f)X_1^2 - R_1X_1}. \quad . \quad . \quad . \quad . \quad (16)$$

Now $t = t_1 + t_2 + t_3$, but $t_1 = t_3$; therefore

$$t = 2t_1 + t_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Substitute the value of t_1 from Eq. (15) and that of t_2 from Eq. (16) in Eq. (17) and the value of t is found to be

$$t = \frac{CX_1}{550P_1} + \frac{550P_1M}{(1-f)X_1^2 - R_1X_1}. \quad . \quad . \quad . \quad . \quad (18)$$

To find the minimum time, t, in which the force X_1 will turn the span 90°, place $\frac{dt}{dX_1} = 0$, or $\frac{dt}{dX_1} = \frac{C}{550P_1} - \frac{550P_1M[2(1-f)X_1 - R_1]}{[(1-f)X_1^2 - R_1X_1]^2} = 0$, or

$$\frac{(1-f)X_1^2 - R_1X_1}{\sqrt{2}(1-f)X_1 - R_1} = 550P_1\sqrt{\frac{M}{C}}. \quad . \quad . \quad . \quad . \quad (19)$$

Equation (19) is analogous to Eq. (7) of Art. 2. It expresses the relation between the tangential force, X_1, the horse-power, P_1, as yet unknown, and the quantities f, R_1, M and C, which are known for any particular problem.

In any ordinary case the total turning time, t, is known.

For a solution, values of X_1 and P_1 must be found that satisfy Eq. (19), and they must also be such that $2t_1 + t_2 = t$, the total time of turning. The most convenient method of solution is to find an approximate value of P_1, then find X_1 by trial and when Eq. (19) is thus satisfied, evaluate $2t_1 + t_2 = t$, and if necessary correct the value of P_1 and repeat the process.

The method of finding a trial value of P, given in Art. 2, fails to indicate the value of P_1 in Eq. (19) with sufficient accuracy. An approximate analysis shows that the first trial value of P_1 may be found from

$$P_1 = \frac{9W_{T_1}(L^2 + B^2)}{35t^3} + \frac{\sqrt{W_T{}^3}}{48t}. \quad . \quad . \quad . \quad . \quad (20)$$

This equation will give only an approximate value for P_1, but will greatly assist in the solution of Eq. (19). In order to indicate the degree of approximation, the constants in Eq. (20) were found from the calculations for a swing span given in Art. 9. When the total time, t, was assumed as 45 seconds, Eq. (20) became

$$P_1 = \frac{9W_{T_1}(L^2 + B^2)}{38t^3} + \frac{\sqrt{W_T{}^3}}{47t}. \quad . \quad . \quad . \quad . \quad (21)$$

When t was taken as 90 seconds,

$$P_1 = \frac{9W_{T_1}(L^2 + B^2)}{29t^3} + \frac{\sqrt{W_T{}^3}}{50t}. \quad . \quad . \quad . \quad . \quad (22)$$

The approximate value of X_1, if f and R_1 are neglected, is found from Eq. (10).

The first trial in Eq. (19) should be made with a value of X_1 from Eq. (10) increased by about 20 per cent.

After X_1 has been found by trial, the values of t, t_1, t_2, and t_3 can be determined in the following order:

t_1 from Eq. (15).
t_2 from Eq. (16).
$t_3 = t_1$.
$t = 2t_1 + t_2$.

4. Resistance to turning a swing span in still air.—A swing span, with equal arms, is opened in still air:

Find the resistance at the rack, and the horse-power required to overcome this resistance, when the velocity of rotation is a maximum.

When the height of the barometer is 30 ins. the U. S. Weather Bureau uses the following formula from which to calculate wind pressure:

$$p = 0.004V^2, \quad \text{or} \quad p = \frac{V^2}{250},$$

in which V is the actual wind velocity, in miles per hour, and p the resulting pressure in pounds per square foot, on a plane surface normal to the direction of the wind.

If the velocity of the wind is given in feet per second, this becomes

$$p = \frac{V_1{}^2}{538}.$$

Let the wind area of a swing span with equal arms, l, be assumed to be a rectangle of length $2l$ and height h; its area, $2hl$, being equal to the combined area of two trusses and one floor system as seen in elevation. Let r = the radius of the pitch circle of the rack and v = the

maximum velocity of the pinion in feet per second at the pitch circle of the rack.

Consider one arm of the span.

The maximum velocity at radius $1 = \frac{v}{r}$.

The maximum velocity at radius $x = \frac{v}{r}x$.

The maximum pressure at radius $x = \frac{v^2}{538r^2}x^2$.

Let a differential element of the wind area $2hl$ be hdx at a radius x. The pressure on this element is

$$dp_1 = \frac{v^2}{538r^2}hx^2dx.$$

The elemental pressure reduced to radius r becomes

$$dp_2 = \frac{v^2}{538r^2}hx^3dx.$$

The total wind pressure on one arm at radius r, is

$$p_2 = \frac{v^2h}{538r^2}\int_0^l x^3dx,$$

or

$$p_2 = \frac{v^2h}{2152r^2}\Big[x^4\Big]_0^l.$$

The total wind pressure on one arm at radius r becomes,

$$p_2 = \frac{v^2hl^4}{2152r^2},$$

and for both arms, or the entire span,

$$p = \frac{v^2hl^4}{1076r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The work to overcome the resistance p, during one second when velocity v is a maximum, is

$$W = \frac{v^3hl^4}{1076r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

One h.p. = 550 ft.-lbs. per second, therefore,

$$\text{h.p.} = \frac{v^3hl^4}{591800r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

5. Resistance to turning a swing span, with equal arms, against a wind of known velocity.—A swing span, with equal arms, is opened against a wind of known velocity:

Find the resistance at the rack, and the horse-power required to overcome this resistance, when the velocity of rotation is a maximum.

Use the notations given in Art. 4 and in addition the following:

Let V_1 = the velocity of the wind in feet per second, assumed normal to the bridge at the instant of maximum velocity of rotation.

The wind velocity at the center of the span = V_1.

The combined velocity at the rack circle = $V_1 + v$ for one arm and $V_1 - v$ for the other.

The combined velocity at a radius of unity = $V_1 + \frac{v}{r}$ for one arm and $V_1 - \frac{v}{r}$ for the other.

The combined velocity at a radius of $x = V_1 + \frac{v}{r}x$ for one arm and $V_1 - \frac{v}{r}x$ for the other.

The unit pressure at radius x,

$$p = \frac{\left(V_1 + \frac{vx}{r}\right)^2}{538} \text{ for one arm and } \frac{\left(V_1 - \frac{vx}{r}\right)^2}{538} \text{ for the other.}$$

The pressure on an element, hdx, at a radius of x, is

$$dp_1 = \frac{\left(V_1 + \frac{vx}{r}\right)^2}{538}hdx \text{ for one arm and } \frac{\left(V_1 - \frac{vx}{r}\right)^2}{538}hdx \text{ for the other.}$$

The pressure on an element, hdx, reduced to the rack circle, becomes

$$dp_2 = \frac{\left(V_1 + \frac{vx}{r}\right)^2}{538r}hxdx \text{ for one arm and } \frac{\left(V_1 - \frac{vx}{r}\right)^2}{538r}hxdx \text{ for the other.}$$

The total rack pressure is

$$p = \int_0^l \left[\frac{\left(V_1 + \frac{vx}{r}\right)^2}{538r}hxdx - \frac{\left(V_1 - \frac{vx}{r}\right)^2}{538r}hxdx\right]. \quad . \quad (26)$$

$$p = \int_0^l \frac{hxdx}{538r}\left[\left(V_1 + \frac{vx}{r}\right)^2 - \left(V_1 - \frac{vx}{r}\right)^2\right] = \int_0^l \frac{hxdx}{538r} \times \frac{4V_1vx}{r},$$

or

$$p = \int_0^l \frac{V_1hv}{134.5r^2}x^2dx = \frac{V_1hv}{403.5r^2}\Big|_0^l [x^3].$$

The total pressure at the rack circle thus becomes

$$p = \frac{V_1 hvl^3}{403.5r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

If V is given in miles per hour in place of feet per second, the value of p becomes

$$p = \frac{Vhvl^3}{275.1r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The work to overcome this resistance, p, during one second, when velocity v is a maximum, is

$$\text{Work} = \frac{Vhv^2l^3}{275.1r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

One h.p. is 550 ft.-lbs. per second, therefore,

$$\text{h.p.} = \frac{Vhv^2l^3}{151{,}305r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

6. Resistance to turning a swing span, with unequal arms, against a wind of known velocity.— A swing span, with unequal arms, is opened against a wind of known velocity:

Find the resistance at the rack, and the horse-power required to overcome this resistance, when the velocity of rotation is a maximum.

Use the notations given in Arts. 4 and 5 for swing spans with equal arms, and the following:

Let l_1 = the length of the long arm and l_2 = that of the short arm;
h_1 = the height of equivalent wind area l_1h_1 of the long arm, and
h_2 = the height of equivalent wind area l_2h_2 of the short arm.

Assume the long arm to turn towards the wind.

By analogy we may write from Eq. (26),

$$p = \int_0^{l_1} \frac{\left(V_1 + \frac{vx}{r}\right)^2}{538r} h_1 x dx - \int_0^{l_2} \frac{\left(V_1 - \frac{vx}{r}\right)^2}{538r} h_2 x dx. \quad . \quad (31)$$

$$p = \frac{h_1}{538r}\int_0^{l_1}\left(V_1 + \frac{vx}{r}\right)^2 x dx - \frac{h_2}{538r}\int_0^{l_2}\left(V_1 - \frac{vx}{r}\right)^2 x dx.$$

Integrating,

$$p = \frac{h_1}{538r}\left[\frac{V_1^2 l_1^2}{2} + \frac{2V_1 v l_1^3}{3r} + \frac{v^2 l_1^4}{4r^2}\right] - \frac{h_2}{538r}\left[\frac{V_1^2 l_2^2}{2} - \frac{2V_1 v l_2^3}{3r} + \frac{v^2 l_2^4}{4r^2}\right].$$

The total pressure at the rack circle thus becomes,

$$p = \frac{V_1^2}{1076r}[h_1 l_1^2 - h_2 l_2^2] + \frac{V_1 v}{807r^2}[h_1 l_1^3 + h_2 l_2^3] + \frac{v^2}{2152r^3}[h_1 l_1^4 - h_2 l_2^4]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

If V is given in miles per hour in place of feet per second,

$$p = \frac{V^2}{500r}[h_1 l_1^2 - h_2 l_2^2] + \frac{Vv}{550r^2}[h_1 l_1^3 + h_2 l_2^3]$$

$$+ \frac{v^2}{2152r^3}[h_1 l_1^4 - h_2 l_2^4]. \quad \ldots \quad (33)$$

The work of p during one second when v is a maximum is,

$$\text{Work} = \frac{V^2 v}{500r}[h_1 l_1^2 - h_2 l_2^2] + \frac{Vv^2}{550r^2}[h_1 l_1^3 + h_2 l_2^3]$$

$$+ \frac{v^3}{2152r^3}[h_1 l_1^4 - h_2 l_2^4]. \quad \ldots \quad (34)$$

One h.p. is 550 ft.-lbs. per second, therefore,

$$\text{h.p.} = \frac{V^2 v}{275{,}000r}[h_1 l_1^2 - h_2 l_2^2] + \frac{Vv^2}{302{,}500r^2}[h_1 l_1^3 + h_2 l_2^3]$$

$$+ \frac{v^3}{1{,}183{,}600r^3}[h_1 l_1^4 - h_2 l_2^4]. \quad \ldots \quad (35)$$

Note that horse-power from Eq. (35) is positive when the long arm, l_1, is turned against the wind. When the short arm, l_2, is turned against the wind, the wind on the long arm helps to turn the bridge and the horse-power becomes negative. When the maximum velocity, v, is reached the horse-power of Eq. (35) is required to hold the span against the wind.

The frictional resistance of the span increases the positive and reduces the negative horse-power.

7. Relations of velocity, time, and space.—A swing span with equal arms is opened in a wind of known velocity:

Find the relations of velocity, time, and space at the rack circle.

Assume that the direction of the wind remains normal to the bridge during the period of acceleration.

Let X have the same value as in Art. 2 or 3;

$(1 - f)X = X_2$ for convenient notation;

R_1 = the constant resistance at the rack circle due to the friction of the center, or the turntable of the bridge, reduced to the rack circle;

R_2 = the constant resistance, at the rack circle, due to the rolling and journal friction of the balance wheels of a center-bearing draw, caused by the overturning effort of the wind pressure;

$R_1 + R_2 = R$ for a center bearing swing span;

$R_1 = R$ for a rim bearing swing span.

In Art. 5 it has been demonstrated that the total rack force, or resistance to turning, due to wind pressure is, Eq. (28),

$$p = \frac{Vhvl^3}{275.1r^2}.$$

For the present investigation, V is assumed constant so that p will vary as v, the velocity at the rack circle.

Let K be the constant coefficient of the variable v, or

$$K = \frac{Vhl^3}{275.1r^2}. \quad \ldots\ldots\ldots \quad (36)$$

From Mechanics, $\alpha = \frac{dv}{dt}$ and $ds = vdt$; in which α = acceleration, v = velocity in feet per second, t = time in seconds, and s = space in feet.

Let M = the mass-equivalent of the rotating span concentrated at the rack circle, then,

$$\alpha = \frac{dv}{dt} = \frac{X_2 - R - Kv}{M},$$

or

$$dt = \frac{Mdv}{X_2 - R - Kv},$$

or

$$t = \int \frac{Mdv}{X_2 - R - Kv},$$

or

$$t = -\frac{M}{K}\log_e (X_2 - R - Kv) + C. \quad \ldots\ldots \quad (37)$$

When $v = 0$, $t = 0$.

$$\therefore C = \frac{M}{K}\log_e (X_2 - R).$$

Substituting in (37).

$$t = \frac{M}{K}[\log_e (X_2 - R) - \log_e (X_2 - R - Kv)],$$

or

$$t = \frac{M}{K}\log_e \frac{X_2 - R}{X_2 - R - Kv}. \quad \ldots\ldots\ldots \quad (38)$$

The value of t can be found from (38) for any value of v, all other quantities being constant.

Note that $\log_e = \log_{10} \times 2.302585 +$.

Eq. (38) may also be written,

$$e^{\frac{Kt}{M}} = \frac{X_2 - R}{X_2 - R - Kv}. \quad \ldots\ldots\ldots \quad (39)$$

In this, e is the base of the Napierian system of logarithms = 2.718282.

From (39),

$$Kv = X_2 - R - \frac{X_2 - R}{e^{\frac{Kt}{M}}} \quad \text{and} \quad v = \frac{X_2 - R}{K}\left[1 - e^{-\frac{Kt}{M}}\right]. \quad (40)$$

Now

$$ds = vdt = \frac{X_2 - R}{K}\left[1 - e^{-\frac{Kt}{M}}\right]dt$$

and

$$s = \int \frac{X_2 - R}{K}\left[1 - e^{-\frac{Kt}{M}}\right]dt = \frac{X_2 - R}{K}\left[t + \frac{M}{K}e^{-\frac{Kt}{M}}\right] + C.$$

When $t = 0$, $s = 0$

$$\therefore C = -\frac{X_2 - R}{K} \times \frac{M}{K}$$

and

$$s = \frac{X_2 - R}{K}\left[t + \frac{M}{K}e^{-\frac{Kt}{M}}\right] - \frac{X_2 - R}{K} \times \frac{M}{K}.$$

Substituting values of t and $e^{-\frac{Kt}{M}}$ from Eqs. (38) and (39), above,

$$s = \frac{X_2 - R}{K}\left[\frac{M}{K}\log_e \frac{X_2 - R}{X_2 - R - Kv} + \frac{M}{K} \times \frac{X_2 - R - Kv}{X_2 - R} - \frac{M}{K}\right],$$

or

$$s = \frac{M}{K}\left[\frac{X_2 - R}{K}\log_e \frac{X_2 - R}{X_2 - R - Kv} - v\right]. \quad \ldots\ldots\ldots \quad (41)$$

The value of s can be found from Eq. (41) for any value of v; all other quantities being constant for any given investigation.

Note.—Whenever $X_2 \gtreqless R + Kv$ the accelerating force will no longer keep the span in motion, and either the horse-power or the gear ratio and time for opening must be increased.

8. Application of Arts. 2, 5, and 7 to an actual swing span.—The significance of the formulae developed in Arts. 2, 5, and 7 is understood most readily by applying them to an actual swing-span design. The general investigation of the design of the power plant may be reduced to the following definite problems:

(1) Determine the source and kind of power that is most expedient.

(2) Determine the time available for opening or closing the bridge.

A_2. Find what number of horse-power must be provided to rotate the span 90° in the time available.

B_2. After the number of horse-power has been found, determine what equivalent force, X, applied at a radius equal to the pitch radius of the rack, will rotate the span 90° in a minimum time; also find the maximum rack force, or tooth load.

C_2. When the force X is known, find the total time of rotation and the times, t_1, t_2, and t_3, of acceleration, uniform motion, and retardation.

D_2. The number of revolutions per minute of the engine becomes known as soon as a selection is made to give the required horse-power. The time of turning also being known, a gear ratio between the engine and the main pinion must be found to fit the conditions.

E_2. Assume that the span is opened in a wind blowing at 50 miles per hour and find the time required to rotate the span through an angle of 90°.

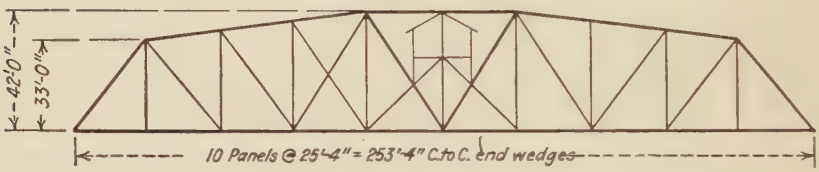


FIG. 2A.

The kind of power is usually determined by local conditions at the bridge site. It may be a steam, gasoline, or oil engine; an electric motor, for direct or alternating current; or man-power, for small bridges, infrequently opened. Whenever current is available and the supply is reliable, electric motors should be used, for they are the most convenient and satisfactory.

Frequently, the time for turning the span is fixed by the requirements of the Government. If the time given is inadequate, or if no time is given, a satisfactory time, not so short as to require extravagant machinery, should be agreed upon with the proper authorities. As the maximum power requirement, for an ordinary swing span, is due to the work of acceleration, and as this varies inversely as the cube of the time, care should be taken that the time for opening be not too brief.

Example.—For a numerical example, an investigation of the power requirements for the Hackensack River swing span, on the Central Railroad of New Jersey, will be made.

Figure 2A shows a line elevation of the trusses. This is a double-

track, through, riveted-truss, center-bearing span, 253 ft. 4 ins. between centers of end wedges. The trusses are 29 ft. 8 ins. between centers. The source of power is steam, generated by either of two vertical boilers. The machinery is driven by one 12- by 12-in. Lidgerwood special drawbridge engine. The boilers, engine, and all auxiliary equipment are located in an operator's house between the trusses and above the railway overhead-clearance line.

The time specified for turning the span 90° was 45 seconds.

The weights from which the mass and moment of inertia were determined were as follows:

Estimated weight of steelwork, including that in the operator's house..	1,575,000 lbs.
Tracks (rails and fastenings, ties, guards and planks)	327,000 lbs.
House, exclusive of steel	24,000 lbs.
Engine, transmission machinery, boilers, tanks, water, and coal	64,000 lbs.
Total weight of the span	1,990,000 lbs.
Center cone, rack segments, wedge seats, latch catches, grillages, and parts that do not turn with the span	109,000 lbs.
Total weight on the center discs	1,881,000 lbs.
Weight of material near the center, including house and all equipment in it and machinery concentrated near the center, which may be neglected when calculating the mass equivalent (usually about 10 per cent)	181,000 lbs.
Weight used for mass equivalent	1,700,000 lbs.

This may be assumed to be spread uniformly over a rectangle of a length equal to the distance between centers of the end wedges and a width equal to the distance between centers of the trusses.

Problem A_2. Horse-power required.

Assumptions:

Disc pressure = 3000 lbs. per square inch.

f_1 = turning friction in per cent of the turning weight, and assumed to act at $\frac{2}{3}$ of the disc radius from the center of the disc;

f = machinery friction in per cent of the engine force.

Data:

W_{T_1} = weight used for mass equivalent expressed in net tons = 850;

W_T = total turning weight expressed in net tons = 940;

L = length c. to c. end wedges expressed in feet = 253.3;

B = width c. to c. trusses expressed in feet = 29.7;

t = specified time for turning 90° expressed in seconds = 45;

P = Necessary number of horse-power to turn the span 90° in 45 seconds;

f_1 = 15 per cent;
f = 25 per cent.

The minimum horse-power to open the span 90° in time, t, is given by Art. 2, Eq. (8), as

$$P = \frac{W_{T_1}(L^2 + B^2)}{6t^3} + \frac{\sqrt{W_T{}^3}}{36.5t}. \quad . \quad . \quad . \quad . \quad (42)$$

Solving Eq. (42) with the above data,

$$P = 100.9 + 17.5 = 118.4.$$

The engine adopted for this bridge will deliver 125 h.p., but for this theoretical analysis 119 h.p. will be used.

Problem B_2. Force X.

Notation:

P = number of horse-power = 119;
W_1 = weight used in finding mass equivalent in pounds = 1,700,000;
W = total weight on center expressed in pounds = 1,881,000;
L and B denote the same dimensions as in Problem A_2;
r = radius of the pitch circle of the rack = 18.675 ft.;
M = mass equivalent concentrated at the rack,

$$= \frac{W_1}{gr^2}\left(\frac{L^2 + B^2}{12}\right) = \frac{1{,}700{,}000}{32.2 \times (18.675)^2}\left(\frac{\overline{253.3}^2 + \overline{29.7}^2}{12}\right)$$

$$= 820{,}523;$$

C = $\frac{1}{4}$ the rack circle = $\dfrac{18.675 \times 2 \times \pi}{4}$ = 29.335 ft.;

d = diameter of disc = 30 ins., $\frac{2}{3}$ of disc radius = $\frac{1}{3}d$ = 10 ins.
f_1 and f denote the same friction coefficients and have the same numerical values as in Problem A_2.
R_1 = frictional resistance at the rack due to the center disc,

$$= Wf_1\frac{\frac{1}{3}d}{r \times 12} = 1{,}881{,}000 \times 0.15 \times \frac{10}{18.675 \times 12} = 12{,}590 \text{ lbs.};$$

X = a tangential force acting at the radius r, delivered by the engine, with no deduction for friction in the mechanism between the engine and the rack.

The value of X is found from Eq. (7), of Art. 2, which is

$$\frac{(1 - f^2)X^2 - R_1{}^2 - 2fR_1X}{\sqrt{(1 - f^2)X - fR_1}} = 550P\sqrt{\frac{2M}{C}}. \quad . \quad . \quad . \quad (43)$$

This equation is most readily solved by trial. Evaluating the second side,

$$550P\sqrt{\frac{2M}{C}} = 550 \times 119\sqrt{\frac{2 \times 820{,}523}{29.335}} = 15{,}478{,}500.$$

If f and R_1 are neglected, an approximate X may be found from the following equation:

$$X = 10\sqrt[3]{\frac{605P^2M}{C}} = 62{,}110.$$

For the assumed friction values this approximate X is usually about 10 per cent. too small. Therefore, try $X = 62{,}110 \times 1.10 = 68{,}321$.

Evaluating first side of Eq. (43), using $X = 68{,}321$,

$$\frac{0.9375 \times \overline{68{,}321}^2 - \overline{12{,}590}^2 - 2 \times 0.25 \times 12{,}590 \times 68{,}321}{\sqrt{0.9375 \times 68{,}321 - 0.25 \times 12{,}590}}$$
$$= 15{,}347{,}500,$$

which does not quite agree with the value of the second side of the equation, and indicates that X should be a little larger.

Evaluating first side of the equation with $X = 68{,}700$, gives 15,490,000, which is sufficiently close to the value of the second side. Therefore $X = 68{,}700$.

This bridge has two main pinions. The maximum stress on one pinion, assuming no overload on the engine, occurs during the period of retardation, and is,

$$\text{pinion stress} = \frac{(1+f)X}{2} = \frac{1.25 \times 68{,}700}{2} = 42{,}937 \text{ lbs.}$$

Problem C_2. Times t, t_1, t_2, and t_3.

Art. 2, Eqs. (6), (4), (1), and (3) give the following expressions for t, t_1, t_3, and t_2, respectively:

$$t = \frac{CX}{550P} + \frac{550PM}{(1-f^2)X^2 - 2fR_1X - R_1^2}. \quad . \quad . \quad . \quad (44)$$

$$t_1 = \frac{550MP}{X[(1-f)X - R_1]}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

$$t_3 = \frac{(1-f)X - R_1}{(1+f)X + R_1}t_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$$t_2 = \frac{MC}{[(1-f)X - R_1]t_1} - \frac{t_1 + t_3}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Solving (44) gives $t = 44.80$ seconds, against 45 seconds agreed upon with the owner.
Solving (45) gives $t_1 = 20.08$ seconds.
Solving (46) gives $t_3 = 7.90$ seconds.
Solving (47) gives $t_2 = 16.78$ seconds.
$t = t_1 + t_2 + t_3 = 44.80$ seconds.

Problem D_2. Gear ratio.
From Art. 2, Eqs. (1) and (4),

$$v = \frac{550P}{X}.$$

Substituting,

$$v = \frac{550 \times 119}{68{,}700} = 0.9528 \text{ feet per second,}$$

which is the velocity of the main pinion around the rack circle at the end of the time t_1, during time t_2, and at the beginning of time t_3.

Let s_1, s_2 and s_3 be the distances along the rack circle over which the main pinion passes during the times t_1, t_2 and t_3, respectively.

During time t_1 the main pinion travels

$$\frac{0.9528}{2} \times 20.08 = 9.56 \text{ ft.} = s_1.$$

During time t_2 the main pinion travels

$$0.9528 \times 16.78 = 15.99 \text{ ft.} = s_2.$$

During time t_3 the main pinion travels

$$\frac{0.9528}{2} \times 7.94 = 3.78 \text{ ft.} = s_3.$$

$s_1 + s_2 + s_3 = C = 29.33$ ft.

The main pinion has a pitch diameter of 21 ins.; hence when it moves at a velocity of 0.9528 ft. per second around the rack circle, it turns at

$$\frac{0.9528 \times 60 \times 12}{21 \times \pi} = 10.4 \text{ revolutions per minute.}$$

The engine shaft turns at 150 revolutions per minute; therefore, the gear ratio between the engine shaft and the main pinion shaft should be $\frac{150}{10.4} = 14.42$.

Problem E_2. Turning in a wind.—Figure 2B is a diagram illustrating the conditions at the center of the span, showing the general dimensions and the amount, direction, and assumed position of the wind pressure.

Let V = wind velocity in miles per hour = 50. The resulting pressure is therefore $0.004V^2 = 10$ lbs. per square foot.

R_1, X, r and v denote the same quantities as in Problems B_2 and D_2.

R_2 = the constant resistance at the rack due to the rolling and journal friction of the balance wheels.

The coefficient of rolling friction = $\frac{0.004}{\sqrt{r_2}}$ where r_2 = radius of balance wheels = 18 ins. (One surface of the roller has contact with the track.)

Radius of balance-wheel journals = 2.625 ins.

R = the total resistance at the rack due to friction of the center and the balance wheels; then, $R = R_1 + R_2$.

X_2 = the net effect of the engine force, X, at the rack after deducting the friction of the intermediate mechanism; then $X_2 = X(1 - f) = 68{,}700 \times 0.75 = 51{,}525$ lbs.

t_4 = the time in seconds consumed, and s_4 = the distance in feet that the main pinion traverses, during the period of acceleration.

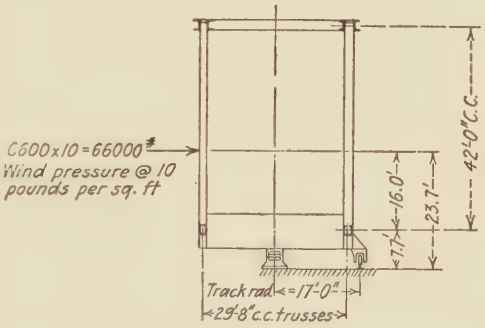


FIG. 2B.

Also, let the wind area be taken as the combined area of two trusses and one floor system as seen in elevation = 6600 sq. ft.

The center of wind pressure causing overturning is usually about $\frac{3}{8}$ of the distance from the center of the bottom chord to the center of the top chord at the middle of the span, which in this bridge is $\frac{3}{8}$ of 42 = about 16 ft.

$$R_2 = 66{,}000 \times \frac{23.7}{17.0} \times \frac{0.004}{\sqrt{18}} \times \frac{17}{18.675} + 66{,}000 \times \frac{23.7}{17} \times 0.06 \times 2.625 \times \frac{1}{18} \times \frac{17}{18.675}$$

$$= 66{,}000 \times \frac{23.7}{18.675}\left(\frac{0.004}{\sqrt{18}} + \frac{0.06 \times 2.625}{18}\right) = 644.$$

$$R = R_1 + R_2 = 12{,}590 + 644 = 13{,}234 \text{ lbs.}$$

The velocity of the main pinion around the rack circle at the end of acceleration may be written,

$$v = \frac{(1-f)X - R}{M} t_1 = \frac{0.75 \times 68{,}700 - 13{,}234}{820{,}523} \times 20.08$$

$$= 0.9368 \text{ ft. per second.}$$

Article 7, Eqs. (38) and (41) give expressions for the values of t_4 and s_4 as follows:

$$t_4 = \frac{M}{K} \log_e \frac{X_2 - R}{X_2 - R - Kv} \quad \ldots \ldots \ldots \quad (48)$$

$$s_4 = \frac{M}{K}\left[\frac{X_2 - R}{K} \log_e \frac{X_2 - R}{X_2 - R - Kv} - v\right]. \quad \ldots \quad (49)$$

Assuming the wind area, 6600 sq. ft., to be a rectangle of a length $2l$ = the distance c. to c. of the end wedges, 253.3 ft., and of a height h, we find

$$h = \frac{6600}{253.3} = 26 \text{ ft.}$$

The value of K is,

$$K = \frac{Vhl^3}{275.1 \times r^2} = \frac{50 \times 26 \times \overline{126.65}^3}{275.1 \times \overline{18.675}^2} = 27{,}530.$$

$$\frac{X_2 - R}{X_2 - R - Kv} = \frac{51{,}525 - 13{,}234}{51{,}525 - 13{,}234 - 27{,}530 \times 0.9368} = 3.063.$$

$\text{Log}_e\, 3.063 = (\log_{10} 3.063) \times 2.3026 = 0.48615 \times 2.3026 = 1.1193.$
Substitute proper values in Eq. (48).

$$t_4 = \frac{820{,}523}{27{,}530} \times 1.1193 = 33.36 \text{ seconds} = \text{the time of acceleration.}$$

Substitute proper values in Eq. (49).

$$s_4 = \frac{820{,}523}{27{,}530}\left[\frac{51{,}525 - 13{,}234}{27{,}530} \times 1.1193 - 0.9368\right] = 18.485 \text{ ft.}$$

The distance s_4 is the space over which the main pinion travels during the period of acceleration. Let s_5 = the space covered during uniform motion and let s_6 = that during retardation.

s_6 is the same distance as s_3 of Problem D_2; then

$$s_5 = C - (s_4 + s_6) = 29.335 - (18.485 + 3.78) = 7.07 \text{ ft.}$$

$$t_5 = 7.07 \div 0.9368 = 7.55 \text{ seconds.}$$

The time for turning the span in a 50 mile wind is therefore,

t_4 = time of acceleration	= 33.36 seconds.
t_5 = time of uniform motion	= 7.55 seconds.
t_6 = time of retardation	= 7.94 seconds.
t = total time	= 48.85 seconds.
The total time with no wind	= 44.80 seconds.
Increased time due to 50 mile wind or 9.04 per cent. of 44.8 seconds.	= 4.05 seconds,

Any increase up to 50 per cent. would not be excessive; therefore, no increase in horse-power is needed to operate the span in a 50-mile wind.

9. Application of Arts. 3, 5, and 7 to an actual swing draw span.— The order of procedure is the same as that given in detail at the beginning of Art. 8. It should, however, be noted that the example in Art. 8 follows the analysis in Art. 2, and is adapted to the use of a steam engine; while the one in this Article follows Art. 3, and is suitable for the same bridge, if it were operated by an electric motor in the same time.

Problem A_3. Horse-power required.

Let all data be the same as given for Problem A_2.

The required horse-power may be found from Eq. (21).

$$P_1 = \frac{9}{38} \times \frac{W_{T_1}(L^2 + B^2)}{t^3} + \frac{1}{47} \times \frac{\sqrt{W_T{}^3}}{t}. \quad . \quad . \quad . \quad (50)$$

Solving with data from Problem A_2, we find,

$$P_1 = 143.70 + 13.63 = 157.33.$$

Use $P_1 = 157$ in this analysis.

Problem B_3. Rack force X_1.

Let all symbols and data be as given in Art. 3 and Problem B_2.

The value of X_1 is found from Eq. (19) of Art. 3.

$$\frac{(1-f)X_1{}^2 - R_1X_1}{\sqrt{2(1-f)X_1 - R_1}} = 550P_1\sqrt{\frac{M}{C}}. \quad . \quad . \quad . \quad . \quad (51)$$

Evaluate the second side of Eq. (51).

$$550P_1\sqrt{\frac{M}{C}} = 550 \times 157\sqrt{\frac{820{,}523}{29.335}} = 14{,}440{,}000.$$

$$\text{Approximate } X_1 = 10\sqrt[3]{\frac{605P_1{}^2M}{C}} = 10\sqrt[3]{\frac{605 \times 157^2 \times 820{,}523}{29.335}}$$

$$= 74{,}716.$$

For the first trial use $74{,}716 \times 1.20 = 89{,}659$, say 89,700.

Evaluate the first side of Eq. (51).

$$\frac{0.75 \times 89{,}700^2 - 12{,}590 \times 89{,}700}{\sqrt{1.5 \times 89{,}700 - 12{,}590}} = 14{,}047{,}500.$$

This is too small; therefore increase X_1 for a second trial to 91,200 and evaluate the first side of Eq. (51). It equals 14,442,500.

This differs from the second side of Eq. (51) by 2500 and is of sufficient accuracy; therefore,

$$X_1 = 91{,}200 \text{ lbs.}$$

The maximum pinion stress is

$$\frac{(1-f)X_1}{2} = \frac{0.75 \times 91{,}200}{2} = 34{,}200 \text{ lbs.}$$

Problem C_3. Times t, t_1, t_2, and t_3.

From Art. 3, Eqs. (18), (15), (16), and (11a) take the following expressions for the values of t, t_1, t_2 and t_3, respectively:

$$t = \frac{CX_1}{550P_1} + \frac{550P_1M}{(1-f)X_1^2 - R_1X_1}. \quad . \quad . \quad . \quad . \quad (52)$$

$$t_1 = \frac{550P_1M}{(1-f)X_1^2 - R_1X_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

$$t_2 = \frac{CX_1}{550P_1} - \frac{550P_1M}{(1-f)X_1^2 - R_1X_1}. \quad . \quad . \quad . \quad . \quad (54)$$

$$t_3 = t_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Eq. (52) gives the value of $t = 44.90$ seconds against 45 seconds agreed upon with the owner.

Eq. (53) gives the value of $t_1 = 13.92$ seconds.

Eq. (54) gives the value of $t_2 = 17.06$ seconds.

Eq. (55) gives the value of $t_3 = 13.92$ seconds.

$t = t_1 + t_2 + t_3 = 44.90$ seconds.

Problem D_3. Gear ratio.

From Art. 3, Eqs. (11) and (15),

$$v = \frac{550P_1}{X_1}.$$

Substituting,

$$v = \frac{550 \times 157}{91{,}200} = 0.9468 \text{ ft. per second,}$$

which is the velocity of the main pinion around the rack circle at the end of the time t_1, during the time t_2, and at the beginning of time t_3.

Let s_1, s_2, and s_3 be the distances along the rack circle over which the main pinion passes during the times t_1, t_2, and t_3, respectively.

During time t_1 the main pinion travels

$$\frac{0.9468}{2} \times 13.92 = 6.59 \text{ ft.} = s_1.$$

During time t_2 the main pinion travels

$$0.9468 \times 17.06 = 16.15 \text{ ft.} = s_2.$$

During time t_3 the main pinion travels

$$\frac{0.9468}{2} \times 13.92 = 6.59 \text{ ft.} = s_3.$$

$s_1 + s_2 + s_3 = C = 29.33$ ft.

The main pinion has a pitch diameter of 21 ins.; hence, when the main pinion moves at a velocity of 0.9468 ft. per second around the rack circle, it turns at,

$$\frac{0.9468 \times 60 \times 12}{21 \times \pi} = 10.335 \text{ r. p. m.}$$

The engine shaft turns at 150 r. p. m.; therefore the gear ratio between the engine shaft and the main pinion shaft should be

$$\frac{150}{10.335} = 14.51.$$

Problem E_3. Turning in a wind.

Let all data be the same as for Problem E_2, except that $X_3 = 68{,}400$.

The velocity of the main pinion around the rack circle at the end of acceleration may be written,

$$v = \frac{(1-f)X_1 - R}{M} t_1 = \frac{0.75 \times 91{,}200 - 13{,}234}{820{,}523}$$

$$\times 13.92 = 0.9358 \text{ ft. per second.}$$

Article 7, Eqs. (38) and (41) give expressions for the values of t_7 and s_7:

$$t_7 = \frac{M}{K} \log_e \frac{X_3 - R}{X_3 - R - Kv}. \quad \ldots\ldots\ldots \quad (56)$$

$$s_7 = \frac{M}{K}\left[\frac{X_3 - R}{K} \log_e \frac{X_3 - R}{X_3 - R - Kv} - v\right]. \quad \ldots \quad (57)$$

The value of K from Problem E_2 is 27,530.

$$\frac{X_3 - R}{X_3 - R - Kv} = \frac{68{,}400 - 13{,}234}{68{,}400 - 13{,}234 - 27{,}530 \times 0.9358} = 1.876.$$

$\text{Log}_e\ 1.876 = (\log_{10}\ 1.876) \times 2.3026 = 0.27323 \times 2.3026 = 0.6291.$

Substitute proper values in Eq. (56).

$$t_7 = \frac{820{,}523}{27{,}530} \times 0.6291 = 18.75 \text{ seconds} = \text{the time of acceleration.}$$

Substitute proper values in Eq. (57).

$$s_7 = \frac{820{,}523}{27{,}530}\left[\frac{68{,}400 - 13{,}234}{27{,}530} \times 0.6291 - 0.9358\right] = 9.678 \text{ ft.}$$

s_7 is the distance the main pinion travels during acceleration. Let s_8 = the space covered during uniform motion and s_9 = that during retardation.

s_9 is the same distance as s_3 of Problem D_3, then

$s_8 = C - (s_7 + s_9) = 29.335 - (9.678 + 6.59) = 13.067$ ft.

$t_8 = 13.067 \div 0.9358 = 13.96$ seconds.

The times for turning the span in a 50 mile wind are therefore,

t_7 = time of acceleration	= 18.75 seconds.
t_8 = time of uniform motion	= 13.96 seconds.
t_9 = time of retardation	= 13.92 seconds.
t = total time	= 46.63 seconds.
The total time with no wind	= 44.90 seconds.
Increased time due to a 50-mile wind	= 1.73 seconds,
or 3.85 per cent. of 44.9 seconds.	

Any increase up to 50 per cent. would not be excessive; therefore, no increase in horse-power is needed to operate the span in a 50-mile wind.

Note from Art. 7 that whenever $X_2 \gtrless R + Kv$, the accelerating force will no longer keep the span in motion and either the horse-power, or the gear ratio and time for opening, must be increased.

10. Summation of the results from Arts. 8 and 9.—A summation of the results from the calculations in Arts. 8 and 9 is given in Table 2A. Columns 2 and 3 compare the results from the theories of Arts. 2 and 3 for a time of 45 seconds, while Columns 4 and 5 give the same information for a time of 90 seconds.

In each case, the theory developed in Art. 3 requires more power than that given in Art. 2, while the rack force, or main-pinion tooth load, is materially reduced. The method of Art. 2 is the proper one to use for a reversing steam engine. That of Art. 3 is suitable for a non-reversing engine or an electric motor with a brake. For the sake of the comparison, the same speed of the motor was assumed for each analysis. In an actual design under Art. 3, for an electric motor, the speed of the motor would be more than 150 r.p.m., and would require

a corresponding increase in the gear ratio to turn the bridge in the same time.

TABLE 2A

Time for Turning 90°	45 Seconds		90 Seconds	
Theory Used in Calculations (1)	Art. 2 (2)	Art. 3. (3)	Art. 2. (4)	Art. 3. (5)
Assumed r.p.m. of engine	150	150	250	250
Horse-power required	119	157	23	30
Force X or X_1	X,68,700 lbs.	X_1,91,200 lbs.	X,29,400 lbs.	X_1,37,350 lbs.
Rack-tooth load, all pinions	85,875 lbs.	68,400 lbs.	36,750 lbs.	28,013 lbs.
Acceleration, t_1	20.08 sec.	13.92 sec.	37.30 sec.	23.50 sec.
Uniform motion, t_2	16.78 sec.	17.06 sec.	45.90 sec.	42.91 sec.
Retardation, t_3	7.94 sec.	13.92 sec.	7.10 sec.	23.50 sec.
Total time t	44.80 sec.	44.90 sec.	90.30 sec.	89.91 sec.
Gear ratio, main pinion to rack	14.42	14.51	53.25	51.85
Time against 50-mile wind			requires larger horse-power or change in gear ratio to open in a 50-mile wind.	
Acceleration, t_1	33.36 sec.	18.75 sec.		51.85 sec.
Uniform motion, t_2	7.55 sec.	13.96 sec.		21.80 sec.
Retardation, t_3	7.94 sec.	13.92 sec.		23.50 sec.
Total time, t	48.85 sec.	46.63 sec.		96.88 sec.
Increased time due to 50-mile wind	4.05 sec.	1.73 sec.		6.97 sec.
Increase in per cent. of time for no wind	9.04	3.85	$X < R+Kv$	7.78

For any ordinary motor-operated swing span, Art. 3 should be used. The rack-tooth load is 20 per cent less, while the horse-power is 32 per cent more than would be required in accordance with Art. 2. The rack-tooth load fixes the design of the mechanism from the rack to the motor. While the power required is more, it should be remembered that it is used only during actual movements of the span. Fifty openings per day would use current for a total of only 75 minutes. Modern electric motors have a large excess starting and running torque, which would not cause injurious heating in short runs of 45 seconds. The full demand on the motors only occurs during acceleration, of for about one-third of each run. It would, therefore, be proper to select

a motor of smaller normal torque than the one called for by Column 3. Taking advantage of these conditions, the theory given in Art. 3 would result in economy in construction, maintenance, and operation.

The analysis in Columns 4 and 5 brings out the same conditions when the time for turning is 90 seconds. Column 4 also shows the interesting fact that the bridge would not turn in a 50-mile wind unless a larger motor was used or the gear ratio and time for turning were increased. The expression for acceleration, which leads to Eq. (37), is

$$\alpha = \frac{X_2 - R - Kv}{M}.$$

Whenever $R + Kv \gtrless X_2$, this expression becomes zero or negative; hence the span would stop moving or the wind would drive it backward.

11. Minimum time for raising or lowering a vertical-lift span a given distance with a given horse-power.—In a vertical-lift span, the resistance to motion, due to the friction of the counterweight-sheave trunnions in their bearings and to the stiffness of the ropes, which must be bent over the sheaves, is so great that the span is quickly retarded after power is cut off. For ordinary operations, no brake is needed to stop the bridge, but a brake should be provided to guard against accidents. The first point of the controller should control the brake, and the second should be a coasting point, thus giving easy control to the operator.

It will be assumed that the bridge is operated by an electric motor capable of exerting an excess above the normal torque during acceleration, and that no brake is applied during retardation.

Let P = the number of horse-power;

W = the weight of the span plus an equivalent weight of counterweight;

M = the mass of the span and its counterweights = $\frac{W}{g}$;

C = the distance, in feet, that the span is to be lifted;

t_1 = the time of acceleration, in seconds;

t_2 = the time of uniform velocity, in seconds;

t_3 = the time of retardation, in seconds;

t = the total time of lifting or lowering the span, in seconds;

X = a tangential force, acting at the pitch line of the counterweight sheaves, delivered by the motor, with no reduction for friction in the mechanism between the motor and the point of application of X, and no allowance for excess torque in the motor;

n = the assumed excess torque of the motor, expressed as a fractional part of X;

f = the frictional resistance of all mechanism from the motor to the point of application of X, expressed as a fractional part of X;

R = the frictional resistance of the counterweight-sheave trunnions, and the guide rollers, plus the resistance due to the stiffness of the counterweight ropes.

The accelerating force then is, $X(1-f)(1+n)-R$.

The velocity at the end of acceleration is $\frac{X(1-f)(1+n)-R}{M}t_1$.

The velocity at the beginning of retardation, with the current cut off from the motor and with no brakes applied, is $\frac{R}{M}t_3$.

These two velocities are equal; hence,

$$\frac{X(1-f)(1+n)-R}{M}t_1 = \frac{R}{M}t_3. \quad . \quad . \quad . \quad . \quad (58)$$

The average velocity multiplied by the total time equals the distance traveled, or,

$$\frac{R}{M} \times \frac{t_3}{2}(t_1+t_3) + \frac{R}{M}t_3t_2 = C. \quad . \quad . \quad . \quad . \quad . \quad (59)$$

The work performed by the motor in the time t_1 must equal the work of the force X during the same time; hence,

$$\frac{550P}{2}t_1 = X\left[\frac{X(1-f)(1+n)-R}{M}\right]\frac{t_1}{2}t_1. \quad . \quad . \quad . \quad (60)$$

From Eq. (60),

$$t_1 = \frac{550PM}{X[X(1-f)(1+n)-R]}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

By sutstitution in Eq. (58),

$$t_3 = \frac{550PM}{RX}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

Substituting in Eq. (59),

$$t_2 = \frac{CX}{550P} - \frac{550PMX(1-f)(1+n)}{2RX[X(1-f)(1+n)-R]}. \quad . \quad . \quad . \quad (63)$$

$$t = t_1 + t_2 + t_3,$$

or,

$$t = \frac{550PM}{X[X(1-f)(1+n)-R]} + \frac{CX}{550P} - \frac{550PMX(1-f)(1+n)}{2RX[X(1-f)(1+n)-R]} + \frac{550PM}{RX}.$$

This reduces to

$$t = \frac{CX}{550P} + \frac{550PM(1-f)(1+n)}{2R[X(1-f)(1+n) - R]} \quad . \quad . \quad . \quad . \quad (64)$$

To evaluate Eq. (64) for a minimum time, t, place $\frac{dt}{dX} = 0$ and solve for X.

$$\frac{dt}{dX} = \frac{C}{550P} - \frac{550PM(1-f)^2(1+n)^2}{2R[X(1-f)(1+n) - R]^2} = 0;$$

hence,

$$X = \frac{R + 550P(1-f)(1+n)\sqrt{\frac{M}{2RC}}}{(1-f)(1+n)}$$

$$= \frac{R}{(1-f)(1+n)} + 550P\sqrt{\frac{M}{2RC}} \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Substituting this value of X in Eq. (64) and solving for P,

$$P = \frac{CR}{550(1-f)(1+n)\left[t - \sqrt{\frac{2MC}{R}}\right]} \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Equations (65) and (66) can be solved directly from definite data for any given structure.

12. Application of Art. 11 to a vertical-lift span.—The investigation of the power requirements of a vertical-lift span, in accordance with the analysis developed in Art. 11, may be divided into a series of definite problems:

(1) Determine the source and kind of power that is most expedient.

(2) Determine the time available for opening or closing the bridge.

A_{11}. Find what number of horse-power must be provided to lift the span the necessary distance in the available time.

B_{11}. After the horse-power has been found, determine what force, X, applied at the pitch circle of the counterweight sheaves, will perform the work of lifting in the given time.

C_{11}. When the force X is known, find the times, t_1, t_2, and t_3, of acceleration, uniform motion, and retardation.

D_{11}. The normal number of revolutions per minute of the motor becomes known as soon as the motor is selected. The time of lifting also being known, the gear ratio between the counterweight sheaves and the motor may be found to fit the given conditions.

Example.—For a numerical example, an investigation of the power requirements for the design of a double-track, railway, through, riveted, silicon-steel, truss span will be made. The bridge was designed for a Class M46 live load. The tension unit-stress basis was 24,000 lbs. per square inch in net sections. The span is 216 ft. between centers of end bearings and of the form shown in Fig. 2C. The required under clearance was 165 ft., and the vertical movement of the span 158.5 ft.

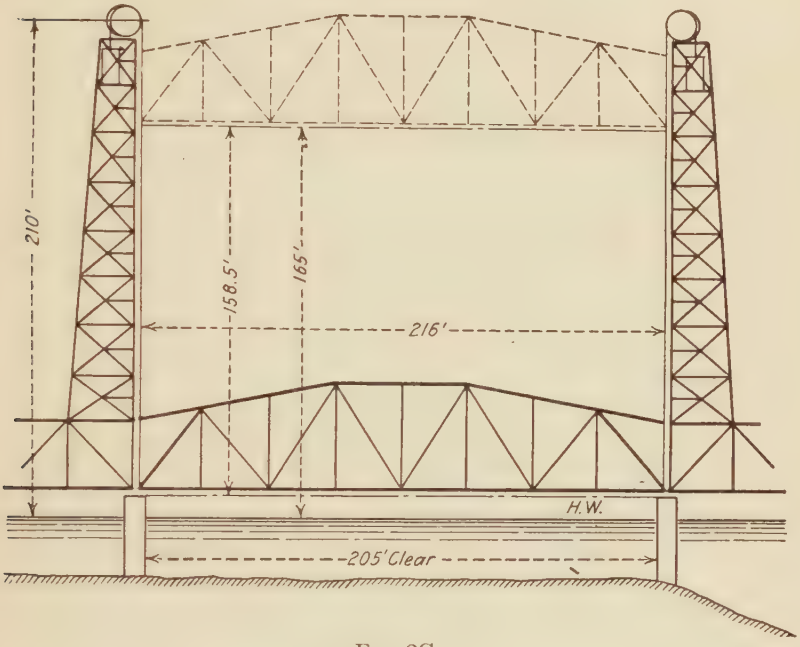


FIG. 2C.

The time for lifting was 90 seconds, including the time for unlocking the ends of the span. The available electric current was 3-phase, 60-cycle, 440-volt alternating current.

The superstructure was proportioned in accordance with the "General Specifications for Steel Railroad Bridges," of the American Railway Engineering Association, dated 1920, except for the use of silicon steel, with appropriate unit stresses. The counterweight ropes, sheaves, trunnions, and mechanism were proportioned in accordance with the author's "Specifications for Movable Bridges," which are given in full in Appendix A.

Data.

The estimated weights are as follows:

Structural steel in the lift span	1,100,000 lbs.
Rails, fittings, ties, guards, and plank footways, 220 ft. at 1200 lbs. per linear foot	264,000 lbs.
Counterweight ropes from the span to the sheaves with the bridge closed	33,000 lbs.
Wire-rope fittings on the lift span	10,000 lbs.
Total	1,407,000 lbs.

Counterweight ropes:

Three sheaves, each carrying four counterweight ropes, are assumed at each front corner of the two towers; hence the load on each rope is 1,407,000 ÷ 48, or ..... 29,312 lbs.

On account of the stresses in the ropes due to bending them over the sheaves, it is desirable to use sheaves of reasonable diameter, moreover, large sheaves afford more clearance for the counterweights within the towers; hence sheaves 12 ft. in pitch diameter will be assumed. Under the requirements of the Specifications, Arts. 413, 721, and 722, the allowable load on Monitor steel wire ropes $1\frac{5}{8}$ ins. in diameter is ..... 33,772 lbs.

Mass to be moved:

Weight to be counterbalanced	1,407,000 lbs.
Counterweight, including steel frame and equalizers.	1,407,000 lbs.
Assume that the counterweight ropes and fittings are also counterbalanced during movement	43,000 lbs.
Total weight moving vertically	2,857,000 lbs.
The mass moving vertically is 2,857,000 ÷ 32.2, or,	88,726 units
The counterweight sheaves must also be accelerated. Their weight is 12 × 12,750 = 153,000 lbs., and their combined mass at the center line of the ropes is	2,594 units
The total mass to be accelerated is	91,320 units

Value of R:

The resistance due to bending the ropes around the sheaves is (Spec., Art. 620), $1{,}407{,}000 \times 0.45 \frac{1.625}{144}$, or ..... 7,145 lbs.

(Taken in this case without reduction of c for large ratio of $\frac{D}{d}$).

The resistance due to turning the trunnions, assumed to be 7 ins. in diam., in their bearings is (Spec., Art. 620), $(2{,}857{,}000 + 153{,}000) \times 0.10 \times \frac{7}{144}$, or ..... 14,630 lbs.

The area exposed to wind normal to the span is about 5500 sq. ft. The pressure per square foot corresponding to a wind velocity of 50 miles per hour is, $P = 0.004V^2$ lbs. = 10 lbs.

The total side pressure is $5500 \times 10 = 55{,}000$ lbs.

Assume guide wheels 18 ins. in diameter with $3\frac{1}{4}$ in. axles. The rolling friction (Spec., Art. 620) is,

$55{,}000 \times \frac{0.004}{\sqrt{9}}$, or ..... 73 lbs.

The axle friction (Spec., Art. 621) is,

$55{,}000 \times 0.06 \times \frac{3.25}{18}$, or ..... 596 lbs.

The total value of R then becomes ..... 22,444 lbs.

Efficiency of mechanism:

Assume the lost work from the motor to the pitch circle of the sheaves to be, $f = 0.20$

It is not feasible to operate a motor, protected by circuit breakers, during acceleration without exceeding the normal torque. Assume the excess torque to be, $n = 0.33$

Time:

The total time for opening the bridge is ..... 90 sec.

The time for unlocking the ends will be assumed as 15 sec.

The time available for lifting the span is ..... 75 sec.

Problem A_{11}. *Horse-power required.*

Summation of data:

M = mass to be moved ..... 91,320 units

C = vertical distance that the bridge is lifted .... 158.5 ft.

R = the frictional resistance at the pitch line of the ropes ..... 22,444 lbs.

f = the frictional losses in the gear trains 0.20

n = the excess motor-torque during acceleration.. 0.33

t = the available time for lifting the span ..... 75 sec.

Insert these values in Eq. (66), and solve for P.

$$P = \frac{158.5 \times 22{,}444}{550(1 - 0.20)(1 + 0.33)\left[75 - \sqrt{\frac{2 \times 91{,}320 \times 158.5}{22{,}444}}\right]} = 155.6 \text{ h.p.}$$

It is assumed that the bridge will be operated by a motor on each tower. The minimum horse-power of each must be 77.8. The General Electric Company build a type MTC motor, in frame No. 5352, that has a normal rating of 80 h.p., and that is suitable for this service. Two such motors will be assumed in subsequent calculations.

Problem B_{11}. *Force* X.

Substitute the data in Eq. (65).

$$X = \frac{22{,}444}{(1-0.20)(1+0.33)} + 550 \times 160\sqrt{\frac{91{,}320}{2 \times 22{,}444 \times 158.5}} = 31{,}063 \text{ lbs.}$$

Problem C_{11}. *Find times* t_1, t_2, *and* t_3.

Substitute in Eq. (61).

$$t_1 = \frac{550 \times 160 \times 91{,}320}{31{,}063[31{,}063(1-0.20)(1+0.33) - 22{,}444]} = 24.39 \text{ sec.}$$

Substitute in Eq. (63).

$$t_2 = \frac{158.5 \times 31{,}063}{550 \times 160} - \frac{550 \times 160 \times 91{,}320 \times 31{,}063(1-0.20)(1+0.33)}{2 \times 22{,}444 \times 31{,}063[31{,}063(1-0.20)(1+0.33) - 22{,}444]} = 38.99 \text{ sec.}$$

Substitute in Eq. (62).

$$t_3 = \frac{550 \times 160 \times 91{,}320}{22{,}444 \times 31{,}063} = 11.53 \text{ sec.}$$

Total time, $t_1 + t_2 + t_3$ $= 74.91$ sec.

Problem D_{11}. *Gear ratio.*

From Eq. (58), the velocity at the pitch circle of the sheaves at the end of acceleration, during uniform motion, and at the beginning of retardation, is $\frac{R}{M}t_3$, or

$$v = \frac{22{,}444}{91{,}320} \times 11.53 = 2.834 \text{ ft. per second.}$$

The counterweight sheaves must rotate at

$$\frac{2.834 \times 60}{12 \times \pi} = 4.511 \text{ r.p.m.}$$

The armature of the 80-h.p. motors selected rotates at 600 r.p.m. synchronous speed, or at 570 r.p.m. under its normal load.

The gear ratio required is $\frac{570}{4.511} = 126.35$.

Ice load on the bridge.

It has been assumed that there is no ice load on the bridge. In northern climates, ice would have to be considered. See Art. 617, *B*2 of the Specifications, which fixes the ice load at $2\frac{1}{2}$ lbs. per square foot on 85 per cent of the area of a quadrilateral, of which the width is the distance between the centers of the trusses, and the length is that of the floor of the moving span.

Area of the span = $32 \times 220 \times 0.85$	=	5,984 sq. ft.
Area of the tops of two counterweights = $28 \times 10 \times 2$	=	560 sq. ft.
Unbalanced ice area	=	5,424 sq. ft.
Unbalanced weight of ice = 5424×2.5	=	13,560 lbs.
Unbalanced weight at each tower	=	6,780 lbs.
The increased torque required from the motor by the ice load is $\frac{6{,}780 \times 12 \times \pi \times 4.511}{2 \times \pi \times 570(1 - 0.20)}$	=	402 lbs.
The normal torque of the motor is		738 lbs.
In the power calculations this was increased by 33 per cent, or		246 lbs.
The required maximum torque then is		1,386 lbs.
The starting torque of the motor is		1,480 lbs.
The maximum running torque is		1,610 lbs.

The motor can run two minutes under a torque of 1480 lbs. without injurious heating, while the time of acceleration is but 24.39 seconds; hence it is concluded that the motor would operate the bridge with the ice load, but at a reduced velocity.

The starting torque is but 6.8 per cent above that theoretically required, which is a narrow margin of safety. In a northern climate, the selection of the next size motor would be wise.

CHAPTER III

GENERAL ARRANGEMENTS AND DETAILS OF MACHINERY

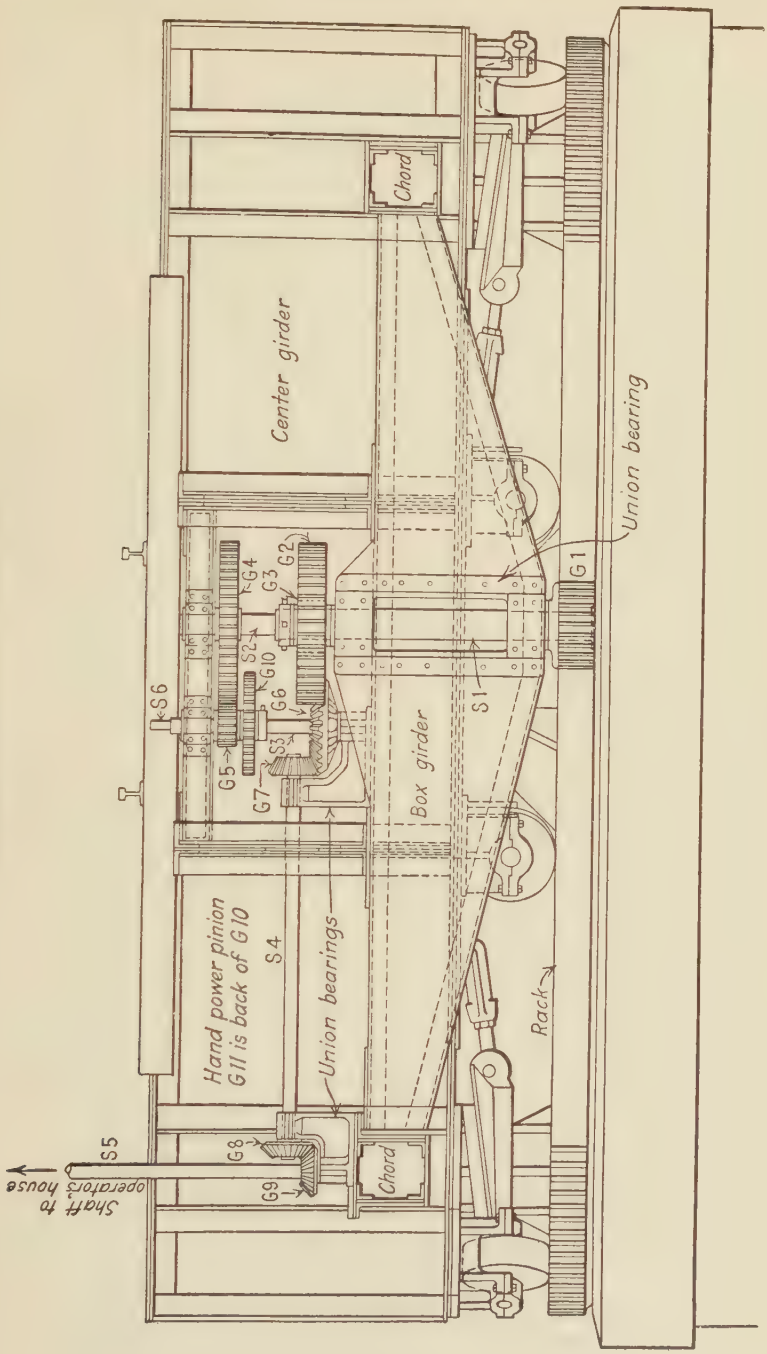
1. General.—Chapter I states some of the principles which apply to the general arrangements of movable-bridge machinery. No set of rules can be given which will apply equally well to all bridges. In some cases, the structure can be arranged in such a manner as to simplify the machinery, and in others the machinery must be designed to fit the structure. It is seldom possible to design an ideal combination of superstructure and machinery. The engineer should try to approach the ideal conditions as closely as possible, and it will usually be found that time spent in such efforts is a wise investment for his clients. Economy in operation, maintenance, and repairs should be considered as thoroughly as minimum first cost.

Several examples of the general arrangements and details of machinery will be given. They are not all ideal, but it is hoped that they may serve to call attention to arrangements that have well served their purpose on actual structures.

2. Turning machinery for a single-track swing span.—Figure 3A is an illustration of the general arrangement of the turning train of an ordinary single-track, through, railway swing span. The power plant is in the operator's house at the top of the span. A vertical shaft in the plane of one truss is driven by a pair of bevel gears at the top of the truss. Another pair on transverse shaft *S*4 delivers the power to a third pair near the center of the bridge. From this point the train consists of spur gears on vertical shafts, terminating in main pinion *G*1.

Union bearings are used for all bevel gears and the main pinion shaft. The main-pinion bearing is bolted to a heavy transverse box girder, attached to the lower chords of the trusses and to the stringers, and thoroughly braced. Shaft *S*2 is directly behind *S*1, and its bearings are on separate structural supports. Hand-turning shaft *S*6 is behind *S*3 and rotates whenever the power-driven gear train is operated.

The only shaft passing through a girder is *S*4. The hole in this stringer should be made large enough to clear bevel gear *G*7 when shaft *S*4 is taken out. The web would be reinforced around the hole if necessary for strength.



SINGLE-TRACK RAILWAY SWING SPAN.

FIG. 3A.

In a case of this kind it is hardly feasible to dispense with bevel gears. They are all on the smaller shafts, where the loads can readily be carried by gears of ordinary size. The heavy tooth loads are all taken by spur gears.

While this is not an ideal design, it has been satisfactory in operation.

3. Turning machinery for a highway swing span.—Figures 3B and 3C show sections and a plan of the turning and a part of the wedging machinery of a highway swing span, 257 ft. between centers of end bearings, for a roadway 21 ft. wide, with one standard-gauge electric railway track in the center of the roadway. The truss outline is shown in Volume I, Chapter III, by Fig. 3J (*c*). This is a center-bearing bridge with the control apparatus in a small operator's house on brackets outside of the truss and opposite the double center cross girder. The 15-h.p. turning and 11-h.p. wedge-driving electric motors are under the floor, on structural steel supports in the middle panels of the span. The center has three discs.

The arrangement of the turning mechanism is similar to that of the railway bridge described in Art. 2, Fig. 3A. The compact arrangement of the gear trains is shown in Fig. 3C. The end wedges are operated from a central, longitudinal shaft, extending the whole length of the span. Both the center and end wedges are driven by the same motor, which is shown in the figures. The principal speed reduction in the wedging train is made by a worm and wheel in the middle panel.

This group of machinery is notable for its simplicity and the small number of its parts. The floor was so shallow that some of the shafts had to pass through the stringers. Union bearings were not used, but the machinery was well aligned and operated smoothly.

4. Kalan Bridge, Columbia River Crossing, Union Pacific System.—Figures 3D to 3H, inclusive, are general plans of the machinery of the Kalan Bridge, over the Columbia River, on the Union Pacific System. It is 280 ft. between centers of end supports and is a single-track, center-bearing swing span, designed for modern loading and specifications. An outline of the trusses is shown in Volume I, Chapter III, Fig. 3C (*c*). The reactions and weights also are given in Table 3D, line (*c*).

The bridge is operated by electric motors below the floor and controlled from a house on top of the middle panel of the bridge. To provide against failure of the current, the house also contains an auxiliary gasoline engine and the necessary control, reversing, and transmission mechanism.

The electric supply is 3-phase, 60-cycle, 220-volt, alternating current. The normal output of the turning and wedging motors is 35

h.p., and each motor has a solenoid brake on the armature shaft. The gasoline auxiliary has a nominal rating of 53 h.p. and can deliver about 75 h.p. at 1000 r.p.m. This selection was made in order to make sure that the torque would be great enough to start and accelerate the bridge from rest. This engine is of the 4-cylinder, 4-cycle type. As such engines are designed to run at high speeds, their starting torque is small, and they are easily stalled during acceleration of the bridge. A large fly wheel was used, and the clutches are of the disc type, so that they can slip without injury and pick up the load gradually, while the engine is running at normal speed. When of ample size, properly applied, and well maintained and operated, this type of engine is a suitable primary source of power for ordinary drawbridges, as well as for an auxiliary supply.

The Kalan Bridge is center-bearing, the center having two discs 24 ins. in diameter, the lower of hard steel and the upper of hard bronze. The balance-wheel track-and-rack is fitted with lugs, which extend into the masonry and form secure anchorages against the tractive force from the main turning pinions. The turning and wedging gear trains are mounted as units wherever feasible. The main-pinion and the bevel-gear shafts run in union bearings. The wedge-driving mechanism is made so that the wedges cannot be over-driven. A crank in each gear train will pass its dead center and withdraw the wedges if the motor is not stopped when the wedges are fully driven. The principal speed reduction in these gear trains is by worms and wheels, the worms running in an oil bath within closed housings. The end latches are of the counterbalanced, automatic type. The longitudinal shafts driving the end-lifting wedges are connected by self-centering jaw couplings. The short shafts have flanged couplings, the bolt heads and nuts being inside of outer cylindrical flanges. This is a safety provision to prevent any chance of an attendant's clothing being caught by the revolving heads and nuts.

Figures 3D, E, and F show general details of the turning and wedging gear trains.

The arrangement of the auxiliary power plant is shown in Fig. 3G, and the gasoline engine, to a larger scale, in Fig. 3H. The engine, friction clutch, and reversing gears are mounted on a single bed plate. They were mounted, adjusted, and tested in the shop and shipped as a unit. Two interlocked jaw clutches provide means for driving either the turning or wedging trains. When in middle position, neither is engaged with the transverse shaft. A compound, reversible band brake is mounted on the bridge-turning shaft. The hand-control levers are at the operator's station, in convenient position for quick and certain

HIGHWAY SWING SPAN.

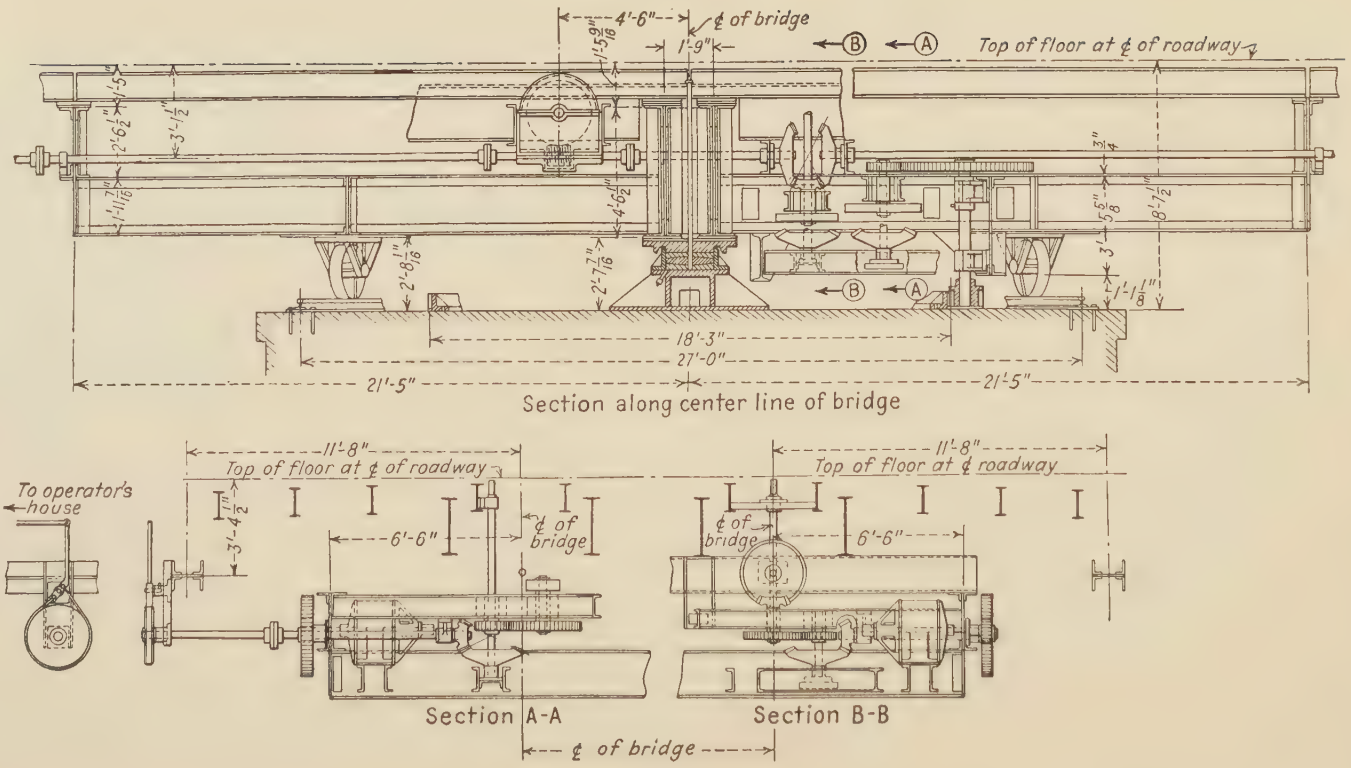
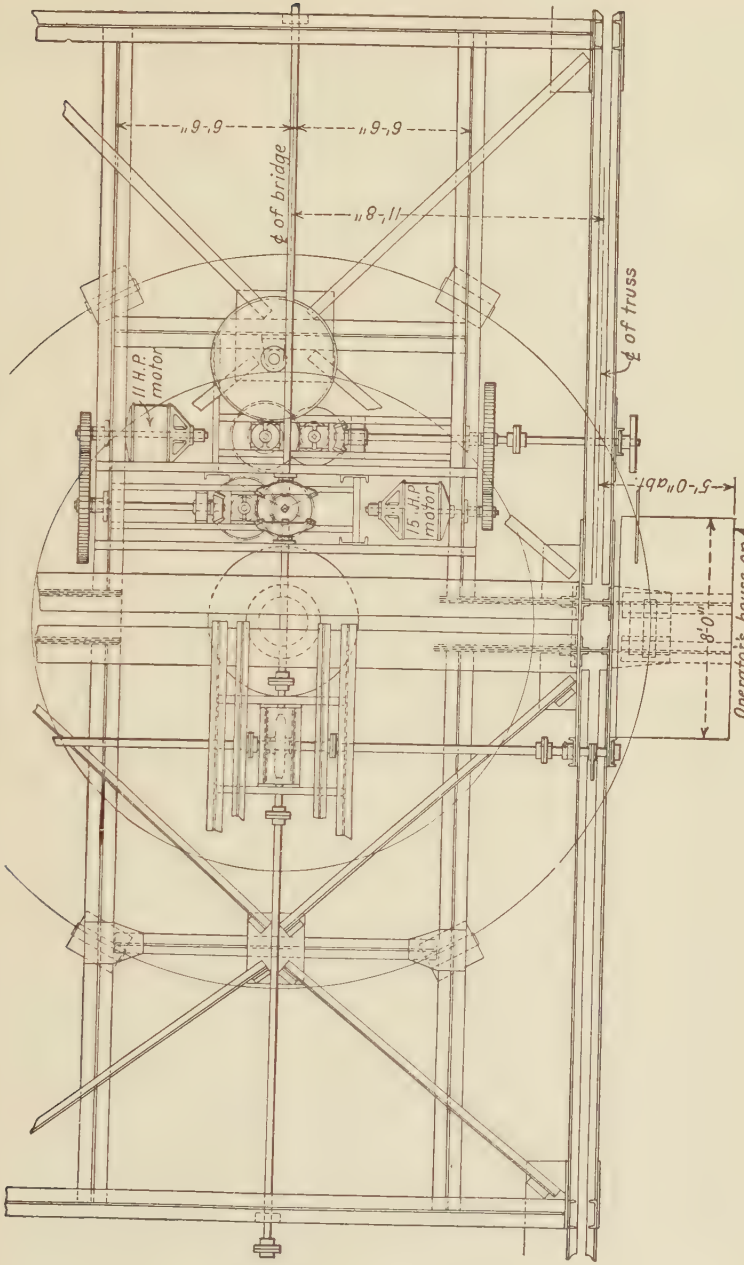


FIG. 3B.



HIGHWAY SWING SPAN.

FIG. 3C.

operation. The brake is operated by a pedal. The engine is cooled by a large radiator and fan of the ordinary truck type.

Shims were provided for field adjustment of the center, the end and center wedges, and all machinery bearings. Many of the holes in the structural-steel machinery supports were left blank and then drilled in the field after the machine parts had been aligned. Others were punched small at the shop and reamed to full size with the bearings in place.

This bridge was finished during 1923.

5. Middletown Bridge, N. Y., N. H., & H. R. R.—Figures 3I and 3J (*a*) and (*b*) show a part of the machinery of the Middletown Bridge, on the N. Y., N. H., & H. R. R. The truss outline is shown Volume I, Chapter III, Fig. 3C (*e*) and the weights and reactions are given in Table 3D, line (*e*).

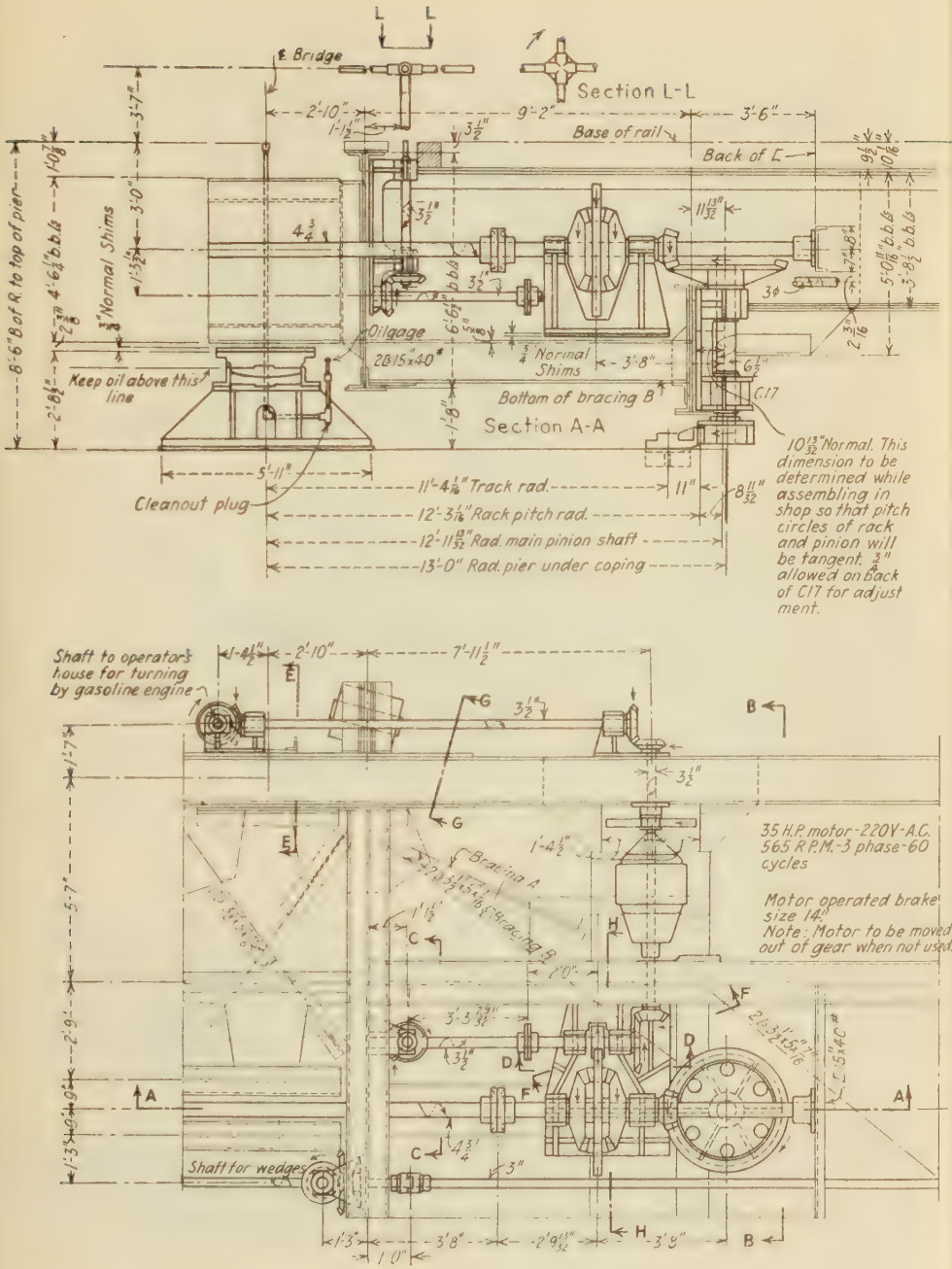
This bridge is operated by electric motors, and the machinery is of special interest because the principal speed reductions in the wedging trains are accomplished by screws with nuts moving in guides. Figure 3I shows the arrangement at the center. In this case the wedges move transversely to the bridge. A plan and sections of the end-lifting trains and latch are given in Fig. 3J (*a*) and (*b*). The figures need no explanation.

This bridge was finished during 1910.

6. Sacramento River Bridge.—This is a double-track, double-deck structure across the Sacramento River, at Sacramento, California, on the lines of the Southern Pacific Company. The outline of the trusses is shown in Volume I, Chapter III, Fig. 3E (*d*), and the reactions and weights are given in Table 3F, line (*d*). This is the heaviest center-bearing swing bridge that has been built up to 1926. It is believed that longer and heavier spans of this type are practicable and that no insuperable difficulties would be encountered in their design, either in the superstructure or in the operating machinery.

The machinery of the Sacramento Bridge presented some interesting problems due to the great weight to be accelerated in turning and the large reactions on the wedges.

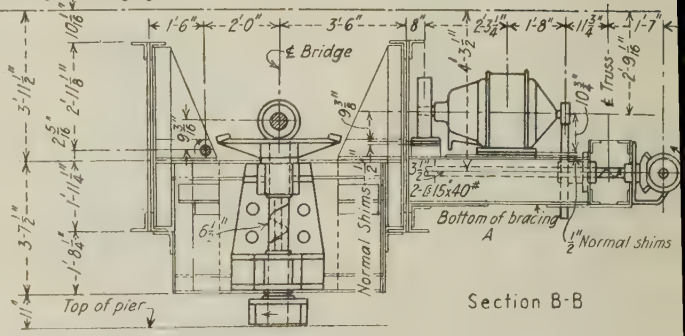
Turning mechanism.—Figure 3K (*a*) and (*b*) shows some of the more important details of the bridge-turning mechanism. The power is furnished by two 80-h.p., 425 r.p.m., mill-type electric motors, one located on each of the middle lower chords, which are bracketed out to give enough space for the motors. The accelerating force required was so great that four main pinions were used, two on each side of the bridge, placed symmetrically about the center line. The estimated

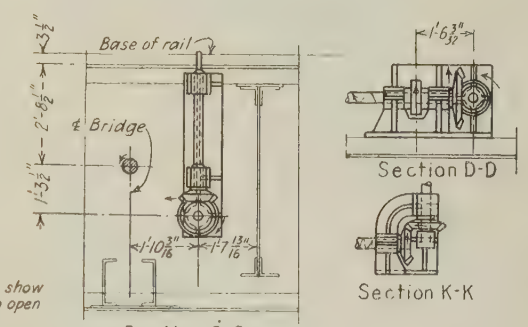

Section L-L
℄ Bridge
Base of rail
Back of L
Section A-A
Oilgage
Keep oil above this line
Bottom of bracing B
Cleanout plug
11'-4 1/16" Track rad.
12'-3 1/16" Rack pitch rad.
12'-11 13/32" Rad. main pinion shaft
13'-0" Rad. pier under coping
10 13/32" Normal. This dimension to be determined while assembling in shop so that pitch circles of rack and pinion will be tangent. 3/4" allowed on back of C17 for adjustment.

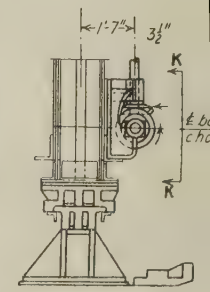
Shaft to operator's house for turning by gasoline engine
35 H.P. motor-220V-A.C. 565 R.P.M.-3 phase-60 cycles
Motor operated brake size 14"
Note: Motor to be moved out of gear when not used.
Bracing A
Bracing B
Shaft for wedges
℄ bridge

Erection Notes

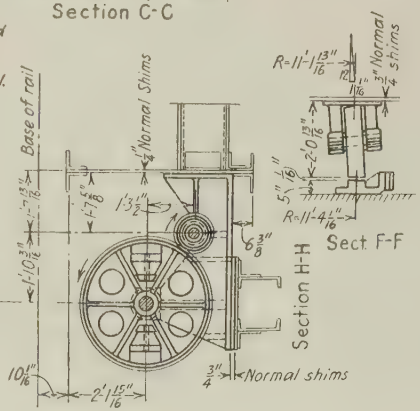
Erector to see that all bearings are shimmed so that gears mesh properly.
Before operating bridge thoroughly oil all bearings and other surfaces.
Erector should open bridge once by hand-power before applying motor-power.
Shim balance wheel bearings so that there is not more than 1/16" clearance between wheel and top of track when bridge is swinging.







General notes: Arrows show direction of turning to open bridge clockwise.
All parts of operating machinery connecting to or supported by struct. steel are to be assembled complete in shop together with the supporting struct. steel. Drill holes for bearings with gears meshing properly and with all other parts in their correct positions. Match mark wherever necessary.



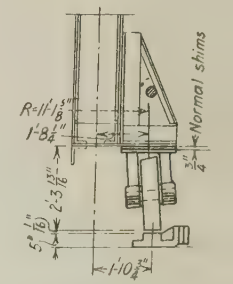


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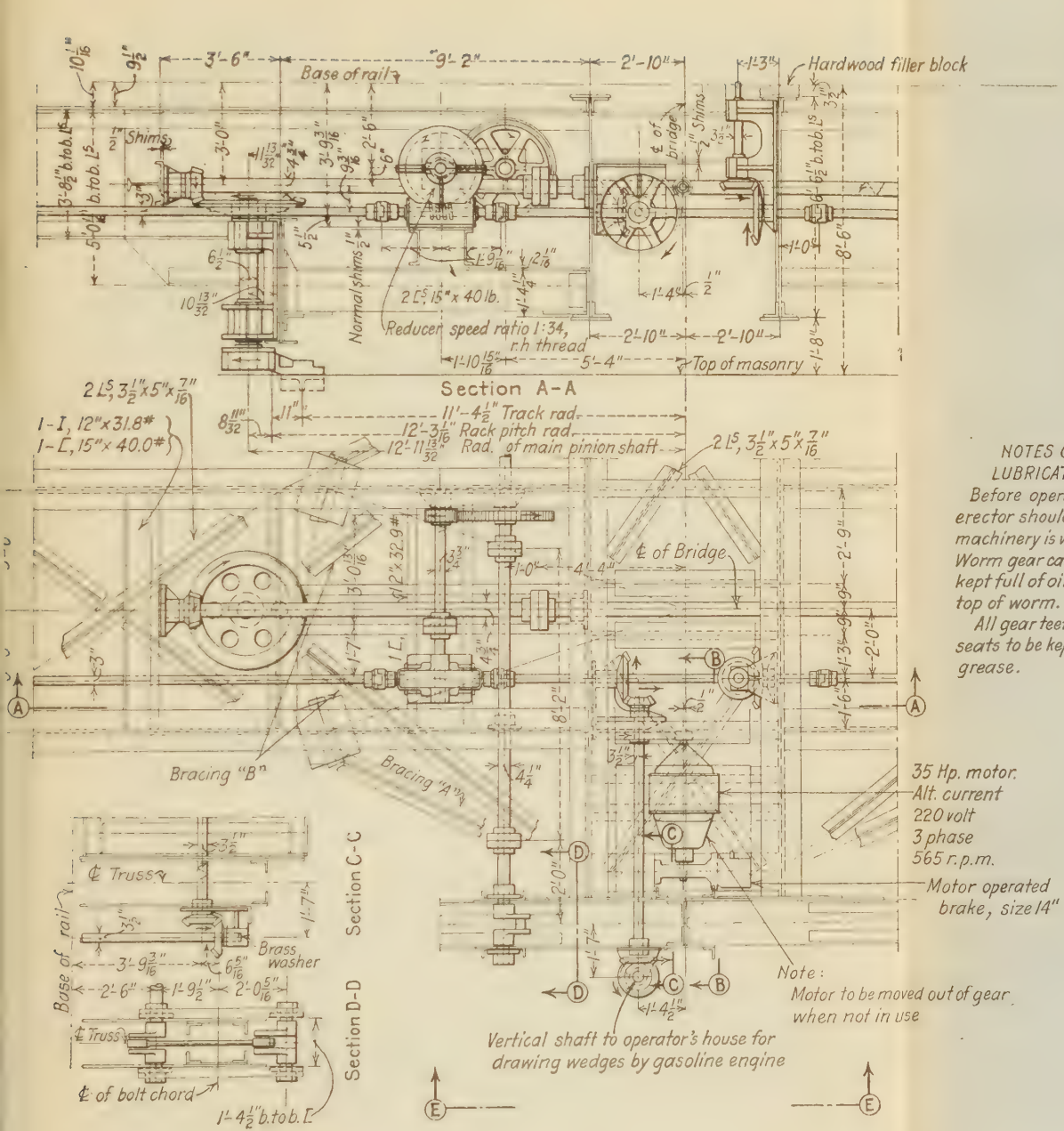
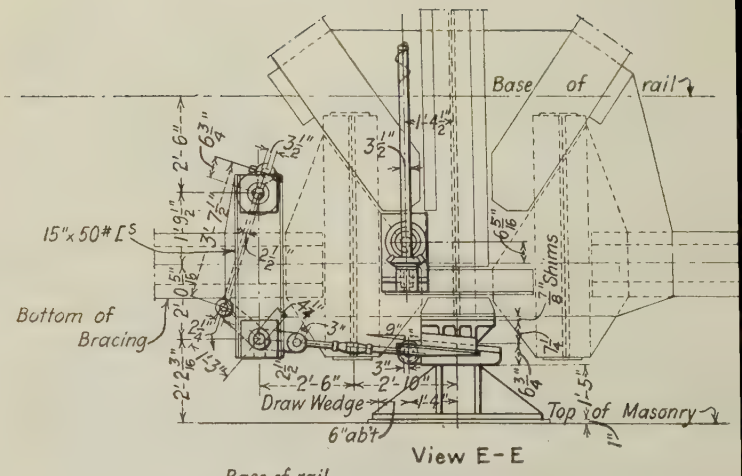
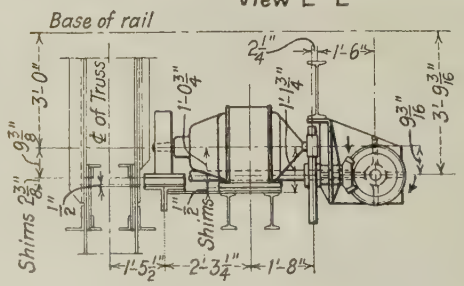
FIG. 3D.







NOTES ON LUBRICATION:

Before operating bridge erector should see that all machinery is well lubricated. Worm gear cases should be kept full of oil to level of top of worm.

All gear teeth and wedge seats to be kept cool with grease.

ERECTOR'S NOTE:

Shim balance wheel bearings, so that there is not more than 1/16 clearance between wheel and top of track when bridge is swinging.

Extreme care must be used in adjusting center wedges, so that they do n come to a solid bearing until conn. rod and lever arm are in a straight line.

Erector to see that all bearings are shimmed so that gears mesh properly. operating bridge all bearings and wearing surfaces should be thoroughly oiled.

Machinery is shown in position with bridge closed. Arrows on machinery indicate direction of turning for drawing wedges.

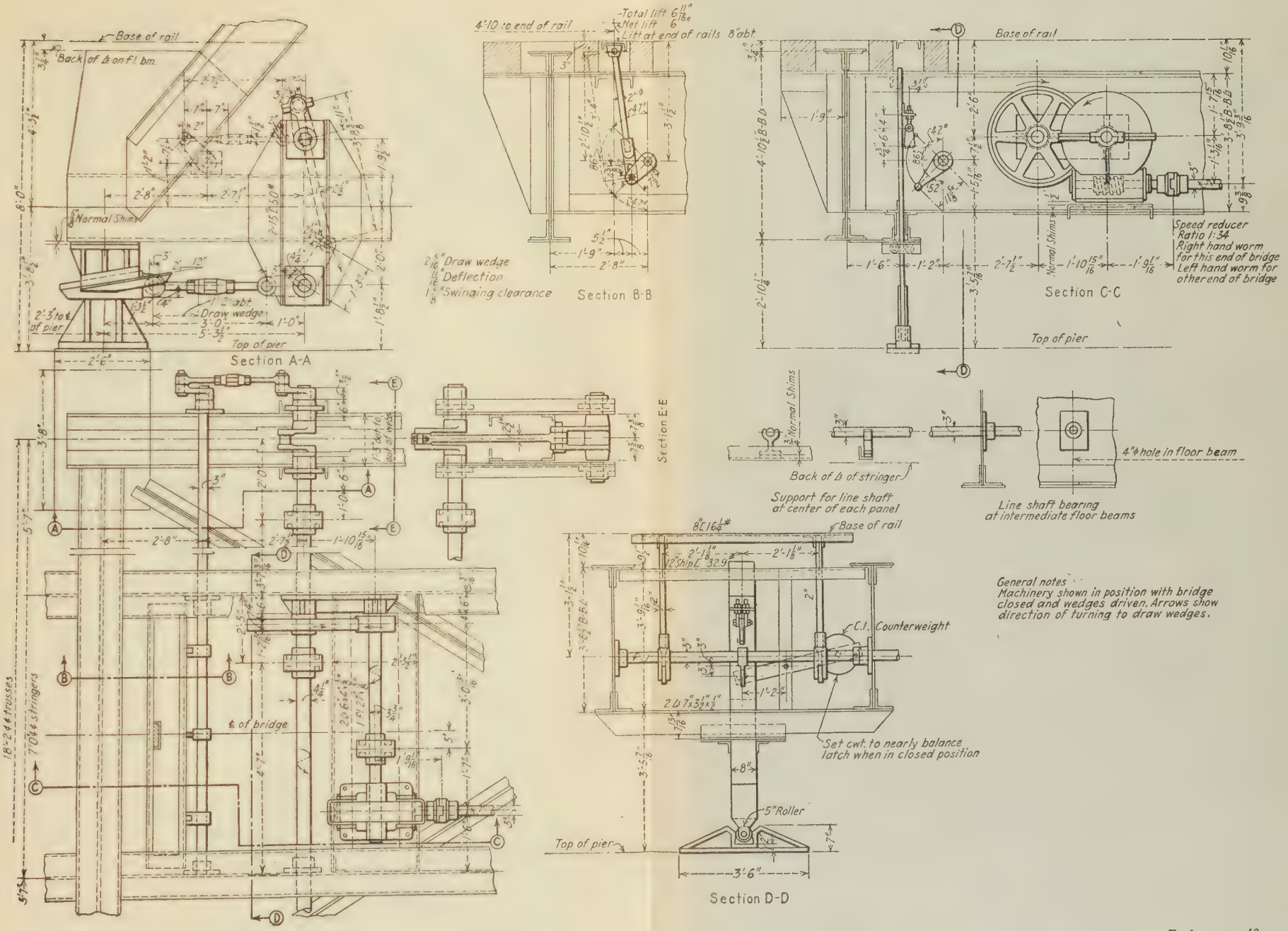
Shims under bearings provided as noted.

NOTE TO SHOP:

All parts of the operating machinery connected to or supported by st al steel are to be assembled completely in shop together with the supporting tural steel. Drill holes for bearings with gears meshing properly and all o parts in their correct position. Match mark where necessary.

To face page

FIG. 3E.


Base of rail
Back of ∠ on fl. bm.
Normal Shims
Draw wedge
Top of pier
Section A-A
4'-10 to end of rail
Total lift 6 11/16"
Net lift 6 7/8"
Lift at end of rails 8" abt
Draw wedge
Deflection
Swinging clearance
Section B-B
Base of rail
Speed reducer
Ratio 1:34
Right hand worm for this end of bridge
Left hand worm for other end of bridge
Section C-C
Top of pier
Normal Shims
Back of ∠ of stringer
Support for line shaft at center of each panel
4"ϕ hole in floor beam
Line shaft bearing at intermediate floor beams
Section E-E
18"-2 ∠'s trusses
7'0" ∠'s stringers
℄ of bridge
Base of rail
C.I. Counterweight
Set cwt. to nearly balance latch when in closed position
5" Roller
Top of pier
Section D-D
General notes
Machinery shown in position with bridge closed and wedges driven. Arrows show direction of turning to draw wedges.

To face page 46.

FIG. 3F.

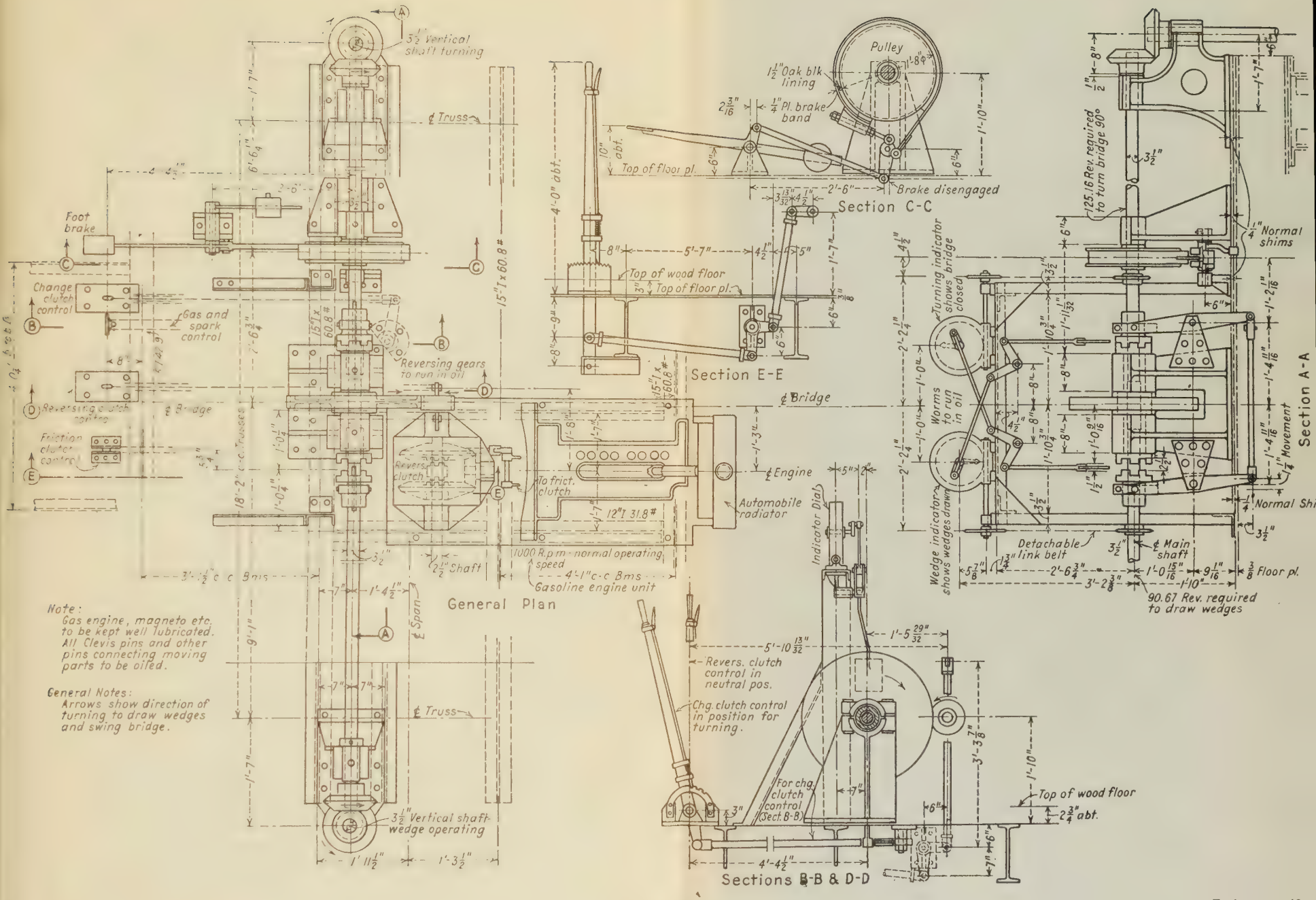
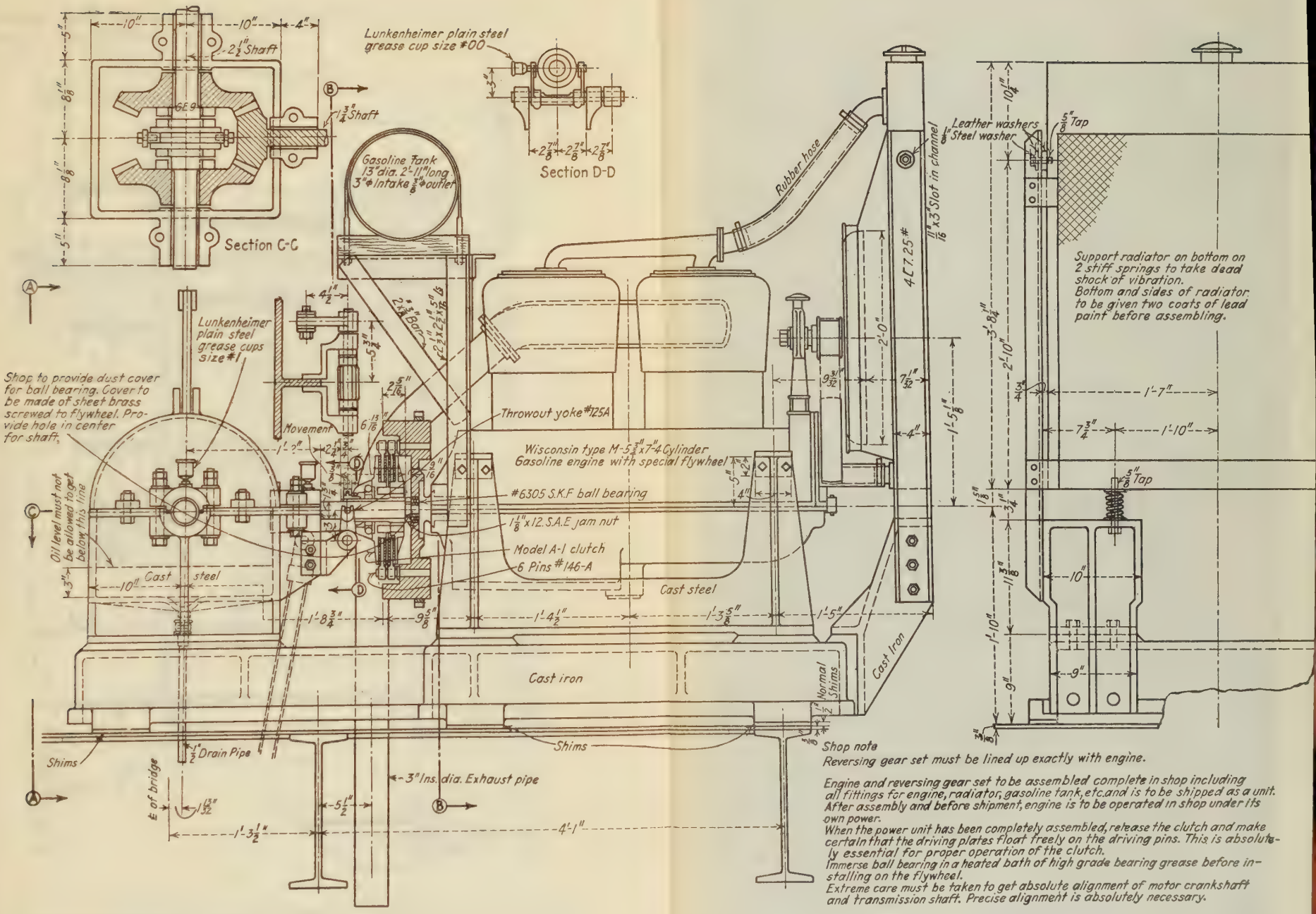

Foot brake
Change clutch control
Gas and spark control
Reversing gears to run in oil
Reversing clutch control
Friction clutch control
℄ Bridge
℄ Truss
℄ Span
Reverse clutch
To frict. clutch
℄ Engine
Automobile radiator
1000 R.p.m - normal operating speed
4'-1" c-c Bms - Gasoline engine unit
3½" Vertical shaft turning
½" Shaft
3½" Vertical shaft - wedge operating
General Plan
Note:
Gas engine, magneto etc. to be kept well lubricated. All Clevis pins and other pins connecting moving parts to be oiled.
General Notes:
Arrows show direction of turning to draw wedges and swing bridge.
Pulley
1½" Oak blk lining
¼" Pl. brake band
Brake disengaged
Section C-C
Top of floor pl.
Top of wood floor
Section E-E
Indicator Dial
Revers. clutch control in neutral pos.
Chg. clutch control in position for turning.
For chg. clutch control (Sect. B-B)
Top of wood floor
Sections B-B & D-D
Turning indicator shows bridge closed
Worms to run in oil
Wedge indicator shows wedges drawn
Detachable link belt
℄ Main shaft
Floor pl.
90.67 Rev. required to draw wedges
125.16 Rev. required to turn bridge 90°
¼" Normal shims
Section A-A

Fig. 36.

To face page 46.


Section C-C
Section D-D
Lunkenheimer plain steel grease cup size #00
Gasoline Tank 13" dia. 2'-11" long 3" ϕ Intake ⅜" ϕ outlet
Rubber hose
Leather washers
Steel washer
Slot in channel
Support radiator on bottom on 2 stiff springs to take dead shock of vibration. Bottom and sides of radiator to be given two coats of lead paint before assembling.
Lunkenheimer plain steel grease cups size #1
Shop to provide dust cover for ball bearing. Cover to be made of sheet brass screwed to flywheel. Provide hole in center for shaft.
Movement
Throwout yoke #125A
Wisconsin type M-5¾"x7-4 Cylinder Gasoline engine with special flywheel
#6305 S.K.F ball bearing
1⅛"x12. S.A.E jam nut
Model A-1 clutch
6 Pins #146-A
Oil level must not be allowed to get below this line
Cast steel
Cast iron
Normal Shims
Shims
½" Drain Pipe
3" Ins. dia. Exhaust pipe
E of bridge
Shop note
Reversing gear set must be lined up exactly with engine.
Engine and reversing gear set to be assembled complete in shop including all fittings for engine, radiator, gasoline tank, etc. and is to be shipped as a unit. After assembly and before shipment, engine is to be operated in shop under its own power.
When the power unit has been completely assembled, release the clutch and make certain that the driving plates float freely on the driving pins. This is absolutely essential for proper operation of the clutch.
Immerse ball bearing in a heated bath of high grade bearing grease before installing on the flywheel.
Extreme care must be taken to get absolute alignment of motor crankshaft and transmission shaft. Precise alignment is absolutely necessary.

To face page 46.

FIG. 3H.

tractive effort at each pinion is 55,300 lbs., and each pinion has 15 teeth of 5-in. pitch with $9\frac{1}{2}$-in. face.

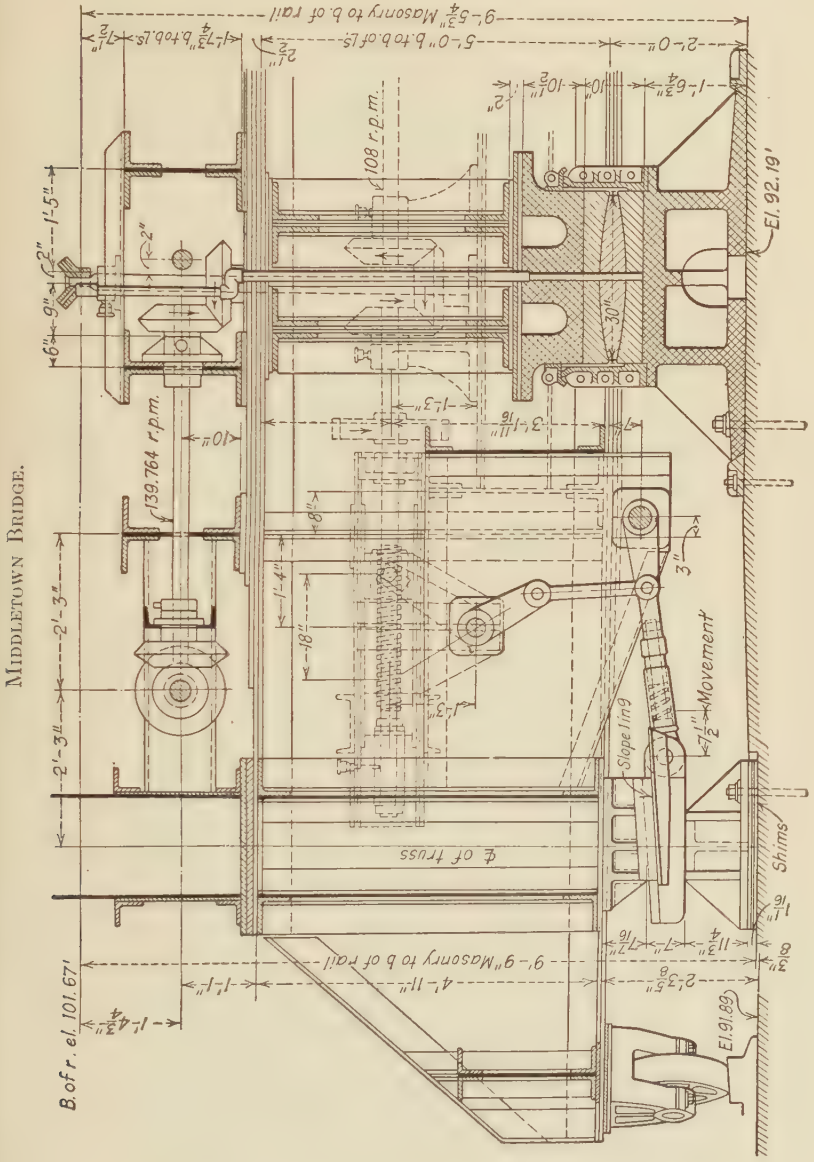
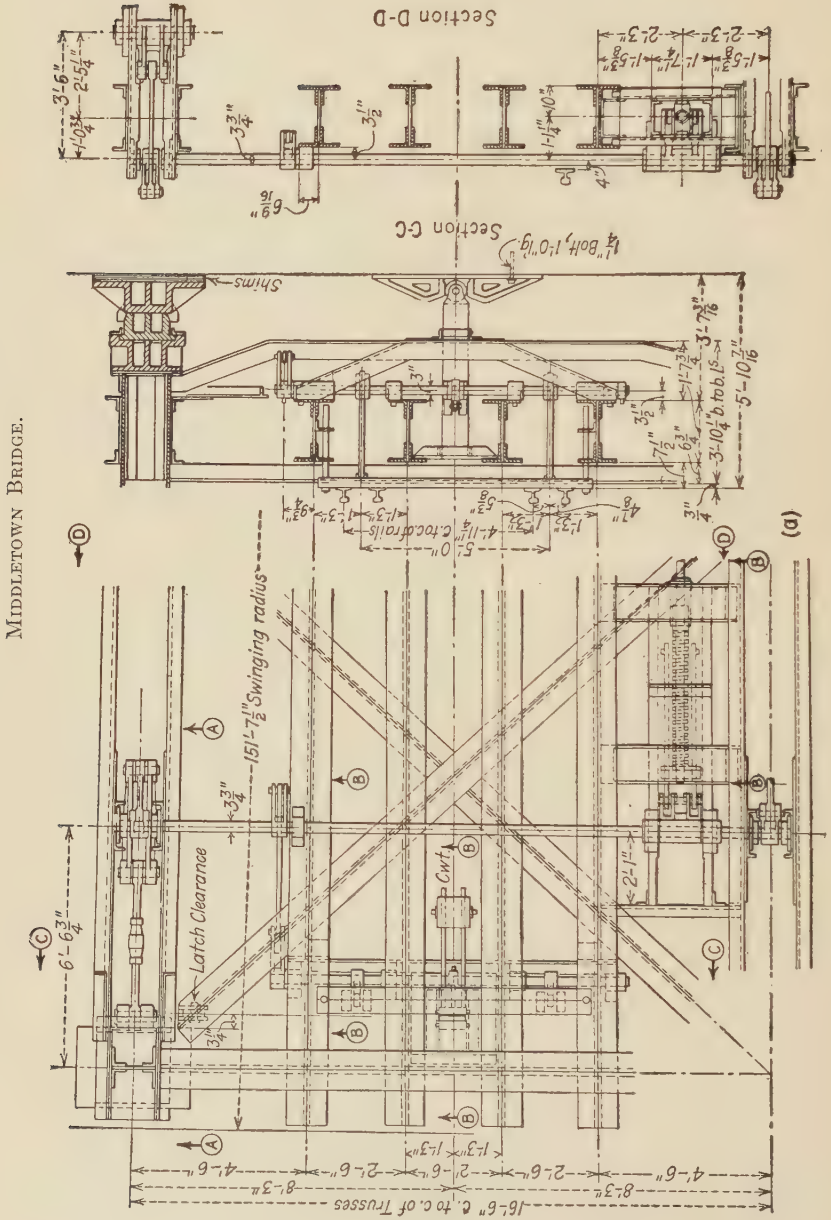


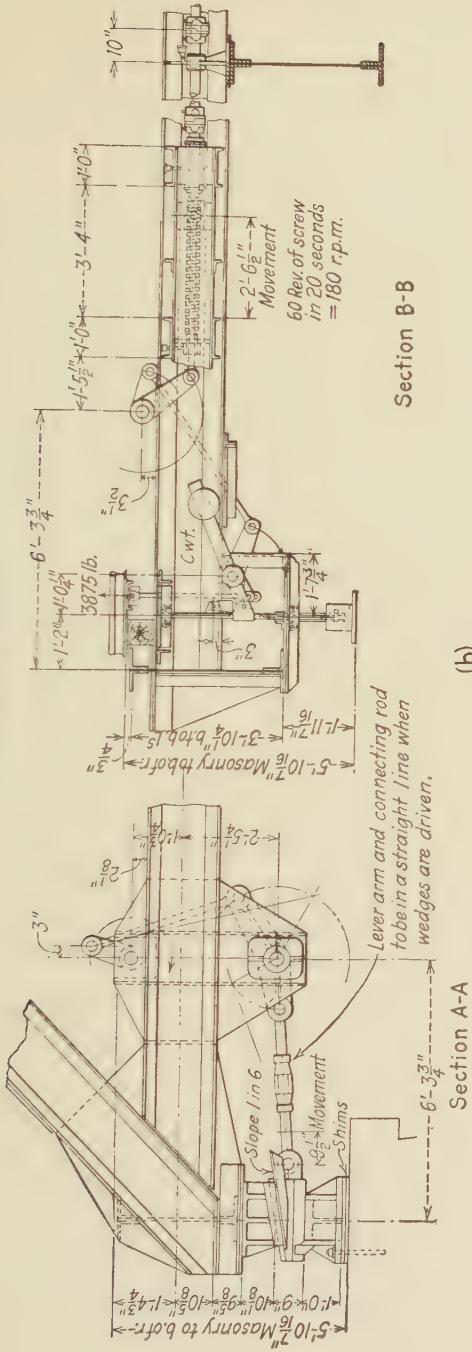
FIG. 31.

Each motor drives a transverse shaft, extending nearly to the center of the bridge, which drives a short longitudinal shaft by bevel gears.



MIDDLETOWN BRIDGE.

MIDDLETOWN BRIDGE



(b)

FIG. 3J.

This shaft drives another counter shaft by spur gears. The latter shaft passes through a floorbeam and drives a vertical shaft by bevel gears. This vertical shaft has a bevel-gear equalizer and also carries two pinions, which drive two trains of spur gears, with vertical shafts, each train terminating with a main pinion engaging the rack, which is anchored to the masonry. These gear trains are compactly arranged, and the bearings, shafts, and gears are accessible for easy maintenance and repairs. Only one shaft on each side of the center passes through a main member of the bridge.

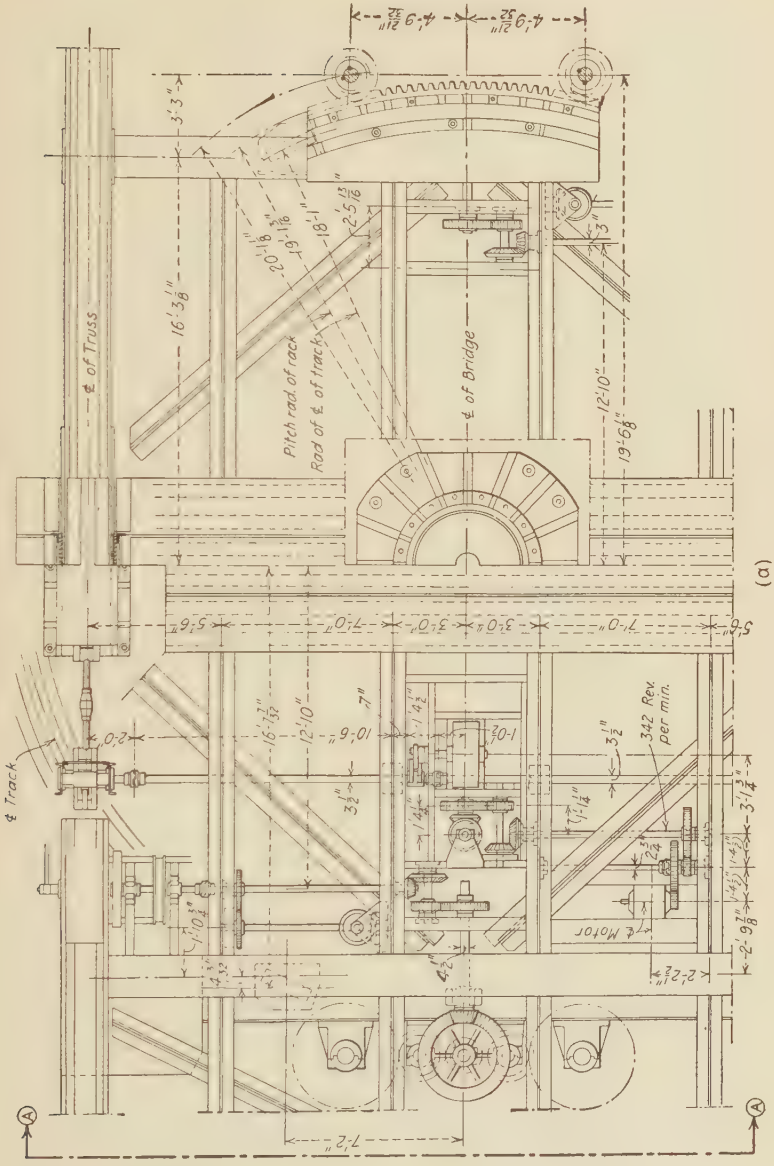
A full-torque band brake is mounted on each of the counter shafts adjacent to the motors. They are actuated from the operator's house through a system of rods, levers, and bell cranks, operated by a hand lever with a quadrant latch and catch, to hold the brakes as applied.

The operator's house contains the control apparatus, also an auxiliary gasoline engine and dynamo, to supply current when the outside supply fails. The bridge can also be turned by man-power when no electric current is available.

Wedging mechanism.—The center wedging mechanism is partly shown in Fig. 3K (*a*), and other details in Fig. 3L (*a*) and (*b*). The center wedges are driven by a 7½ h.p., 800-r.p.m., crane-type electric motor. It is under the railway floor in the first panel from the center. There are four speed reductions, the last by a worm and wheel, the worm running in an oil bath in a closed housing. The worm-wheel shaft drives a transverse rocking shaft by a link and bell crank. This shaft drives the wedges by the linkage shown in (*b*).

The live-load reaction on each center wedge is 1,528,000 lbs. and the wedges are the largest that have been used for this purpose. They are adjusted so that at the end of the stroke, when the toggle joint formed by the wedge pin, the thrust link, and the bell crank are in a straight line, there is no dead-load uplift, the wedges just making contact.

One of the end-wedge, rail-lift, and latch trains is shown in Fig. 3M (*a*), and a larger detail in (*b*). At each end of the span, the wedging train is driven by a 40-h.p., 520-r.p.m., crane-type motor. The wedges are driven by links articulated to segmental gears. The device acts as a toggle, locking the wedges when they are fully driven. The wedge pedestals have a lug on the outside which extends above the wedge seat. The inside of the lugs and the ends of the wedges are tapered, so that the bridge is brought to exact central position when the wedges are fully driven. This device supplements the centering by means of the automatic end latches. The design used on this bridge requires the use of limit switches to make sure that over-driving does not occur. Indicators also were used in the operator's house.

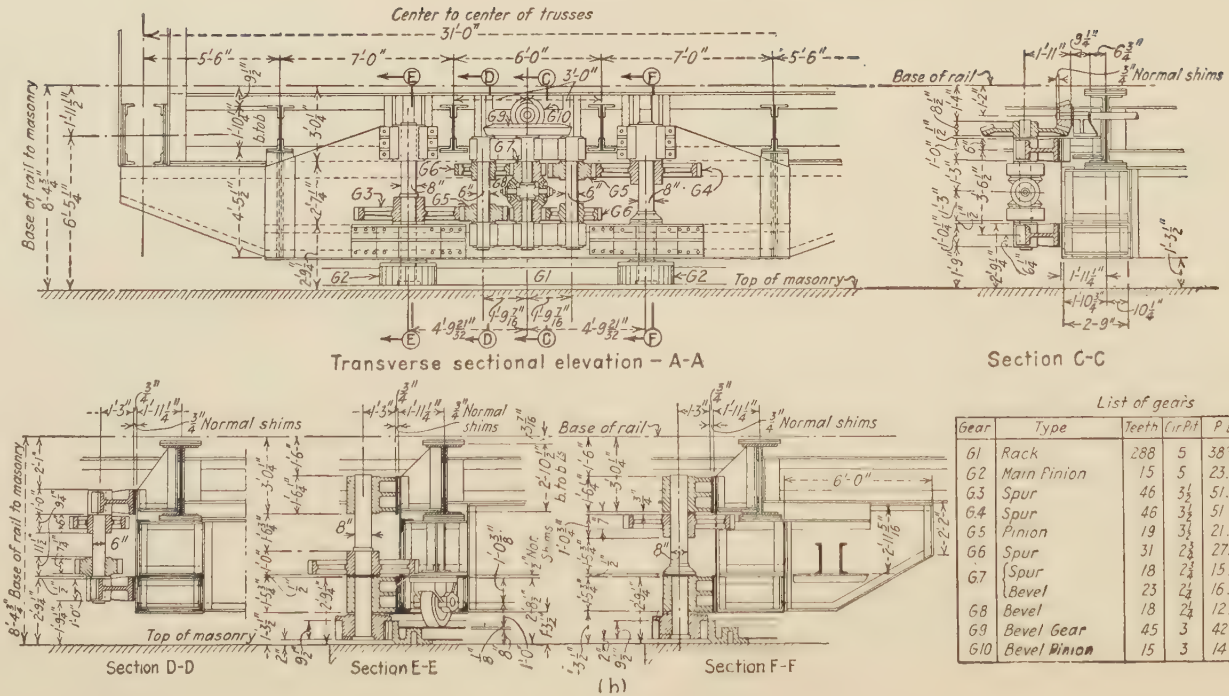


SACRAMENTO RIVER BRIDGE.

(a)

FIG. 3K (a).

SACRAMENTO RIVER BRIDGE.



List of gears

Gear	Type	Teeth	Cir. Pit.	P. Dia.	Face	Bore
G1	Rack	288	5	38-2 3/8"	9	
G2	Main Pinion	15	5	23.873	9 1/2	8
G3	Spur	46	3 1/2	51.248	7	8
G4	Spur	46	3 1/2	51.248	7	8
G5	Pinion	19	3 1/2	21.167	7 1/4	6
G6	Spur	31	2 3/4	27.136	6	6
G7	Spur	18	2 3/4	15.756	6 1/2	5 3/4
	Bevel	23	2 3/4	16.472	4	5 3/4
G8	Bevel	18	2 3/4	12.955	4	3
G9	Bevel Gear	45	3	42.971	6 1/2	5 3/4
G10	Bevel Pinion	15	3	14.324	6 1/2	4 1/2

FIG. 3K (*b*).

The center and end wedges can be manually operated when electric current is not available.

The draw span weighs about 6,748,000 lbs. and each end reaction is 868,000 lbs. The span is 390 ft. 3 ins. between centers of end bearings.

SACRAMENTO RIVER BRIDGE.

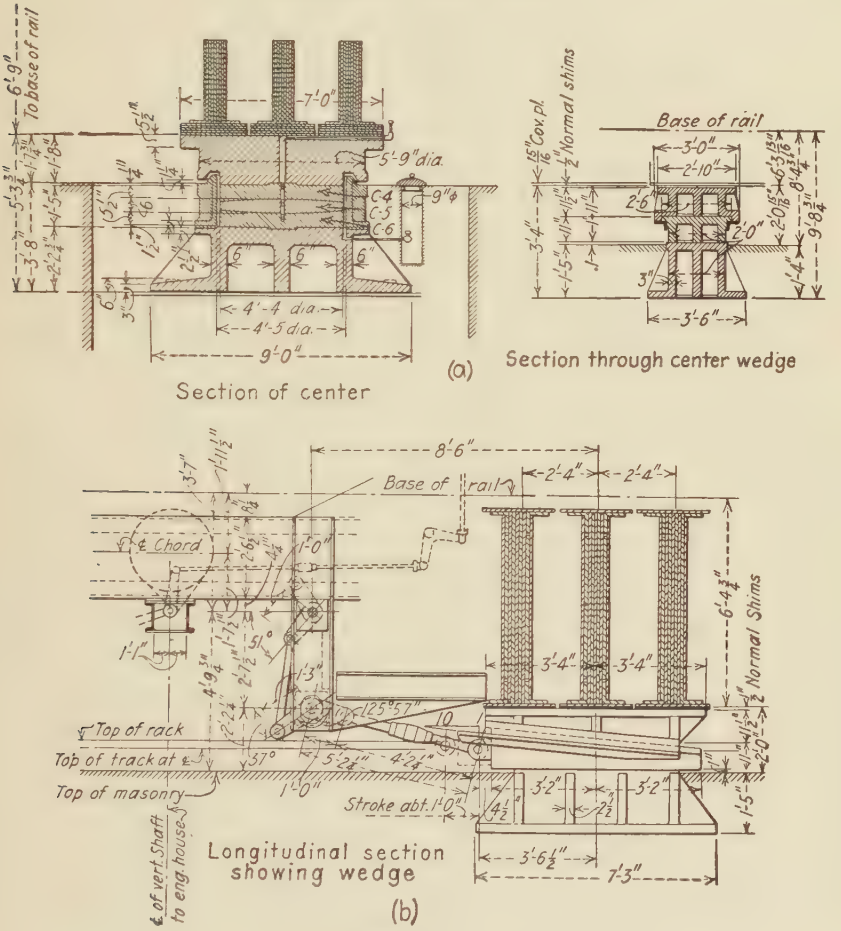


FIG. 3L.

The bridge can be unlocked and the ends lowered in 30 seconds, and the span turned 90° in 2 minutes, the total time for opening, or closing, being $2\frac{1}{2}$ minutes.

The bridge was finished during 1911, and has operated satisfactorily in all respects.

SACRAMENTO RIVER BRIDGE.

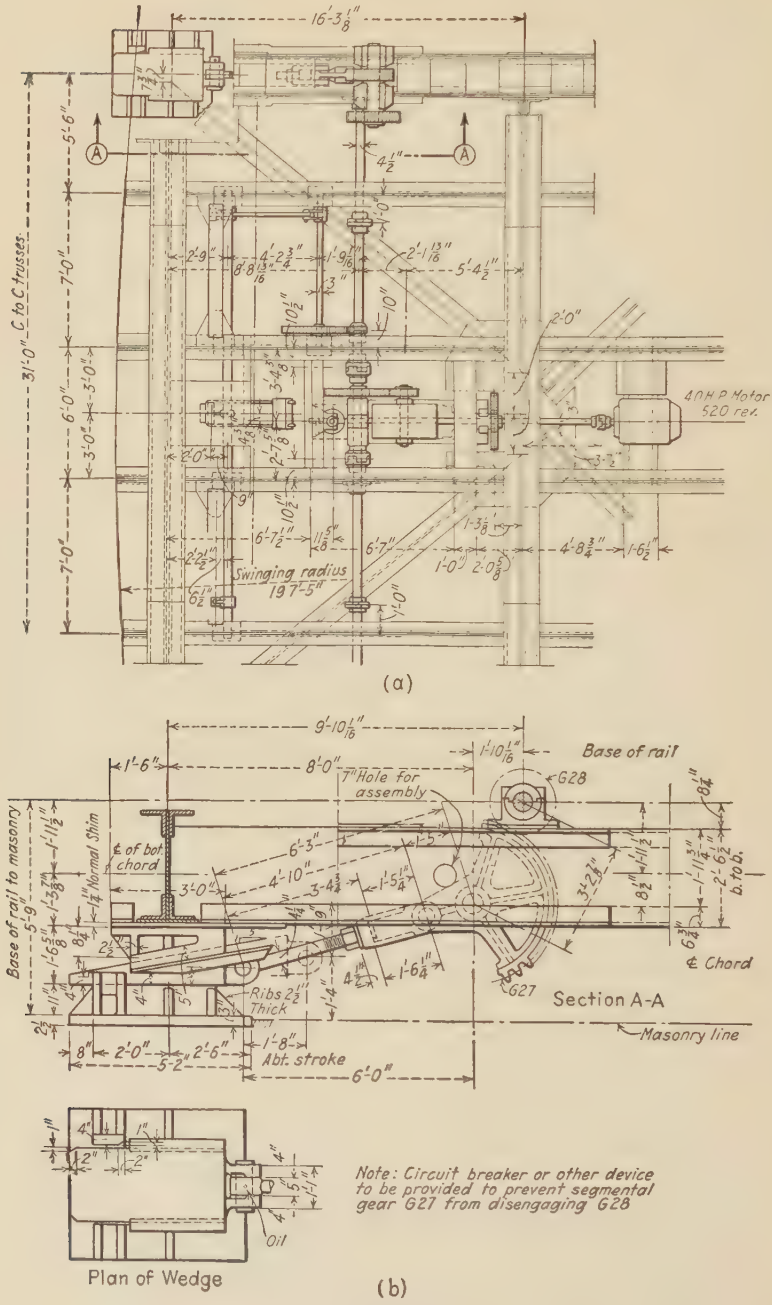


FIG. 3M.

7. Kinzie Street-Kinnickinnic center- and rim-bearing drawbridge. Figures 3N (*a*) and (*b*) and 3O show the center, drum, and turning machinery of a rim-bearing drawbridge that has an unusual history. In 1884, the Edge Moor Iron Company built a bridge across the Chicago River, at Kinzie Street, for the Chicago, Milwaukee, and St. Paul Railway. This bridge was unusual in that the center of rotation was under one truss of a double-track structure. The Sellers roller center, shown in Fig. 3N (*a*), was used under this bridge until 1892, when the bridge was changed to a swing span rotating about its center. The roller center was stored until 1897 and was then found to be in perfect condition. It was purchased by the Chicago and Northwestern Railway, and installed under their Kinzie Street bridge, which was a double-track span 181.5 ft. long. The drum and turning machinery shown in the figures was installed on this bridge. This bridge was taken down in 1906 and the center and machinery were transferred to the Kinnickinnic River bridge of the Chicago and North Western railway at Milwaukee, where both the center and the machinery are still in service, and are reported to be in excellent condition and never to have given any trouble. The center is forty-two years old and has been in service thirty-seven years, under three bridges, and is subject to a load, when turning, of about 600,000 lbs.

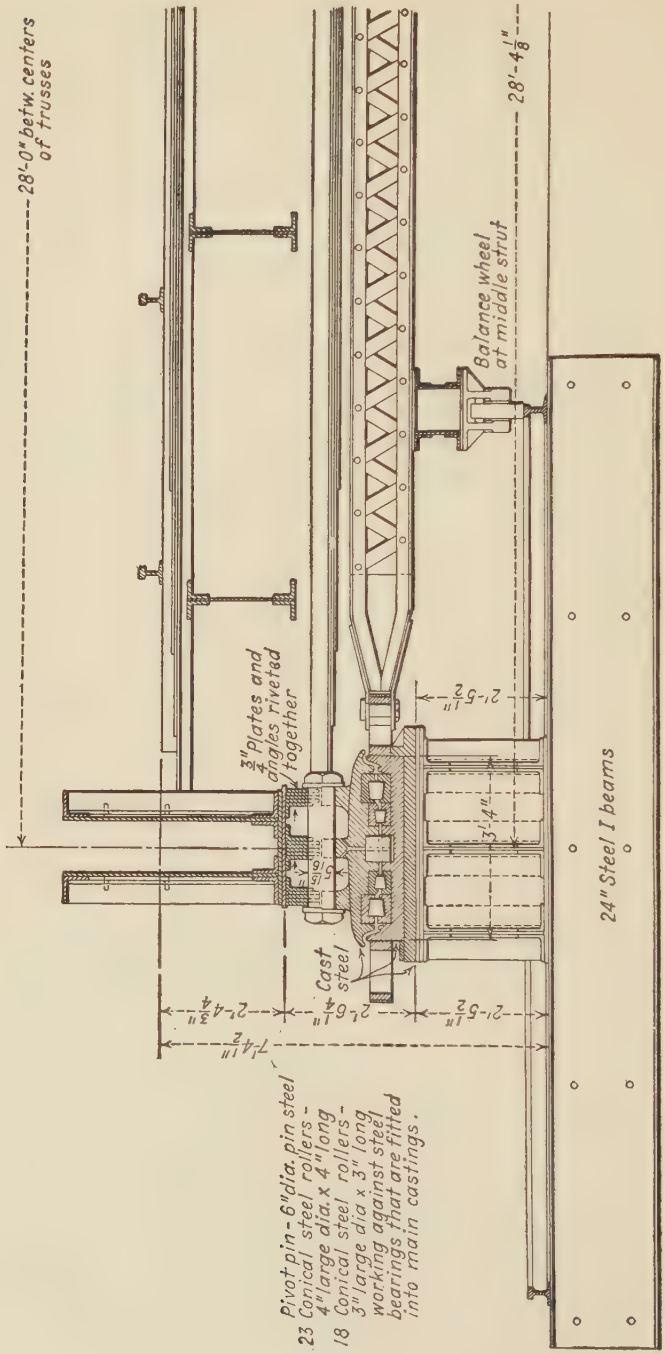
This center illustrates some important principles of design. The load upon the top of the center is delivered centrally by a bearing of small radius, in this case a pin. This is a vital point for a successful roller center. The rollers are short, their length being equal to their larger diameter. No such rollers should be longer, and it would be better to limit their length to not more than their mean diameter. The rollers are loose in their raceways. Ordinarily this is not good practice. The success of this center in this respect was due to the fact that the rollers practically occupied the entire circumference of the raceways, and hence could not get out of radial alignment.

The Sellers practice was to make both the rollers and the raceways from tool steel of quite an ordinary grade, and there is no evidence that this practice was not followed in this case. It is not probable that either the rollers or the raceways were tempered.

The secret of the long service is that a properly loaded, well-designed, and carefully built center was made from good, but not unusual, material, and that it was well maintained in service.

All of the bridges in which the center was used were across busy channels, requiring far more than an average number of daily openings.

The load upon the rollers may be expressed by the formula, $W = 1325\ dl$, in which d is the diameter and l the length of the


28'-0" betw. centers of trusses
28'-4 1/8"
Balance wheel at middle strut
3/4" Plates and angles riveted together
Cast steel
2'-6 1/4"
2'-4 3/4"
2'-5 1/2"
2'-5 1/2"
7'-4 1/2"
3'-1 1/4"
24" Steel I beams
Pivot pin - 6" dia. pin steel
23 Conical steel rollers - 4" large dia. x 4" long
18 Conical steel rollers - 3" large dia x 3" long working against steel bearings that are fitted into main castings.

Kinzie Street—Kinnickinnic Bridge.

Fig. 3N (a).

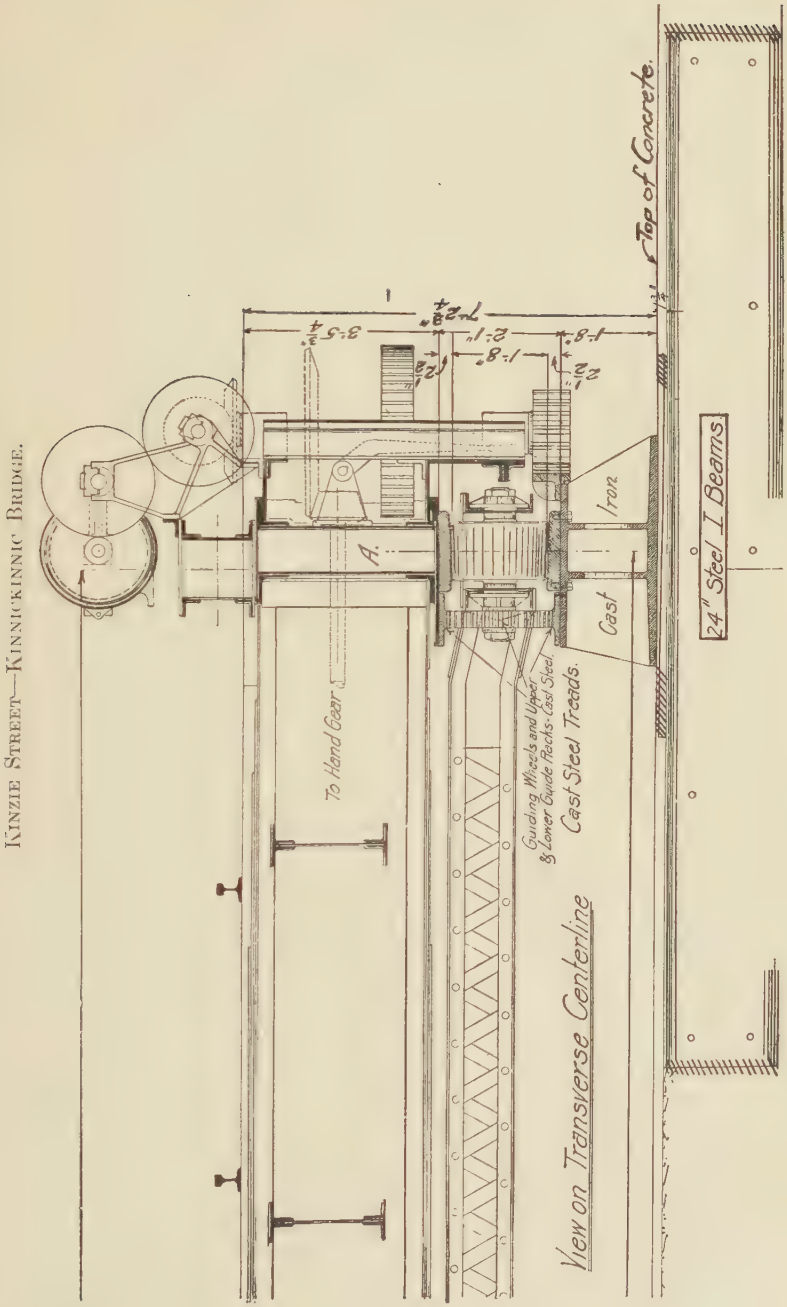

Kinzie Street—Kinnickinnic Bridge.
Top of Concrete.
24" Steel I Beams
Cast Iron
Cast Steel Treads.
Guiding Wheels and Upper & Lower Guide Racks. Cast Steel.
To Hand Gear
A.
View on Transverse Centerline

Fig. 3N (*b*).

rollers, in inches, and W is in pounds. This assumes that the loads were distributed proportionately to the products, dl, of the rollers. Probably the inside ring of the smaller rollers carried more than this average load.

The record of the turning machinery is only a little less notable. It was first installed on the Chicago and North Western Railway bridge

Kinzie Street—Kinnickinnic Bridge.

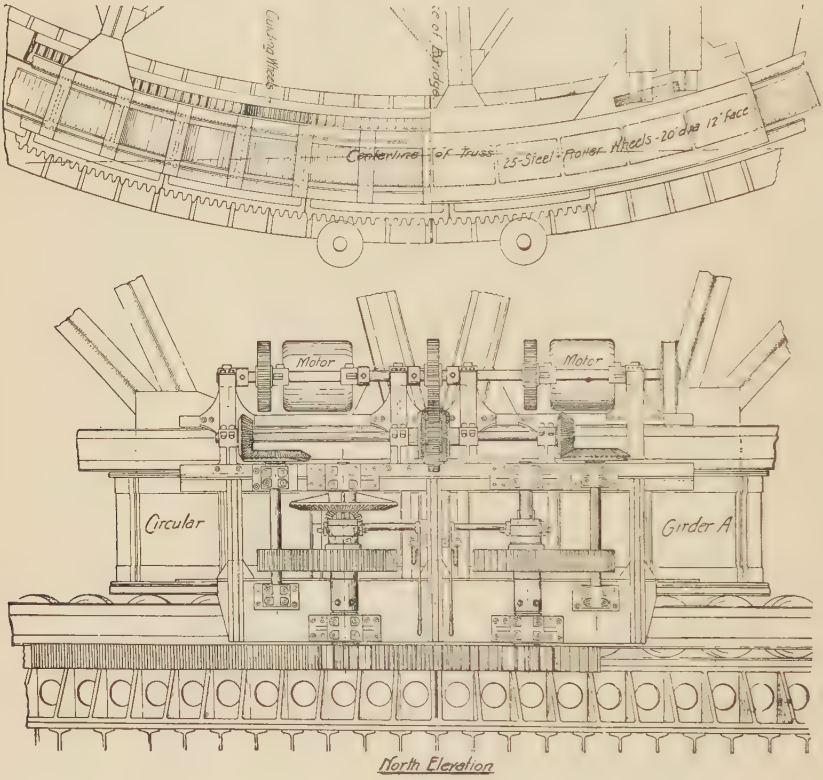


Fig. 30.

at Kinzie Street, in 1897, and was used there until it was removed to Milwaukee in 1897, and it is still in use and in good condition. It has been in service twenty-nine years.

The main-pinion shafts are mounted on a unit bracket, rigidly attached to the drum, making accurate alignment feasible. The loads from the motors are equalized between the two gear trains. Union bearings, rigidly attached to the superstructure, carry the counter and

equalizer shafts. All of the mechanism is accessible for oiling, maintenance, and repairs. As in the case of the center, good design, accurate workmanship and installation, and adequate maintenance are responsible for the long and satisfactory service.

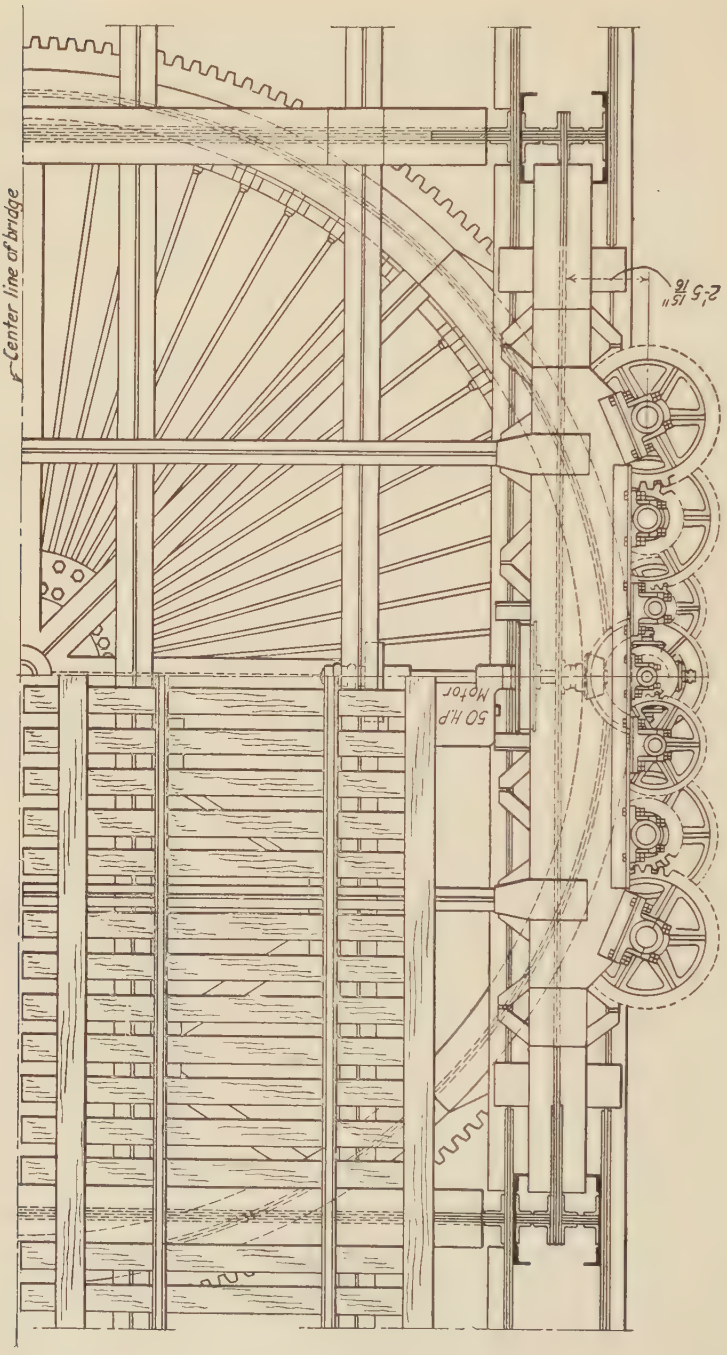
The author recently had an opportunity to inspect another drawbridge for which well-designed machinery was furnished and properly maintained. This was the Point Street Bridge, in Providence, R. I. It was designed and built by Mr. Charles Macdonald, M. Am. Soc. C. E., in 1872, and was still in service and in good operating condition of the end of 1925, after fifty-three years. The bridge is now inadequate for the greatly increased traffic and must soon be replaced on this account; but the machinery, after some repairs, would be good for many years more if the bridge were not overloaded.

8. Turning machinery of a modern rim-bearing swing span.—Figure 3P shows a plan and Fig. 3Q an elevation of one of the simplest arrangements of turning machinery that has been used on rim-bearing swing spans. The mechanism shown was used on the Columbia River bridge of the Spokane, Portland, and Seattle Railway, and was designed by Mr. Byron B. Carter for Mr. Ralph Modjeski, Chief Engineer. Similar machinery was also used on the Willamette River bridge, on the same line, and on other large bridges designed by the same engineer.[1] An outline of the trusses of the Willamette River 521-ft. swing span is given in Volume I, Chapter III, Fig. 3II (*d*), and the reactions and weights are shown in Table 3H, line (*d*).

The Columbia River span is a double-track structure, 462 ft. between centers of end bearings, and the trusses are similar in arrangement to those of the Willamette span. The entire center-pier load, however, is carried by the drum, the center girders distributing it equally to eight points on the drum. Two main pinions are equalized symmetrically about the transverse center line of the span on each side of the bridge. The bevel gear of the equalizer is driven directly from the counter shaft of the motor by a bevel pinion. The vertical equalizer shaft has a pinion at each end, engaging a train of spur gears with three speed reductions to the main pinion shaft. The upper pinion drives one of the main shafts, and the lower pinion drives the other. The motor and gear trains are duplicated on the other side of the bridge.

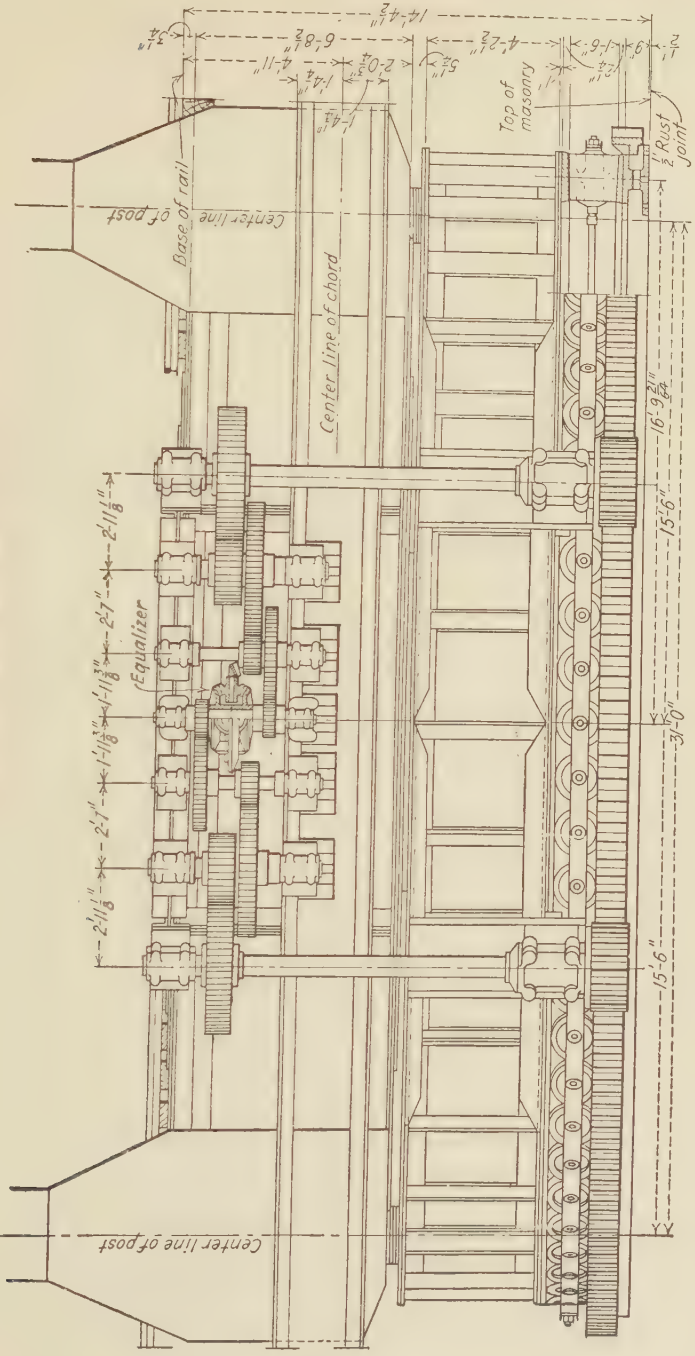
The spur gears are all between the shaft bearings. The upper bearings are supported on a special bracket, riveted to the top of the longitudinal center girder, and the lower set is carried by the lower

[1] "The Operating Machinery of the Willamette River Drawbridge, near Portland, Oregon," by Byron B. Carter. *Jour. of the Western Soc. of Engrs.*, Vol. XXI, p. 529, 1916.



COLUMBIA RIVER BRIDGE.

FIG. 3P.



COLUMBIA RIVER BRIDGE.

FIG. 3Q.

chord section, built as a part of the same girder. All of the bearings are fully accessible for oiling, maintenance, and repairs.

The time for turning the span 90° is 2 minutes.

The electric supply is 3-phase, 33-cycle, 600-volt alternating current. The two turning motors are 50 h.p., normal rating. The end lift and rail locks are driven by smaller motors at the ends of the span. The arrangement of the end gear trains is given in Volume I, Chapter X, Art. 8, Fig. 10J. Ordinarily, the current is taken from an outside source of supply. To provide for emergencies, the control house contains a 125-h.p., 3-cylinder, vertical gasoline engine, connected to a dynamo, to generate current when the outside supply is cut off.

This bridge was finished during 1908.

9. Chicago Type bascule bridge. *Operating machinery.*—The general arrangement of the operating machinery of the simple trunnion, double-leaf, double-deck bascule bridge, carrying Michigan Boulevard across the Chicago River, in Chicago, is shown in Figs. 3R and 3S. General data concerning this bridge are given in Volume I, Chapter IV, Art. 7, and in Table 4A.

Each leaf of the bridge is operated by two 100-h.p. motors, with two additional motors in reserve. The motors are located in a machinery room under the lower street level and back of the anchor pier. They are arranged, both mechanically and electrically, so that any two can be used for operation. The layout of the mechanism is shown in Fig. 3R. Pinions on extensions of the armature shafts engage a master gear containing a bevel-gear equalizer on a transverse shaft, extending across the entire width of the bridge. At each end a miter gear drives a longitudinal shaft, extending to the grouped reduction gearing, the first shaft of which is driven by a pair of miter gears. The bevel-gear shafts turn in union bearings at their ends and are supported at intermediate points by ordinary pedestal bearings. Figure 3S shows enlarged details of one of the groups of three pairs of reduction gears, the first driven by the miter gears, and terminating in the main-pinion shafts. These gears are mounted on large cast-steel union-frames. The first and second shafts are each fitted with powerful, adjustable, double-block brakes.

The movements of the bridge are controlled from operating rooms at each end of the bridge outside of the fixed approach sidewalks.

Emergency brakes.—Figure 3T shows the general arrangement and Fig. 3U the details of the electro-pneumatic emergency brakes. They are supplementary to the two double-block brakes in each operating gear train. At each corner of the bridge a $\perp$-shaped brake rail is anchored to the masonry of the counterweight pit. The rails are

gripped by brake shoes on toggles, actuated through levers by pneumatic cylinders, 10 by 10 ins., the air supply being regulated by magnetic valves, controlled from the operator's house. The brakes were

Michigan Boulevard bridge.

Operating Machinery.

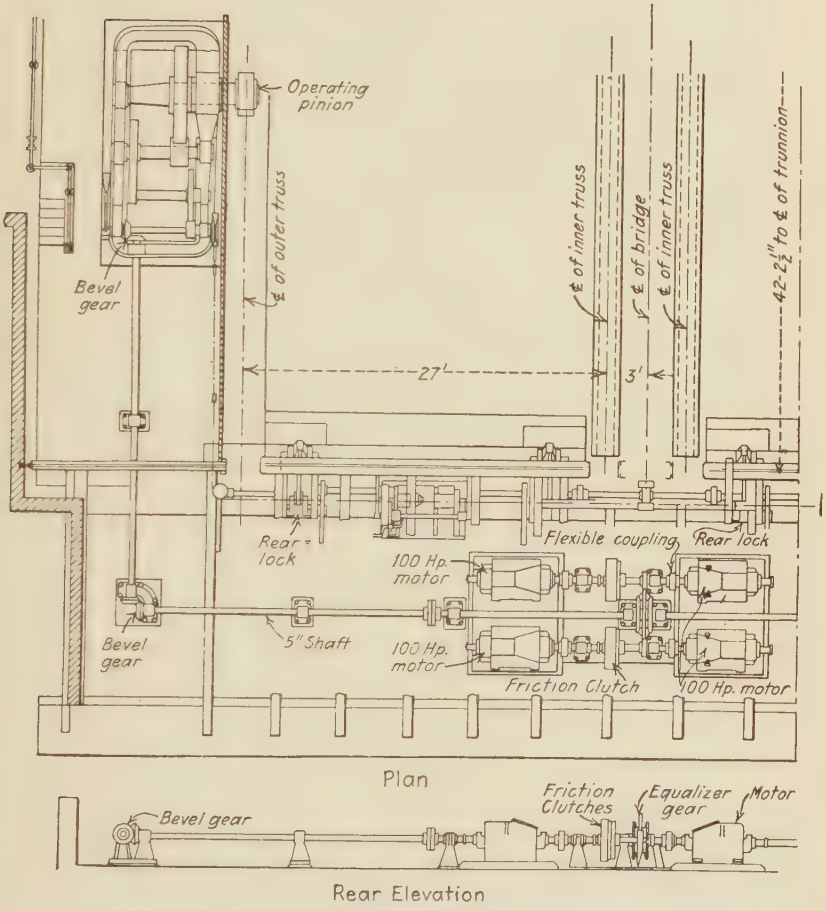


Fig. 3R. *Eng. News-Rec.*

designed to hold the leaves against a wind pressure of 20 lbs. per square foot on the vertical projection of the leaves. Under this condition a force of 95,000 lbs. is required at each brake, corresponding to 6000 lbs. at the cylinder. The coefficient of brake friction was assumed at 0.30.

MICHIGAN BOULEVARD BRIDGE.
OPERATING MACHINERY.

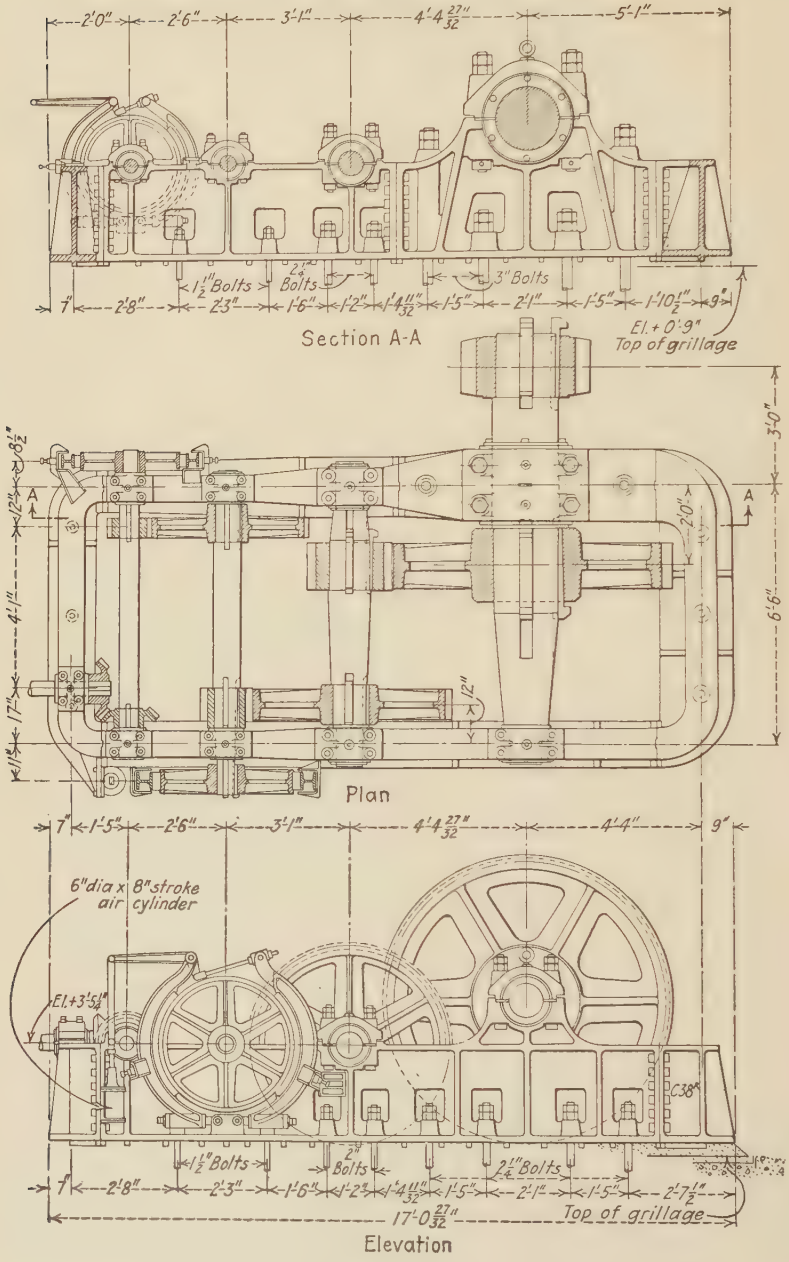
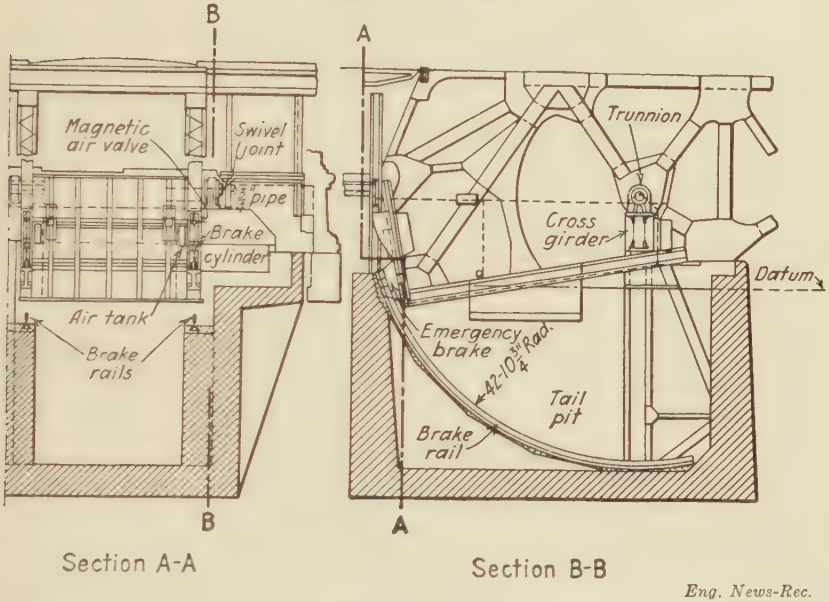


FIG. 38.

Heel locks.—The shore end of each of the four trusses has a heel lock, designed to carry a live load of about 138,000 lbs. Fig. 3V shows the details. Each lock is operated by a 10-h.p. motor, through a gear train on the anchor pier, which actuates a toggle engaging a cast-steel seat on the rear of the counterweight box.

Center locks.—Center locks are provided on the end of each truss, to make sure that the two leaves of the bridge deflect equally under

MICHIGAN BOULEVARD BRIDGE.
EMERGENCY BRAKE.



Eng. News-Rec.

FIG. 3T.

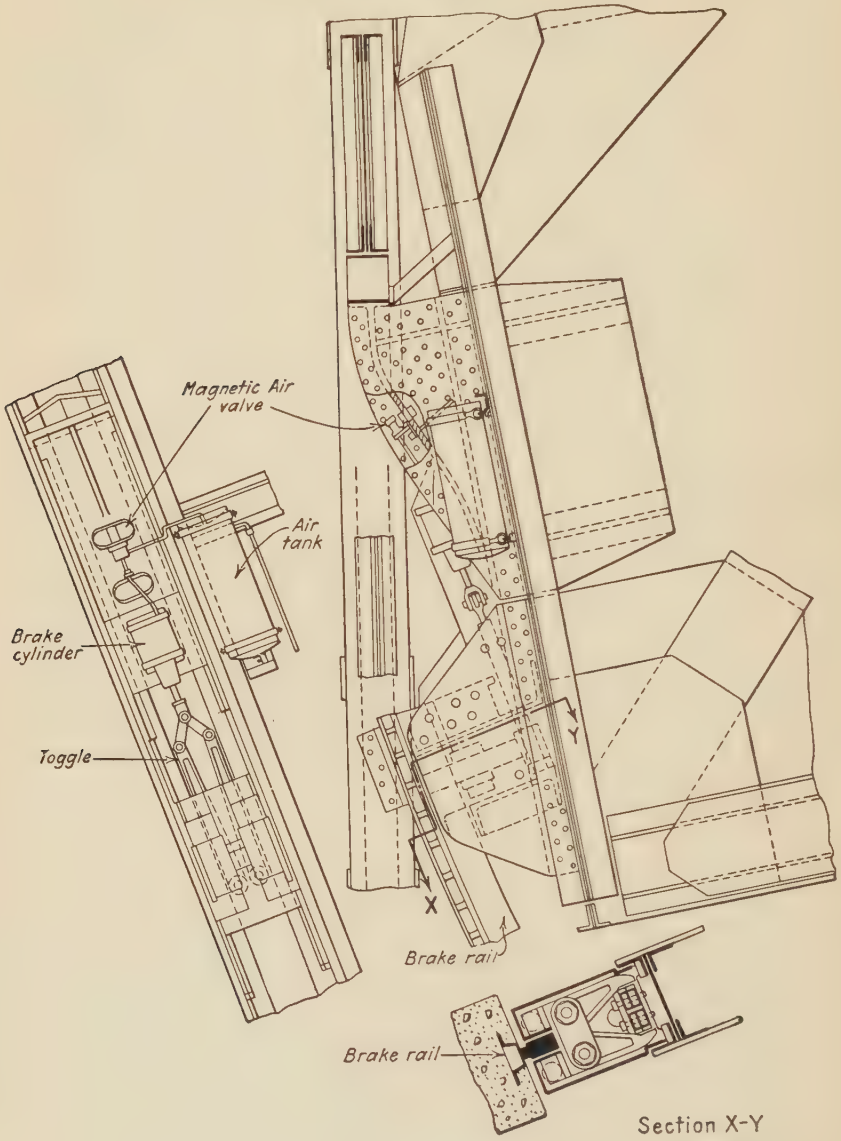
unsymmetrical live loads. They are similar in principle to that shown in Fig. 3W, which was used on another bridge.

Safety devices.—On account of the unusually heavy traffic, special safety devices were provided. Assume that the bridge is closed and carrying traffic. The center and rear locks are closed, the roadway and sidewalk gates are open, the brakes may or may not be set; and current is only available for the river signals and the service and signal lights.

The center locks are under the control of the north operator. He gives the first signal. The operators and gatemen exchange signals, to call all of the men to their posts. Upon receiving return signals, the north operator again signals and the operators and gatemen each ring large hand-operated bells and start the warning bells and flasher

MICHIGAN BOULEVARD BRIDGE.

EMERGENCY BRAKE.



Eng. News-Rec.

FIG. 3U.

MICHIGAN BOULEVARD BRIDGE.
REAR LOCK.

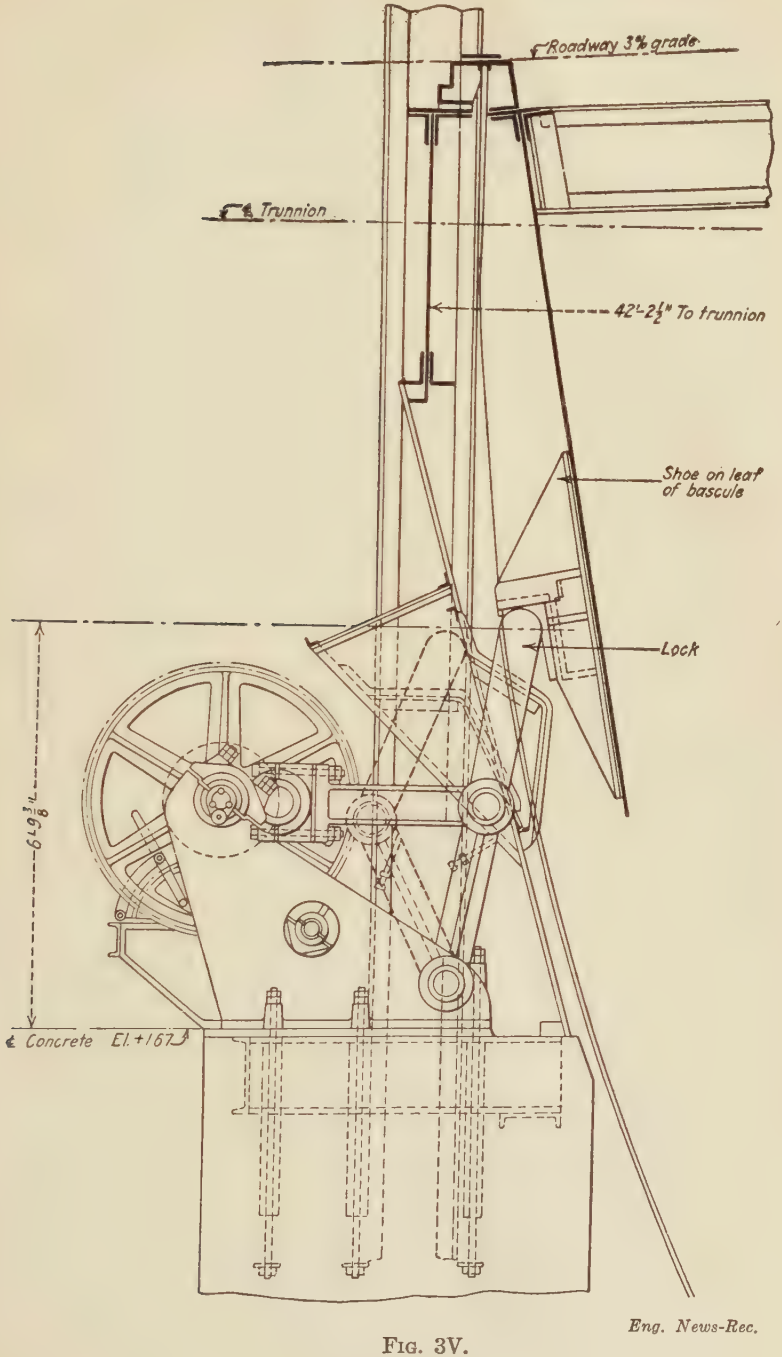


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FIG. 3V.

THREE BRIDGES IN SEATTLE.
CENTER SHEARLOCK.

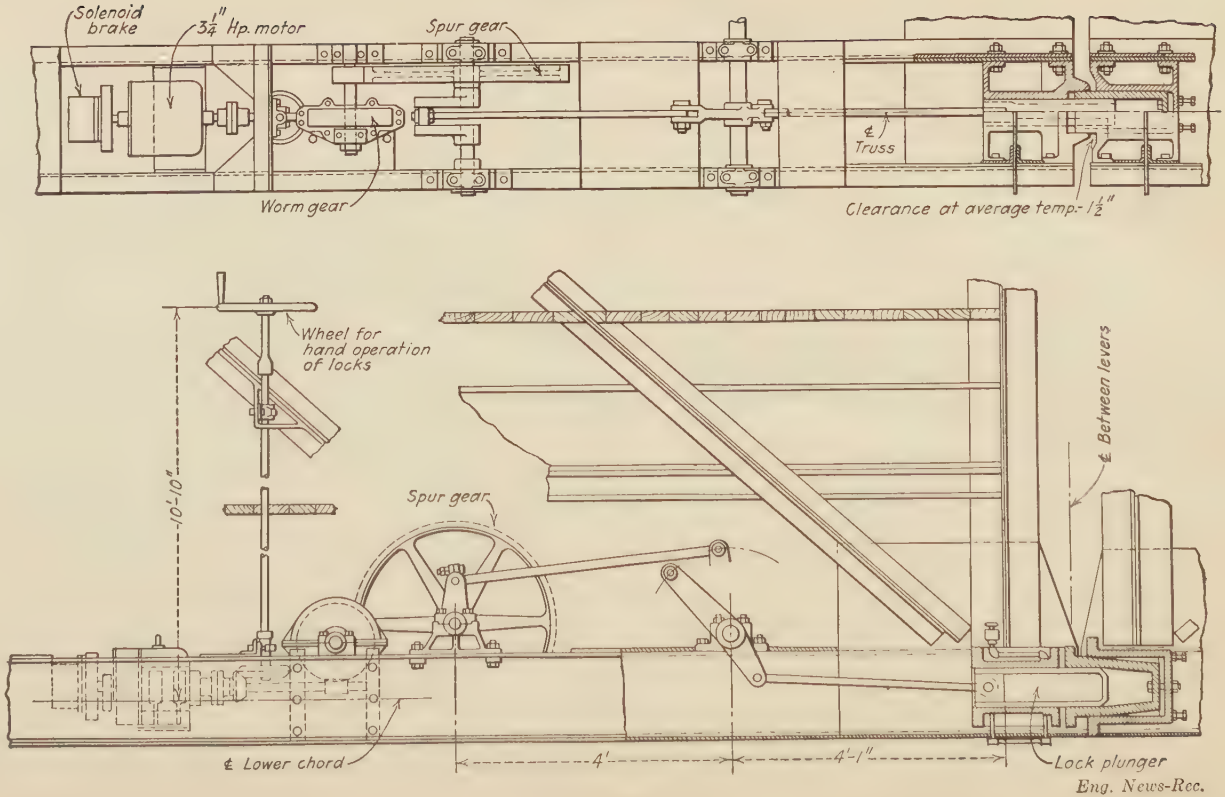

FIG. 3W.

motors, by means of snap switches. When these switches are closed, current is available for the operation of the gates, closing that section of the roadway at which the signals are displayed. When the gates are fully closed, current is made available for operating the flexible barrier at the part of the roadway closed by the gates. The barrier is then lowered. Next, the sidewalk gates are closed. When traffic has cleared the bridge, the exit gates and barriers are closed in the same order. When all of the barriers are down, the interlocking switches are closed automatically. The center locks can be withdrawn during the time required to clear the bridge of traffic, but can only be closed after the rear locks are seated. When the center locks are fully opened, an interlocking contact is closed and the rear locks on the north and south ends can be opened independently. When they are fully open, current is made available to operate the main motors, provided that the clutches and switches are in proper relation for the two motors for which the clutches and switches are set. If they are not properly set, no control current is available. When they are correctly set, the operator pulls the reverse lever, on the main motor master-controller, in the direction that the bridge is to move. This unlocks the power drum handle, making current available for opening the leaves.

This bridge was opened for traffic in May, 1920.

10. Strauss bascule bridges.—Bascule drawbridges are used at sites requiring a wide range of designs, and the operating machinery varies with the type of the superstructure. Space will admit of giving only the latest arrangements developed by the Strauss Bascule Bridge Company.

Operating machinery.—The Wishkah River bridge at Aberdeen, Washington, is a single-leaf, through-truss, heel-trunnion bascule for a 125-ft. clear channel, carrying a 27-ft. city street with two sidewalks 5 ft. wide. Figure 3X gives the arrangement of the mechanism in the machinery house. The leaf is operated by two 35-h.p. motors, each equipped with a full-torque solenoid brake. The two armatures are coupled, and a pinion midway between the motors engages the spur gear of an equalizer in a transverse shaft which drives the two main pinions through a pair of intermediate shafts. The pinions on the equalizer shaft are on cantilever ends, and the gears on the intermediate shafts are between the bearings. In many bridges the main pinions are on cantilever ends of their shafts, but in this case they are between the bearings, with the large driving spur gears on cantilever ends.

The equalizer shaft has two emergency solenoid brakes. The shaft bearings are all bolted to structural-steel supports, and union bearings are not used.

A 40-h.p. gasoline engine, running at 1000 r.p.m., supplies power when electric current is not available. The engine, radiator, clutch, and reversing gears are mounted on a unit-frame. The auxiliary engine

Wishkah River Bridge.
Operating Machinery.

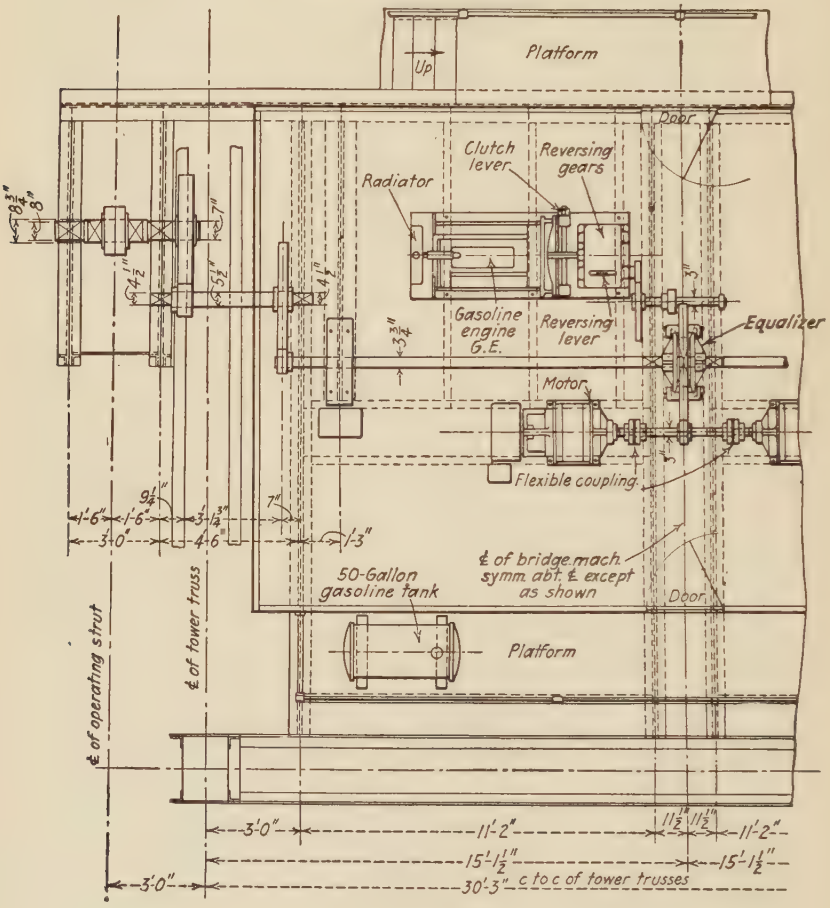

Fig. 3X.

delivers power through a pinion arranged to slip along its shaft into engagement with the equalizer gear.

End locks.—When the leaf is closing, two automatic end locks, shown in Fig. 3Y, contact with their catches, as shown in Fig. No. 3 in Fig. 3Y. As the leaf moves downward, the latch takes the position indicated by Fig. No. 2. When closed, it slips under the catch (Fig.

WISHKAH RIVER BRIDGE.
END LOCK.

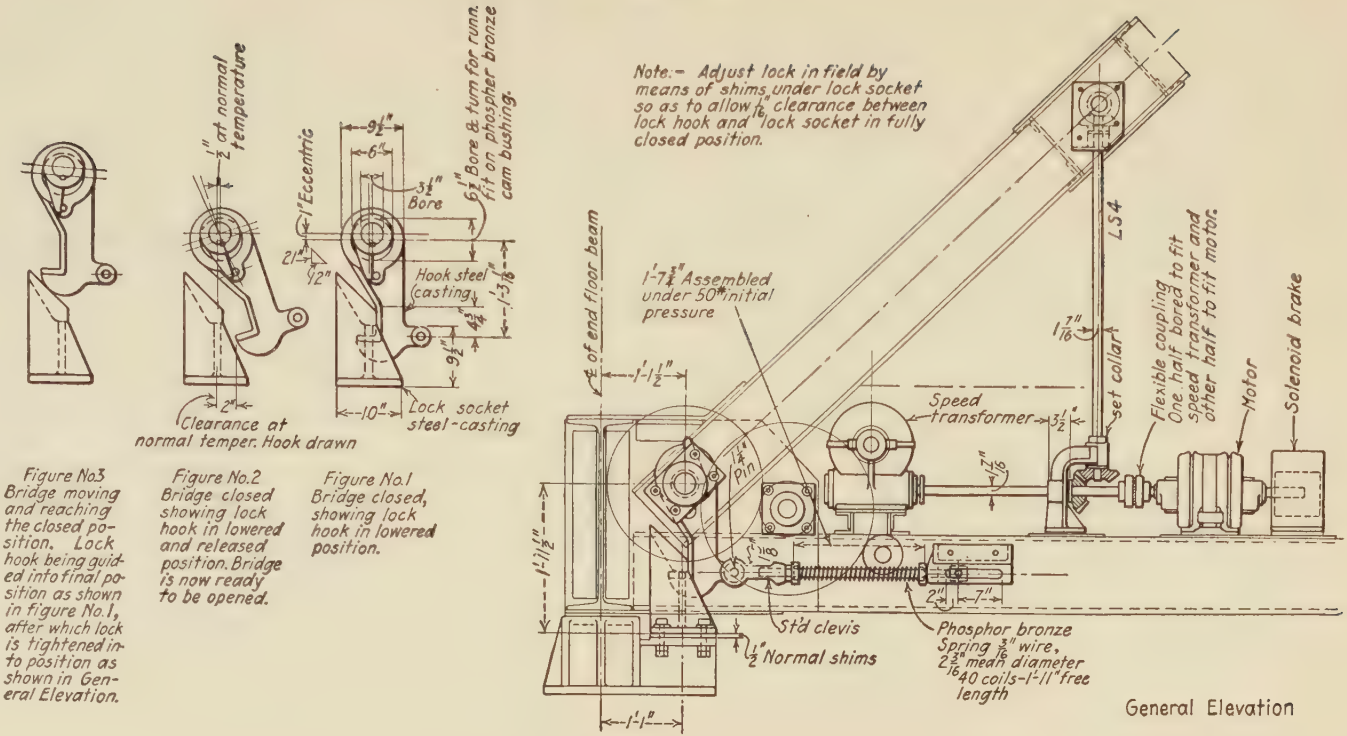

Figure No.3 Bridge moving and reaching the closed position. Lock hook being guided into final position as shown in figure No.1, after which lock is tightened into position as shown in General Elevation.

Figure No.2 Bridge closed showing lock hook in lowered and released position. Bridge is now ready to be opened.

Figure No.1 Bridge closed, showing lock hook in lowered position.

FIG. 3Y.

No. 1) and, finally, is drawn into locked position, as shown in the General Elevation, by an eccentric on the shaft carrying the lock lever. These latches are operated by a 3.2-h.p. motor through a speed reducer, on the armature shaft, which drives a gear train on one side of the bridge. A similar train on the other side is driven by a pinion on the transverse shaft, having a pinion on its other end which engages a spur gear on the directly driven gear train. The end locks may also

WISHKAH RIVER BRIDGE.

PNEUMATIC BUFFER.

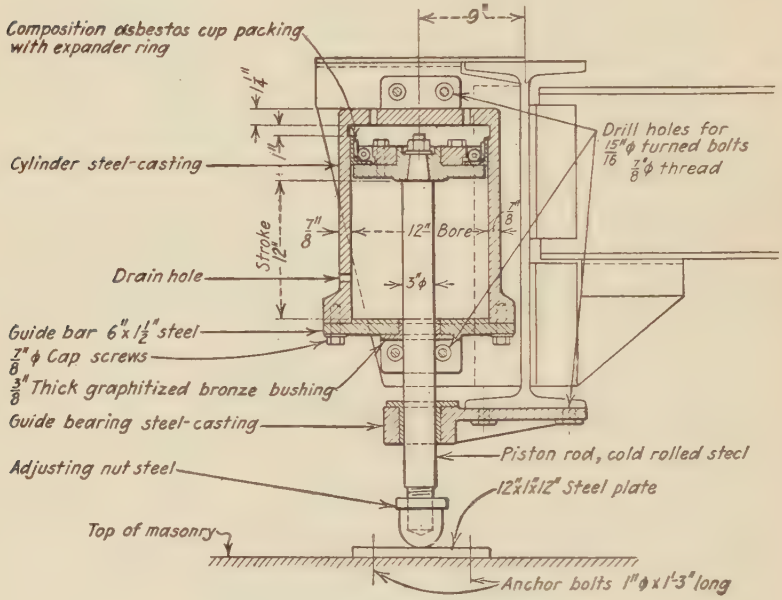

FIG. 3Z.

be operated by a hand winch, mounted on the end post of the leaf, through the vertical shaft *LS*4.

Pneumatic buffer.—A pneumatic buffer, Fig. 3Z, is mounted on the end floorbeam at the center line between the trusses. The cylinder is 12 ins. in diameter with a 12-in. stroke. This buffer enables the operator to seat the bridge gently when closing.

Center lock.—When two bascule leaves are used, they must be connected by a center shear lock to make them deflect together under unsymmetrical live loads. Fig. 3AA shows a patented type used for this purpose. A toggle joint, driven by a motor geared to an eccentric,

RIVER THAMES, KENT CO., ONTARIO.
CENTER SHEAR LOCK.

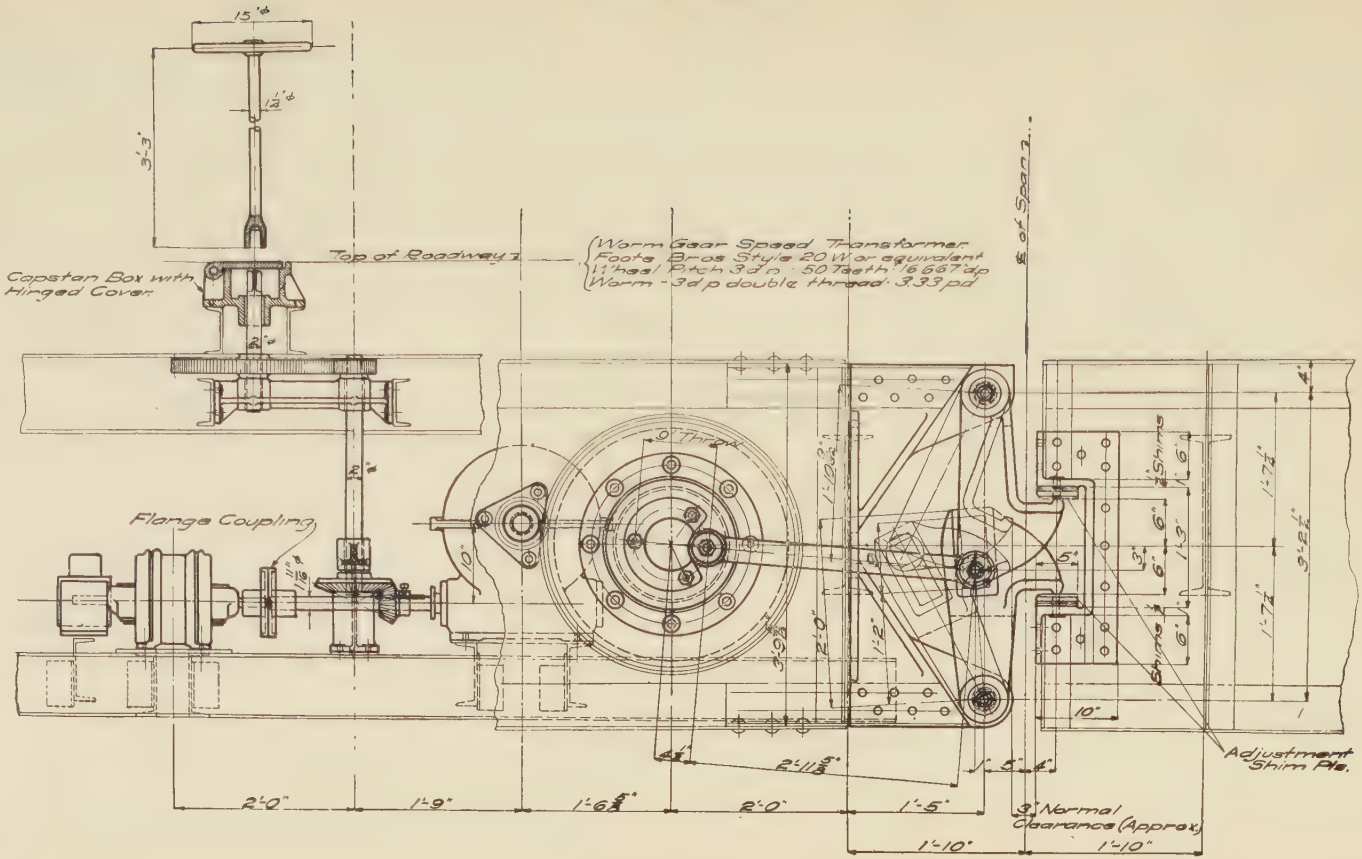

FIG. 3AA.

ST. CHARLES AIR LINE BRIDGE.
TRUNNIONS AND BEARINGS.

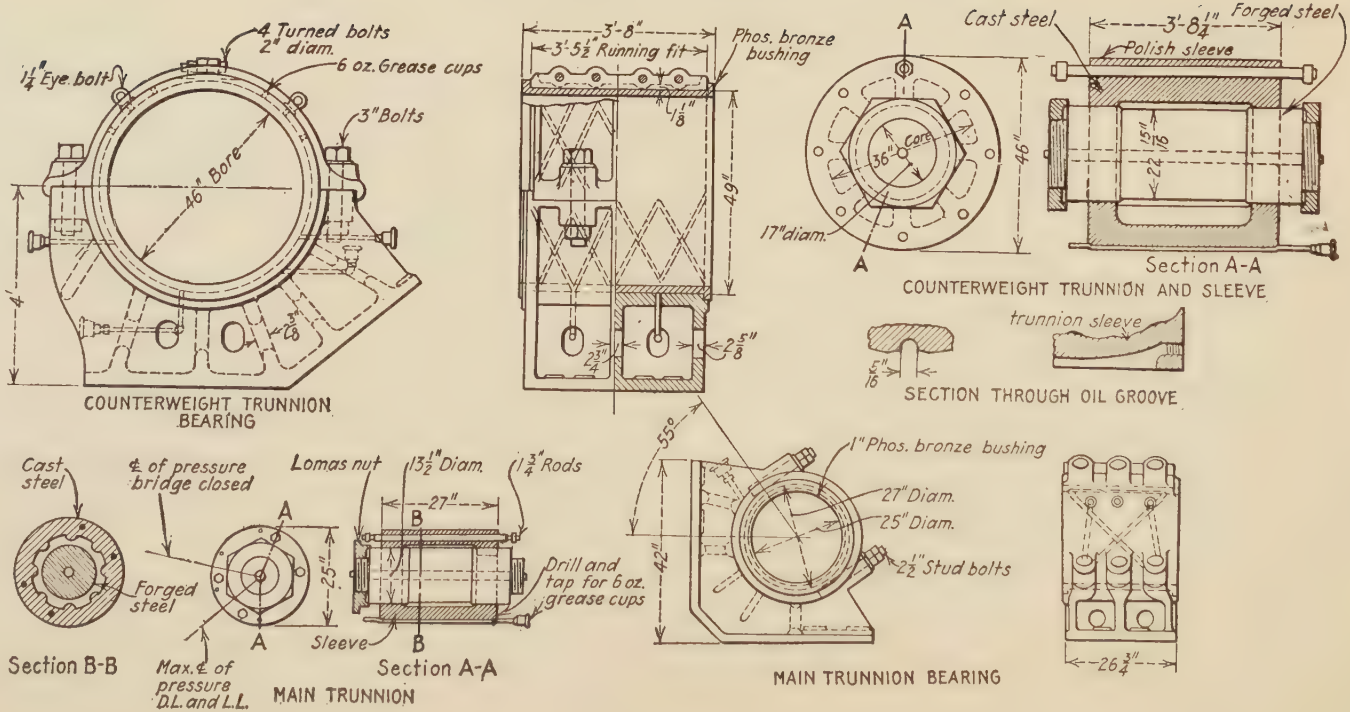

FIG. 3BB.

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on one leaf, has jaws engaging seats on the other leaf. These jaws are driven to a firm bearing on their seats, so that the leaves are positively locked.

Trunnions and bearings.—The design of the trunnions and bearings of bascule bridges requires careful attention. The pressures are heavy, the rotation is slow, and the trunnions only turn through a part of one revolution. Thorough provision for good lubrication is important. Fig. 3BB shows the main and counterweight trunnions and bearings

ST. CHARLES AIR LINE BRIDGE.

EMERGENCY BRAKE.

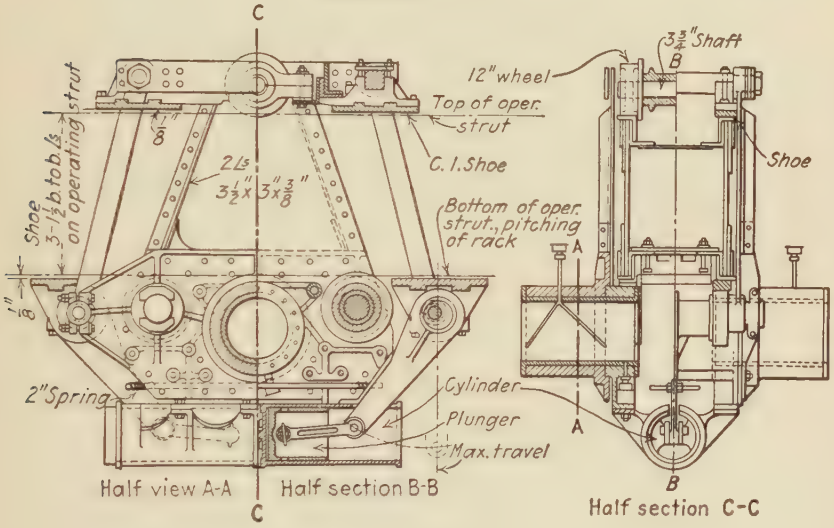


FIG. 3CC.

of the St. Charles Air Line bridge, in Chicago. The general plan of this bridge is shown in Volume I, Chapter IV, Art. 10, Fig. 4X. The counterweight trunnion is the largest yet built. The phosphor-bronze bushings were finished to turn within the bearings, as well as around the trunnions. Double lubrication was thus provided and the turning surface is selective, depending on the lubrication of the surfaces. It is to be noted that ample grease grooves, pipes, and cups were used.

The trunnion, turning within the bearing, is a large cast-steel sleeve, proportioned with respect to the permissible bearing pressure when moving, while the pin through the sleeve makes the attachment to the structural parts, and is proportioned for strength. This gives a compact detail that has been satisfactory.

PISCATAQUA RIVER BRIDGE.

OPERATING MACHINERY.

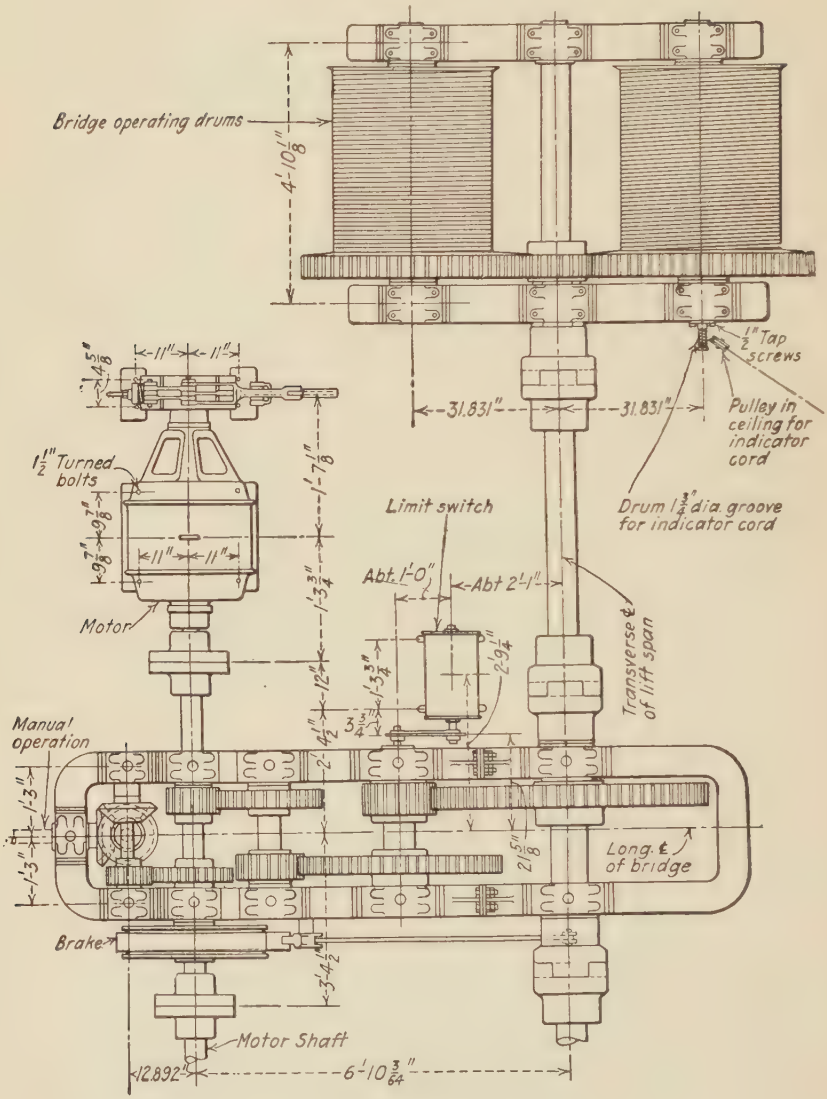


FIG. 3DD.

Emergency brakes.—Figure 3CC is a general drawing of a type of pneumatic emergency brake advocated and used by the Strauss Bascule Bridge Company, on heavy spans. It acts directly upon the top and bottom surfaces of the operating strut and is mounted on the main-pinion shaft. These brakes act on each strut, independently of the operating machinery, and are designed to hold the bridge leaf against the maximum wind pressure in any position.

11. Vertical-lift bridges.—Figure 3DD shows the arrangement of the operating machinery of the Piscataqua River vertical-lift span, at Portsmouth, N. H., designed by Dr. J. A. L. Waddell. A brief description of this bridge is given in Volume I, Chapter V, Art. 3, and a general view, with the draw span raised, is shown in Fig. 5J.

The operating ropes from the ends of the span are wound around two drums at each side of the bridge, only those on one side being shown in the figure. The spur gears at one end of the drums are driven by a master pinion, meshing with both. The parts of the shaft carrying these pinions are connected by self-centering jaw couplings, and the middle section runs in two bearings on a unit-frame, on which the remainder of the gear train is mounted. This unit was completely fitted and adjusted at the shop and shipped to the site assembled.

The motor armature has a full-torque solenoid brake. A manually operated band brake is also mounted on the same shaft. A limit switch, geared to one of the intermediate shafts, breaks the circuit near the fully closed and fully open positions of the span. Manual emergency operation is provided for, the bevel gears and driving shaft being mounted on the unit-frame. This mechanism is an excellent illustration of the advantages of mounting the gearing as a compact unit.

CHAPTER IV

MATERIALS OF CONSTRUCTION

1. Materials in general.—The subject of the materials of construction might be expanded almost without limit if a discussion of all their chemical and physical properties were attempted. The subject is one that would require more space than this entire volume for adequate treatment.

The American Society for Testing Materials, organized in 1898, includes in its membership engineers, chemists, metallurgists, inspectors, manufacturers, and users of the materials of construction. Its purpose is announced in its Charter. "The corporation is formed for the Promotion of Knowledge of the Materials of Engineering, and the Standardization of Specifications and the Methods of Testing." The Society has attained such a high standard of accomplishment that it is now a recognized authority.

The requirements for most of the materials in a movable bridge are given in the Standard Specifications of the Society, and only those pertaining to parts having relative motion will be considered in detail.

2. Specifications.—The following materials should be made in accordance with the current revisions of the Specifications of the American Society for Testing Materials: [1]

Materials.	A.S.T.M. serial designation.
Structural steel	A 7–24.
Carbon-steel forgings	A 18–21, Class C.
Heat-treated carbon-steel forgings	A 18–21, Class G.
Steel castings	A 27–24, Class B, Medium.
Iron castings, except counterweights	A 48–18, Medium.
Boiler steel	A 70–24.
Cold-finished steel	A 54–24.

The requirements for the hard, or case-hardened, steel suitable for the center discs of swing bridges and for similar purposes are given in the Specifications in Appendix A, Art. 802.

[1] Copies of these specifications, in pamphlet form, may be obtained from the American Society for Testing Materials, 1315 Spruce Street, Philadelphia, Pennsylvania.

3. Bearing metals. General.—The ideal condition for all types of bearings, except roller and ball bearings, is that the rotating member be wholly borne by an unbroken film of oil. This can only be realized when there is a nice adjustment of rubbing velocity, pressure, temperature, and the amount of lubricant introduced into the bearing. In the relatively slow-moving, heavy machinery of movable bridges, which is also subject to high pressures, the oil film will seldom be unbroken.

Heating is not a serious consideration in movable bridges, for they must open and close quickly and there is ample time for cooling between consecutive bridge movements.

The most important condition is that all bearings be designed so that a partial film of lubricant is maintained at all times and that the bearing metals be selected so that there is no danger of seizing and abrasion.

Good bearing metals should have the following characteristics:

(1) The metal must be strong enough to carry its load.

(2) It must not cut or abrade the journal.

(3) It should show a small coefficient of friction.

(4) It should not heat readily.

(5) It should give much service with little loss of metal.

(6) It should resist corrosion by the lubricant.

(7) It should work well in the foundry and machine shop.

(8) Most important of all, it should be unlike the metal of the journal.

Among ordinary metals there are but two exceptions to the last requirement: Hardened steel will run well in hardened steel, and cast iron in cast iron, whenever the workmanship is good, the alignment accurate, and the lubrication adequate.

There are two somewhat antagonistic qualities required in a good bearing metal—hardness and plasticity. The structure should consist of a large number of hard points embedded in a plastic matrix. The hard particles should be softer than the metal in the journal, so that the latter will not be cut and scored. The matrix should be plastic enough to yield and permit the hard bearing points to adjust themselves to the journal. As the bearing wears in, the number of hard points of the bearing metal in contact with the journal is increased until the load is properly distributed.

Metallurgists have given much study to the development of alloys having these characteristics.

4. Steel. *Soft steel.*—Steel with carbon up to about 0.25 will not run with similar rolled or cast steel. The journal and bearing will

quickly cut or heat and seize. When run in bronze containing copper and tin only, the soft-steel journal picks up particles of copper, causing a rapid increase in friction. Bronzes containing lead are better than those without, and phosphorus should not exceed about 0.20 per cent. Soft steel will run in soft gray cast iron under ordinary pressures and rubbing velocities; also with various good white bearing metals, commonly known as babbitts. Soft steel should be avoided in the machinery of movable bridges.

Medium steel.—Medium steel, with carbon from 0.25 to 0.35, or preferably from 0.35 to 0.45, is much better for journals than soft steel. When the carbon is 0.40, the steel has the greatest capacity to resist abrasion. Medium-steel journals should never be allowed to run in similar rolled or cast steel. When it is necessary for this combination to be used, the journal should be heat-treated. Medium steel will run well in gray cast iron, bronze with a lead content, and good white metals. It can be used with phosphor-bronzes when the phosphorus is below 0.20.

Hard steel.—Hard steels, like tool steel, heat-treated medium steel, and case-hardened soft steel, can be run in bearings of gray cast iron; bronzes, with or without lead; phosphor-bronze, even when the phosphorus is above 1.00; and some aluminum alloys. It is seldom used in movable bridges as a bearing metal, except for the center discs of swing bridges and sometimes for steps of medium-steel shafts.

Rollers, balls, and the raceways of roller and ball bearings should always be of very hard steel.

Alloy steels.—With the same carbon content, alloy steels wear less than plain carbon steels. They may be used with almost any bearing metal, even the hard phosphor-bronzes. Chromium adds rapidly to hardness and wearing qualities. The best rollers, balls, and raceways for roller and ball bearings are alloys of steel with nickel and chromium. The nickel adds strength and the chromium increases the hardness. This alloy is very sensitive to heat-treatment.

5. Cast iron.—Soft gray cast iron is one of the best bearing metals when the workmanship is good and the alignment is accurate. It does not readily adapt itself to full contact with the journal when out of alignment or roughly finished. It will run well with soft, medium, and hard steel, wrought iron, and even with itself. It has several unusual qualities.

While in service, it takes a glaze which resists cutting. It is porous enough to hold some oil when the supply is cut off. Its granular structure, containing graphite, reduces wear and it runs cool when lubricated. It can be used when highly heated, as for cylinders and

pistons of engines. It is one of the very best metals for piston rings. But cast iron will not run if it is dry. It will then be gradually worn away and reduced to powder.

Chilled cast iron may be used in place of hard or case-hardened steel, and gives excellent service.

Mr. R. K. Le Blond made an eight-year test on four lathe spindles. The combinations of metals are given below.

No. 1. Hardened steel in cast-iron bearings.

No. 2. Steel in babbitted bearings.

No. 3. Hardened steel in bronze bearings.

No. 4. Steel in bronze bearings.

The journals were of 0.60 carbon steel made by the crucible process. The bronze was the bearing bronze specified by the Pennsylvania Railroad Company, containing lead. The merit of the bearings in service was in the order stated above.

William Sellers & Co., and others, have used steel spindles running in cast-iron bearings in machine tools, with uniform success.

6. Bronze.—Bronze linings are extensively used in steel bearings. Bronze is much tougher than the white bearing metals and is better able to withstand high stresses with impact. It is fundamentally an alloy of copper and tin. In this series of alloys the bearing points are $SnCu_3$, or about 9 per cent tin and 91 per cent copper. This constituent is hard and resists wear. The matrix is a eutectic of tin and copper, somewhat softer but still relatively hard. These alloys do not fulfill the definition of a bearing metal as one with many hard bearing particles in a plastic matrix. Both constituent substances are hard enough to score and cut soft steel and wrought iron.

Phosphorus is commonly used as a flux to reduce the oxides in the metal and to prevent gas holes in the castings. When an excess of phosphorus is left in the alloy, it forms a phosphide of copper which is nearly as hard as tool steel. Bronzes containing more than 0.20 per cent. of phosphorus are not suitable for bearing linings except with hard steels.

Lead is added to the alloy when it is to be used with relatively soft steels. The lead does not alloy with the copper and tin but separates out in small globules which should be uniformly distributed through the mass. These soft particles tend to make the metal more plastic and more suitable for ordinary machinery bearings. The lead also reduces the wear in service.

Zinc, up to about 2 per cent., is sometimes added to increase the soundness of the castings. Larger amounts tend to reduce the tensile strength and cause brittleness. When zinc is used in place of tin the

alloy becomes a brass and is not suitable for bearing linings of heavy machinery. It should not be used in movable-bridge machinery.

So-called manganese bronze, which is really a brass of about copper 59, zinc 40, and iron 1 per cent., is not a good bearing metal. The friction is high and the alloy heats and abrades rapidly. It should never be used for movable-bridge bearings. It is well adapted for marine work where high tensile strength is required, and it is only slightly corroded by the action of sea water.

Selection of bronzes.—Four grades of bronze, which will cover the requirements of movable-bridge design, have been selected.

Grade A is for use under pressures exceeding 2000 lbs. per square inch, and is suitable for the center discs of center-bearing swing-bridges. It is too brittle for bearing linings.

Grade B is for use in low-speed bearings under pressures of 2000 lbs. per square inch or less, such as the trunnions of bascule and vertical-lift bridges.

Grade C is for general use in ordinary machinery bearings.

Grade D is suitable for gears, worm-wheel rims, nuts transmitting motion, and in similar parts subject to other stresses than compression.

The chemical and physical properties desired are given in the specifications in Appendix A, Arts. 803 to 808.

7. White bearing metals.—Many alloys of tin, lead, antimony, and copper are used as bearing metals. They are nearly white in color and are quite commonly called babbitts, although few have the composition discovered in 1839 by Isaac Babbitt, the inventor of the recessed lined box. The best of these alloys are excellent bearing metals for a wide range of pressures and rubbing velocities. Unlike bronze, they can be melted in an ordinary ladle, readily poured, and easily anchored in bearings. The babbitting process is the easiest method for making an accurately aligned bearing.

The constituent metals tend to accentuate different qualities in the alloys.

Copper adds toughness and increases plasticity.

Antimony and tin increase hardness and compressive strength but reduce plasticity.

Lead increases softness and plasticity but reduces compressive strength and increases friction.

There are two principal types of white alloys: those that are mainly tin, with compounds of tin and antimony and tin and copper; and those that are mainly lead with tin and compounds of tin and antimony.

8. Copper-antimony-tin alloys.—This group of alloys includes the original babbitt metal. The tin and antimony unite to form SnSb,

which crystallizes in hard but malleable cubes. Tin and copper form $SnCu_3$, which crystallizes in very hard, brittle needles. These two compounds are the bearing points and are embedded in a matrix of tin with some antimony and copper. The matrix is plastic and allows the hard points to adjust themselves quickly to the journal. The alloy used by Babbitt was probably about:

Tin	89.3
Antimony	7.1
Copper	3.6
	100.0

Charpy states that the maximum compressive strength without brittleness is obtained when the mixture is:

Tin	83.3
Antimony	11.1
Copper	5.6
	100.0

The following are quite commonly used in the United States:

	Common	Society of Automotive Engineers
Tin	86	84
Antimony	8	9
Copper	6	7
	100	100

As the melting points of the metals are about 450° F. for tin, 1166° F. for antimony, and 1983° F. for copper, it is very difficult to obtain the specified proportions, on account of the vaporization of the tin and antimony. In a series of tests made by the American Bridge Company, the closest approximation attained was:

Tin	86.3
Antimony	9.0
Copper	4.7
	100.0

This alloy had an elastic limit of 3245 lbs. per square inch, when tested as cylinders one square inch in sectional area and one inch high.

Genuine babbitt metal is the most expensive of this series of alloys but is a very good bearing lining for relatively high pressures and ordinary velocities at the rubbing surface.

9. Antimony-lead alloys.—In the antimony-lead alloy, the eutectic contains 14 per cent of antimony. More than 13 per cent of antimony should be used, for the 87–13 alloy has a relatively high coefficient of friction. When 25 per cent or more of antimony is used the metal is too brittle. When about 18 to 20 per cent is used the antimony crystallizes in well-defined cubes, which serve as the hard bearing points, and it is embedded in a softer matrix of the eutectic of antimony and lead. This alloy has excellent anti-friction qualities and wears well when speeds are high and pressures are low. It is too soft for movable-bridge work, but has the unusual property of cooling without shrinkage. It is excellent for pouring under a pedestal when it is desired to obtain a full bearing between static parts. The pressure in such a case should not exceed 1250 lbs. per square inch.

10. Antimony-tin-lead alloys.—These alloys are much cheaper than the antimony-copper-tin series. They are commonly used in the cheaper grades of machinery. They are not hard enough for general use in movable-bridge mechanism, but would give excellent service in line-shaft bearings where velocities are relatively high and pressures are low.

During 1908, the Otis Elevator Company made a series of tests of various bearing metals, including one that was practically a genuine babbitt. The journal with which the tests were made was 7 ins. in diameter and $5\frac{3}{4}$ ins. long.

The compositions of two of the best alloys tested were as follows:

	Lead base	Tin base
Antimony	9	8.4
Tin	44	89.5
Lead	46	.....
Copper	1	2.1
	100	100.0

A summary of the results of the tests is given in Table 4A.

Within the limits of the tests, there is very little difference in the efficiency of the two alloys. The lead-base alloy is hardened with copper and would give good service if the bearing pressures were not too high. The tin-base alloy would serve for even greater pressures than those of the tests. The cost per unit of weight of the alloy with a lead base is about 58 per cent. of that of the one with a tin base. The unit weight of the lead-base alloy is 118 per cent. of that of the tin-base metal. Thus the net cost per cubic inch of the lead alloy would be about 69 per cent. of that of the tin alloy. The relative weights of bear-

ing metals should always be taken into account when making comparisons of the costs when applied to the bearings, for it is the cost per unit of volume that determines the cost of the alloy as it is used in the bearing.

TABLE 4A

Pressure per square inch	112		270		300		410		507		
Rubbing velocity, feet per minute	1164		751		431		180		119		
Duration of test, hours	3		3		3		3		10		
Bearing Metal	Temp. Rise	Av. Amps	Temp. Rise	Av. Amps	Temp. Rise	Av. Amps	Temp. Rise	Av. Amps.	Temp. Rise	Av. Amps.	Av. Temp. Rise
Lead-base	60	29	54	21	32	17	20	6.3	24	4.95	38
Tin-base	79	29	51	24	38	16.9	17	4.3	16	5.1	40.2

11. Antimony-zinc-tin alloy. An alloy of antimony, zinc, and tin, which was developed at the Pencoyd Iron Works, before 1892, has unusual qualities as a bearing metal. The proportions by weight are as follows:

Antimony	4.75 per cent.
Zinc	31.75 per cent.
Tin	63.50 per cent.
	100.00 per cent.

The specific gravity of the alloy is 6.95 and its weight per cubic foot is 434.7 lbs., which corresponds to 0.2516 lb. per cubic inch. It is one of the lightest of the good bearing metals, and its unit cost is intermediate between that of the genuine babbitts and that of the white metals with a lead base. Its elastic limit in compression is from 3500 to 3800 lbs. per square inch.

A test was made in connection with several other well-known bearing alloys. The Pencoyd metal was stronger, ran cooler, developed less friction, and weighed less per cubic inch than any of the other alloys tested. Its cost per pound was greater than that of most of the

other metals, but when its superior qualities were taken into account it was the most economical of all.

In a report on this alloy, Mr. James L. Bernard, Mechanical Engineer of the American Bridge Company, states:

> We have used it on light loads and high speeds; on medium loads and medium speeds; and on heavy loads and slow speeds. In other words, it has been used in the roll neck bearings of the blooming mill, and all other places down to bushings for high-speed shafts where the loads are comparatively light and the speeds high. It has always stood up better than any other metal that we have ever used.

In rolling-mill practice this metal is used where the rubbing velocity is from 350 to 600 ft. per minute, and under pressures of from 100 to 2000 lbs. per square inch of projected area of the bearing; also in pinion housings carrying from 30 to 50 lbs. per square inch. A pressure of 2000 lbs. per square inch should not be exceeded.

The melting points of the component metals are 1166° F. for antimony, 786° F. for zinc, and 450° F. for tin. The highest melting point is much less than that of copper, which is used in genuine babbitt, and for this reason it is easier to obtain the correct proportions in the alloy.

The antimony and zinc should be melted separately. When both are melted, the antimony should be added to the zinc and the whole stirred with an iron ladle. The tin is then added cold, and, when melted, the mixture should be stirred thoroughly. Before pouring, a little sal-ammoniac should be thrown on the surface to collect the impurities.

It is the opinion of the author that this alloy is the best bearing metal for general use in the operating machinery of movable bridges. It is readily mixed, easily melted, pours well, has little shrinkage, and machines to a smooth surface. On account of these superior qualities, it is the only white bearing metal mentioned in the Specifications in Appendix A. Refer to Art. 809.

12. The physical properties and strengths of metals and alloys.— Some of the physical properties and strengths of the metals and alloys commonly used in the machinery of movable bridges are shown in Table 4B. These data have been compiled from many sources deemed to be reliable.

Different authorities give different data, even for such a simple constant as the weight per cubic foot. These differences are greater in the case of the more subtle constants, like Brinell and scleroscope hardness numbers. The physical constants of a given metal or alloy

may vary appreciably, even when great care is taken in manufacture. Some metals are very sensitive to heat-treatment, and it is unusual to find full data concerning this important condition in connection with the recorded results of tests.

The values given in the table are fair averages of the data investigated, but must not be interpreted as being exact for all conditions of any particular metal or alloy.

TABLE

PHYSICAL PROPERTIES AND

Compiled from

1	2	3	4	5
Metal or Alloy	Approximate Proportions of Alloys	Weight per Cubic Foot, Pounds	Weight per Cubic Inch, Pounds	Melting Point, Degrees C.
Aluminum, commercial		168	0.0974	659
cast				
Antimony		421	0.2436	630
Babbitt, Pencoyd	Sn 63.5, Zn 31.75, Sb 4.75.	435	0.2517	.....
tin base	Sn 88.9, Sb 7.4, Cu 3.7. . .	465	0.2691	238
lead base	Pb 48, Sn 40, Sb 10, Cu 2.	550	0.3183	184
antimonial lead	Pb 87, Sb 13			
Brass, cast	Cu 70, Zn 30±	525	0.3038	925±
rolled		534	0.3090	.....
manganese	Cu 58, Zn 40, Sn 1, Fe 1. .	524	0.3033	
Bronze, gun metal	Cu 90, Sn 10	555	0.3211	912±
aluminum	Cu 89, Al 10, Fe 1	468	0.2708	.....
aluminum	Cu 88, Al 9, Fe 3	468	0.2708	.....
Lumen	Zn 86, Cu 10, Al 4	432	0.2500	.....
Phosphor, Grade A	Cu 80, Sn 20	542	0.3137	.....
Grade B	Cu 80.7, Sn 18.5, P 0.67. .	542	0.3137	.....
Grade C	Cu 80, Sn 10, Pb 10	555	0.3211	900±
Grade D	Cu 88, Sn 10, Zn 2	542	0.3137	.....
Copper, annealed		553	0.3200	1083
cold drawn				
Iron, pure		489	0.2830	1520
cast, common		450	0.2604	1240
gray		445	0.2575	1240
wrought		485	0.2807	1520
wire, annealed				
Lead		709	0.4103	327
wire				
Nickel		551	0.3189	1450
Steel, structural, soft		489.5	0.2833	1510
medium		489.5	0.2833	.....
hard		489.5	0.2833	.....
nickel				
silicon				
spring, tempered				
tool, carbon				
wire, unannealed				
Tin		455	0.2633	232
Zinc		444	0.2569	419

4B

STRENGTHS OF METALS AND ALLOYS

Many Authorities

6 Melting Point, Degrees F.	7 Coefficient of Linear Expansion ×10,000, per Degree F.	8 Yield Point in Tension, Kips	9 Ultimate Strength in Tension, Kips	10 Yield Point in Compression, Kips	11 Ultimate Strength in Compression, Kips	12 Modulus of Elasticity in Tension ÷ 100,000	13 Brinell Hardness Number	14 Shore Scleroscope Number	15 Poisson's Ratio
1218	0.1280	12–25	18–40	.....	.....	10.4	25	4–6	0.130
.....	.....	8.5	12–15	3.5	12±				
1166	0.0587	.....	1						
.....	.....	.....	.....	3.5–3.8	.....	.....	25	10	
460	0.1207	6	10	3	14	.....	26	10	
363	.....	.....	7.5	.....	.....	.....	21		
.....	0.1503	5	8	.....	9	.....	17		
1697±	0.1042	10–11	28–32	6	30	9.0	50–60	7–35	0.350
.....	0.1072	.....	45–70	.....	.....	14.0	50–150	10–15	
.....	.....	30–40	65–75	22–26	.....	13.0	90–110	16–21	
1673±	0.1010	18–20	30–37	13–15	.....	10.0	65–70		
.....	.....	20–26	65–75	16–19	.....	.....	92–96		
.....	.....	35–45	65–80	14–18	.....	.....	95–120		
.....	.....	None	32–36	25	75	12.0	100–115		
.....	.....	18–21	30–33	24–28	100	13.0	130–143		
.....	0.0980	19	38	32	100	.....	140	41	
1652±	0.1050	11	14	12–16	60	.....	60–70	20–25	
.....	0.0907	15–20	30–40	13–16	64	.....	65–74	18–22	
1981	0.0926	.....	30–40	.....	40	15.0	30–40	6–8	0.340
.....	.....	.....	45–62	.....	.....	17.3	60–100	14–20	0.327
2768	0.0672	18	40	19	.....	.....	69	18	
2264	0.0590	6	15–18	.....	80	12.0			
2264	0.0590	.....	18–24	.....	.....	.....	.....	25–45	0.270
2768	0.0670	26	48	.....	.....	25.0	.....	16–18	0.278
.....	.....	27	60	.....	.....	25.0			
620	0.1590	0.3	1.6	.....	.....	0.7	.....	2–5	0.428
.....	.....	.....	2.4	.....	.....	1.0	.....	3.7	
2642	0.0722	20–30	50–60	.....	.....	30.0	80–100	20–30	
2750	0.0610	27.5–32.5	55–65	.....	.....	29.5	113–127	18–20	
.....	0.0670	30–35	60–70	.....	.....	29.5	120–134	19–21	0.303
.....	0.0700	32.5–37.5	65–75	.....	.....	29.5	127–141	20–23	
.....	.....	42.5–50	85–100	.....	.....	29.5	164–185	23–27	
.....	.....	40–47.5	80–95	.....	.....	29.0			
.....	0.0730	40–70	65–110	.....	.....	30.0	127–189	20–31	
.....	0.0730	70–80	100–125	.....	.....	29.0	175–210	Ann. 29–35 Temp. 90–105	
.....	0.0730	100–200	150–250	.....	.....	27.5			
450	0.1276	1.5–1.8	3.5–4.6	.....	6	4.6	.....	8–9	
786	0.1653	4	4–6	.....	18	13.0	.....	8–10	

CHAPTER V

MACHINERY RESISTANCES AND MOTOR TORQUES

1. General.—No sliding surface in machinery is perfectly smooth. Every finished metal surface consists of minute elevations and depressions, their height and depth depending upon the nature of the metals and the care with which they are finished. When a machine is used, some of the roughness is worn away, particularly when the metals in contact have been properly chosen, but there will always remain some residual irregularities. When the surfaces are sufficiently worn and polished to be in their best condition for intimate contact, adhesion also occurs. The laws of adhesion are little known, but enough is known to emphasize the importance of preventing this form of resistance in machinery.

Lubrication is applied to separate the surfaces and to cause their relative motion to take place in the film of oil which separates them, rather than between the surfaces themselves. The ideal condition exists when the entire load is carried by the oil film, or when the journal is oil-borne. In high-speed machinery with low bearing pressures and effective lubrication, this condition can be realized, but it is doubtful whether any journal in the heavy, slow-moving machinery of movable bridges is wholly oil-borne. The only exception might be the journals of motor-armature shafts, which run at high speed under light loads and are continuously lubricated by rings resting on the journals and dipping into a reservoir of oil.

Many experiments have been made which have developed quite definite laws concerning the friction of well-lubricated and oil-borne journals. Those made by Beauchamp Tower,[1] John Goodman,[2] Robert H. Thurston,[3] and Arthur M. Wellington[4] are classics, and

[1] Research Committee on Friction, *Inst. of Mech. Engrs.*, London.
First Report, 1883, pp. 632–652. Journals.
Second Report, 1885, pp. 58–63. Oil Pressure in Bearings.
Third Report, 1888, pp. 173–179. Collar Bearings.
Fourth Report, 1891, pp. 111–140. Pivots.

[2] *Proc. Inst. of C. E.*, London, Vol. LXXXV, p. 380.

[3] "Friction and Lost Work in Machinery and Millwork," John Wiley & Sons, New York, 1898.

[4] "The Economic Theory of Railway Location," Appendices A and B, John Wiley & Sons, New York, 1888.

should be studied by those who wish to become familiar with some of the more exhaustive investigations of friction.

Most of the tests on oil-borne journals give coefficients of friction much smaller than can be expected in movable-bridge machinery.

2. High-pressure bearings.—The bearings of movable-bridge machinery are high-pressure bearings with the journals running at relatively low velocities. The lubrication is not continous or uniform, and frequently the supply of oil is inadequate. Under these conditions the journals are not fully oil-borne, the film being partially broken down. The journals and bearings will then be in partial contact, so that the character of the metal in the journals and the bearing linings, as well as the lubrication, will determine the ultimate loads that can safely be carried.

Very few tests of the friction of movable-bridge machinery are available, and the probable frictional resistances must be determined by comparisons with the results obtained from tests of similar bearings, which usually have been made under much more favorable circumstances than exist in a movable bridge.

Mr. Louis Illmer, M. Am. Soc. M. E., has given the results of an investigation of high-pressure bearings, with partial oil films, in a paper presented to the American Society of Mechanical Engineers.[5]

His results are given in such form that they can be used to determine the approximate coefficients of friction of bearings of the types used in movable bridges. In what follows, his method of analysis will be freely used.

When a well-fitted, smooth journal is liberally supplied with a suitable lubricant, the coefficient of friction is determined by the unit working pressure and the temperature attained by the rubbing surfaces. In the case of an oil-borne bearing running at relatively high velocity, the coefficient of friction, f, may be expressed by the formula:

$$f_{\min} = \frac{2}{5\sqrt{PT}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this expression:

P = the pressure in pounds per square inch on the projected area of the bearing, P being less than the critical pressure, P_c, defined below;

$T = t_1 - 60$ = the virtual temperature for the more common grades of mineral lubricating oil, in degrees F.;

t_1 = the temperature of the rubbing surfaces, which should be limited to 200° F.

[5] High-Pressure-Bearing Research. *Trans. A. S. M. E.*, Vol. 46, p. 833, 1924.

In order to lift the journal in the bearing against a given unit pressure and to permit an oil film to be established, the journal must be run at a certain critical rubbing velocity. The velocity required for this purpose when the common grades of mineral engine oils are used has been found to be approximately that given by the equation:

$$P_c = 140\sqrt[3]{\frac{V}{T}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In this expression,

P_c = the critical pressure, in pounds per square inch of projected area, at or below which a perfect oil film may be maintained under a journal when running at a rubbing velocity of V linear feet per minute.

When the pressure is increased so that the oil film is partly broken, as is the case in most movable-bridge bearings, let the unit pressure be P and the coefficient of friction be f. When the bearing pressure is above the critical pressure, P_c, in addition to breaking the film, it increases the viscosity of the lubricant. Under these circumstances the supply may be inadequate, and the effect of roughness and adhesion is increased. It was found that the effect of these imperfect conditions could be approximately expressed by multiplying the coefficient, f_{min}, by the fourth root of $\frac{P}{P_c}$, as follows:

$$f = f_{\text{min}}\sqrt[4]{\frac{P}{P_c}} = \frac{2}{5\sqrt{P_c T}}\sqrt[4]{\frac{P}{P_c}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this equation, P is greater than P_c in pounds per square inch of projected area. The expression assumes that P remains reasonably constant. Satisfactory values of f under peak loads for brief time intervals may be expected when the maximum unit pressure does not exceed about $1\frac{1}{2}P$ to $2P$, provided that the maximum is below the abrasive-pressure limit.

When the value of P_c from (2) is substituted in (3), the equation becomes approximately

$$f = \frac{1}{100}\sqrt[4]{\frac{P}{VT}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For movable-bridge bearings with heavy mineral oils, the increase in viscosity may be compensated by taking $T = t_1 - 80$ in place of $t_1 - 60$. It may also be assumed that t_1 is constant and is 120° F.

Hence, T in (4) becomes $120 - 80 = 40°$ F. When this value of T is is substituted in (4), the equation becomes

$$f = \frac{1}{250}\sqrt[4]{\frac{P}{V}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For application to a variety of bearings running under different circumstances and of different types, Mr. Illmer introduces two constants, one of circumstance and the other of type, and Eq. (5) then becomes

$$f = \frac{C_1 C_2}{250}\sqrt[4]{\frac{P}{V}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This formula can be used to determine the approximate coefficients of friction of a variety of bearings when suitable values are assigned to C_1 and C_2. The paper recommends the values given in Tables 5A and 5B.

TABLE 5A

VALUES OF CIRCUMSTANCE CONSTANT C_1

C_1	Lubrication	Workmanship	Attendance	Location
1	Oil bath or flooded	High-grade	First-class	Clean and protected
2	Oil, free drop (constant feed)	Good	Fairly good	Favorable (ordinary conditions)
4	Oil cup or grease (intermittent feed)	Fair	Poor	Exposed to dirt or grit or other unfavorable conditions

TABLE 5B

VALUES OF TYPE CONSTANT C_2

1	Rotating journals, such as rigid bearings and crank pins.
1	Oscillating journals, such as rigid wrist pins and pintle blocks.
2	Rotating journals lacking ample rigidity, such as eccentrics and the like.
2	Rotating flat surfaces lubricated from center to circumference, such as annular step or pivot bearings, etc.
2–3	Sliding flat surfaces wiping over the guide ends, such as reciprocating crosshead shoes. Use 2 for relatively long and 3 for short guides.
3–4	Sliding or wiping flat surfaces lubricated from the periphery of outer wiping edge, such as marine thrust bearings and worm gears.
4–6	Long power-screw nuts and like wiping parts over which it is difficult to effect a uniform distribution of lubricant or load.

3. Examples of high-pressure bearings.—A few examples of the application of Eq. (6) to typical movable-bridge bearings will be given.

Example 1.—A horizontal journal 4 ins. in diameter runs in a bearing at 100 r.p.m., and under a load of 600 lbs. per square inch of projected area. Find f.

$$V = \frac{4\pi \times 100}{12} = 104.7 \text{ ft. per minute.}$$

Assume $C_1 = 3$ and $C_2 = 1$, then

$$f = \frac{3}{250}\sqrt[4]{\frac{600}{104.7}} = \frac{3}{250} \times 1.547 = 0.0186.$$

Example 2.—A horizontal journal 4 ins. in diameter runs in a bearing at 30 r.p.m., and under a load of 600 lbs. per square inch of projected area. Find f.

$$V = \frac{4\pi \times 30}{12} = 31.416 \text{ ft. per minute.}$$

Assume $C_1 = 3$ and $C_2 = 1$, then

$$f = \frac{3}{250}\sqrt[4]{\frac{600}{31.416}} = \frac{3}{250} \times 2.09 = 0.024.$$

Example 3.—A vertical main-pinion shaft $7\frac{1}{2}$ ins. in diameter runs at 5 r.p.m., and under a pressure of 800 lbs. per square inch of projected area. Find f.

$$V = \frac{7.5\pi \times 5}{12} = 9.816 \text{ ft. per minute.}$$

Assume $C_1 = 4$ and $C_2 = 1$, then

$$f = \frac{4}{250}\sqrt[4]{\frac{800}{9.816}} = \frac{4}{250} \times 3.001 = 0.048.$$

Example 4.—A trunnion of a bascule bridge 24 ins. in diameter runs at $\frac{1}{4}$ r.p.m., and carries a load of 2000 lbs. per square inch of projected area. Find f.

$$V = \frac{24\pi \times 0.25}{12} = 1.57 \text{ ft. per minute.}$$

Assume $C_1 = 3$ and $C_2 = 1$, then

$$f = \frac{3}{250}\sqrt[4]{\frac{2000}{1.57}} = \frac{3}{250} \times 5.973 = 0.072.$$

Example 5.—A vertical shaft 4 ins. in diameter runs on a step bearing at 100 r.p.m., and carries a load of 200 lbs. per square inch of its end area. Find f.

$$V = \frac{4\pi \times 100}{12} \times \frac{2}{3} = 69.81 \text{ ft. per minute.}$$

Assume $C_1 = 3$ and $C_2 = 2$, then

$$f = \frac{6}{250}\sqrt[4]{\frac{200}{69.81}} = \frac{6}{250} \times 1.301 = 0.031.$$

Example 6.—A center pivot of a swing bridge runs on a disc 36 ins. in diameter at $\frac{1}{6}$ r.p.m., and carries a load of 3000 lbs. per square inch of projected area. Find f.

$$V = \frac{36\pi \times 0.167}{12} \times \frac{2}{3} = 1.047 \text{ ft. per minute.}$$

Assume $C_1 = 2$ and $C_2 = 2$, then

$$f = \frac{4}{250}\sqrt[4]{\frac{3000}{1.047}} = \frac{4}{250} \times 7.315 = 0.117.$$

Example 7.—An end-lifting wedge moves 1 ft. in 15 seconds and carries a load of 1500 lbs. per square inch. Find f.

$$V = 4 \text{ ft. per minute.}$$

Assume $C_1 = 3$ and $C_2 = 2$, then

$$f = \frac{6}{250}\sqrt[4]{\frac{1500}{4}} = \frac{6}{250} \times 4.401 = 0.106.$$

All of these results are seen to be reasonable under the assumed conditions. It will be observed that all of the coefficients are much larger than those determined from experiments on oil-borne journals and pivots; and that even in the cases of the swing-bridge pivot and wedge the coefficients correspond closely to tests made on actual bridges. It may then be concluded that the Illmer formulae may be taken to represent approximately the friction coefficients that may be expected in this class of machinery.

These coefficients are all for running and not starting friction, and this must be remembered when designing the power plant.

4. Turning resistance of swing bridges.—Only a few tests of the resistance of swing bridges to turning forces have been made, but those available cover a range in length, weight, and design sufficient to provide a basis for the exercise of good judgment in making calculations for a new structure.

(1) *Center-bearing swing bridges.*—In each case the resistance to turning will be expressed as a decimal part of the superincumbent weight turned on the center discs and considered as applied tangentially to a circle, the radius of which is two-thirds of the radius of the discs. The authority will be given in each case.

General.

C. Shaler Smith, M. Am. Soc. C. E.[6] Average..... 0.135

C. C. Schneider, M. Am. Soc. C. E.[7] Discs only.

At the start	0.101
To keep moving	0.068

Discs and operating machinery.

At the start	0.173
To keep moving	0.120

Stillwell Avenue Bridge over Coney Island Creek.—This is a double-track electric railway bridge 250 ft. long, weighing 728,000 lbs. The center has three lenticular discs, two of steel and one of bronze. The tests were made by the author.

Discs only.

Minimum to move very slowly	0.119
Maximum to move very slowly	0.181
Average of 13 tests	0.149

Discs and operating machinery.

Minimum to move very slowly	0.135
Maximum to move very slowly	0.197
Average of 13 tests	0.166

Panama Emergency Dam.—Tests were made on the West Emergency Dam at Pedro Miguel, C. Z., and reported by T. B. Monniche, M. Am. Soc. C. E.[8] The structure is in the form of a bob-tailed swing span with the long arm 164 ft. 3 ins. and the short arm 98 ft. long, a total length of 263 ft. 3 ins. The trusses are 32 ft. between centers. The weight was 6,700,000 lbs. and the structure turned on three lenticular discs 43 ins. in diameter, two of steel and one of bronze.

The lubrication was unusually effective. Before each operation

[6] Draw Spans and their Turn-tables. *Trans. A. S. C. E.*, Vol. III, p. 129, 1874.

[7] Movable Bridges. *Trans. A. S. C. E.*, Vol. LX, p. 258, 1908.

[8] Emergency Dams above Locks of the Panama Canal. *Trans. Int. Engrg. Congress*, San Francisco, Cal., 1915, Section III, p. 131.

a heavy grease was forced through a central pipe and between the discs, by means of a large piston in a grease-containing cylinder. The piston was operated by a vertical screw surmounted by a hand wheel about 20 ins. in diameter. The operating machinery was disconnected by removing the main-pinion shafts.

Disc friction, when turning at a uniform velocity.. 0.079

This may be taken as a minimum coefficient of friction for discs with the best of lubrication.

(2) *Rim-bearing swing bridges.*—In each case the resistance to turning will be expressed as a decimal part of the superincumbent weight turned on the roller live ring, and is considered as applied tangentially to a circle the radius of which is the radius of the drum and the track on which the rollers travel. The authority will be given in each case.

General.

C. Shaler Smith, M. Am. Soc. C. E.[6]

Relatively small and light structures..... 0.004 to 0.008

Thames River Bridge, New London, Conn.—This was a double-track, through, pin-connected swing span, 500 ft. long, weighing 2,400,000 lbs. Twenty-three tests were made and the average was used in the calculation of the coefficient of friction. The tests were by Mr. A. P. Boller, Jr., and Mr. H. J. Schumacher.[9]

Collar friction of live ring, f assumed as 0.07....	0.00124 or	31.8 per cent.
Shafts and gears of the operating machinery, coefficients used according to F. Reuleaux..	0.00008 or	2.1 per cent.
Rollers between the drum and track, by difference	0.00257 or	66.1 per cent.
Total. Average of 23 tests.....	0.00389 or	100.0 per cent.

A test also was made with a dynamometer attached to one end of the span, as a check on the others. This test included all resistances and gave the value, 0.00345.

Second Avenue Bridge, New York City.—In a discussion of the paper on the Thames River Bridge,[9] Theodore Cooper, M Am. Soc. C. E., gave the result of a test which he had made on the old Second Avenue Bridge. This was a rim-bearing swing span which weighed 880,000 lbs. It was tested by means of the hydraulic operating rams.

Total friction, including rams, 0.0038.

[9] Experimental Determination of the Rolling Friction in Operating the Draw of the Thames River Bridge, Together with Method for Determining Power to Operate Draw Bridges. *Trans. A. S. C. E.*, Vol. XXV, p. 638, 1891.

145th Street Bridge, New York City.—This is a rim-bearing riveted swing span 300 ft. long and 84 ft. wide, weighing 3,900,000 lbs. Tests were made with a dynamometer attached tangentially at one end of the span, and the resistance of the operating machinery back to the motor was included. Thirty-four separate readings were taken.

The tests were made by Martin Gay, M. Am. Soc. C. E., and the author. The results follow.

Minimum	0.00177
Maximum	0.00394
Average of 34 determinations	0.00247

This is the most modern of the rim-bearing bridges enumerated.

5. Machinery resistances for use in design.—For use in the design of movable-bridge machinery and for determining the power required, the author recommends the coefficients given in Tables 5C and 5D. These are based upon many years' experience in the design of this type of machinery, and are known to give safe results without undue conservatism.

TABLE 5C

MACHINERY RESISTANCES FOR USE IN THE DESIGN OF MOVABLE-BRIDGE MACHINERY

Machine Elements	Coefficients of Friction
Horizontal shafts and journals	0.05
Intermediate vertical shafts and journals	0.06
Vertical main-pinion shafts and journals	0.10
Collar thrust bearings	0.10
Foot-steps for vertical shafts	0.10
Efficiency of gearing, exclusive of bearings:	Per Cent.
Spur gears with cut teeth	97
Spur gears with cast teeth	95
Bevel gears with cut teeth	95
Bevel gears with cast teeth	92
High-grade silent power-transmission chains	97
Well-made pin-connected power-transmission chains	95
Coefficient of efficiency of worm gearing, excluding thrust bearings, $E = \frac{L}{L + 2f\pi R}$ In which: L = the lead of the worm and R its pitch radius, both in inches. For cut worms and wheels, $f = 0.10$.	

TABLE 5D

MACHINERY RESISTANCES FOR USE IN THE DESIGN OF MOVABLE-BRIDGE MACHINERY

Machine Elements	Coefficients of Friction	
	Starting	Running
Trunnions:		
a. Less than one complete rotation	0.15	0.10
b. One or more complete rotations	0.12	0.08
Center discs	0.15	0.12
Wedges:		
a. Against driving, each contact surface	0.15	0.15
b. Against slipping out when under load, each contact surface	0.05	0.00
Rolling friction of bridges having rollers with flanges and for segmental rolling girders	0.009	0.006
Rollers without flanges:		
a. In contact with one surface, only	$\frac{0.004}{\sqrt{r}}$	$\frac{0.004}{\sqrt{r}}$
b. In contact with two surfaces	$\frac{0.008}{\sqrt{r}}$	$\frac{0.008}{\sqrt{r}}$
In *a* and *b*, r is the radius of the roller in inches.		
Stiffness of wire ropes bent around sheaves: Coefficient of the tensile stress in the ropes	$0.45\frac{d}{D}$	$0.45\frac{d}{D}$
In which D is the pitch diameter of the rope on the sheave and d is its diameter.		
Sliding friction between plane surfaces intermittently lubricated, such as guides for vertical-lift spans	0.10	0.10
When designing the machinery for holding the span against the wind pressure and for computing braking effect, only one-half of the above coefficients should be used.		

6. Motors and engines.—A movable bridge may be operated by a steam, oil, or gasoline engine, or an electric motor. The type of power to be adopted will depend upon the location of the bridge, the number of operations per day, the availability of electric current, and the relative reliability and cost of power from the various possible sources of supply.

Formerly, nearly all movable bridges were operated by steam. This is the most reliable of all but is also the most expensive. Steam pressure must be kept up, even when the bridge is closed, and the cost

of wasted fuel and extra labor and attendance is considerable. Boilers, pumps, and water and coal supplies must be maintained in addition to the engine itself. A good water supply is often difficult to secure and impure water may ruin the boilers.

When a reliable supply of electric current is available, electric motors are the most convenient and satisfactory, and the cost of operation, maintenance, and attendance is near the minimum.

A good gasoline engine is usually the most economical of all, so far as the cost of fuel is concerned, but there is likely to be some trouble with small repairs and the maintenance of small parts and the ignition sytem. Careful designing of the machinery is necessary on account of the small torque of the engine at low speeds. The engines now available run in only one direction, and reversing gears, which complicate the machinery, must be used.

In spite of these difficulties and complications, gasoline engines are now commonly used in locations where electric current is not available, and they have been almost uniformly successful. They are also the best and most economical source of auxiliary power for bridges normally operated by electric motors.

The problem of the design of the machinery is varied by the type of power that is adopted. Engine, or motor, torque is the fundamental basis from which the machinery should be proportioned, and this varies with the type of motor.

7. Torque of a direct-current electric motor.—The term, torque, designates the intensity of the force, in pounds, which a motor armature exerts tangentially to a circle of one foot radius. The normal torque of a motor then is

$$\text{Torque} = \frac{\text{h.p.} \times 33{,}000}{2\pi \times \text{r.p.m.}} = \frac{\text{h.p.} \times 5250}{\text{r.p.m.}} \text{ (very nearly).} \quad . \quad (7)$$

For movable-bridge operation, the most suitable of the direct-current motors are the crane and mill types. When the motor is exposed to the weather, one with an enclosed frame should be selected. When fully housed, the open type may be used, but most engineers prefer the enclosed type for this service. The crane-type motors are made of sufficient size and capacity for most movable bridges of ordinary lengths and weights.

These motors are wound so that they have large starting torques, and the running torques increase with the load up to a point where the motor breaks down by burning the insulation. The speed also is variable through wide limits. To prevent accidental overloading of

the machinery and motor, the switchboard should have overload relays that can be set to open when the current exceeds safe limits.

The characteristics of a 35- to 45-h.p., crane-type, direct-current, series-wound motor are shown by the curves in Fig. 5A. The flexibility

Characteristics of a Direct-current Motor.

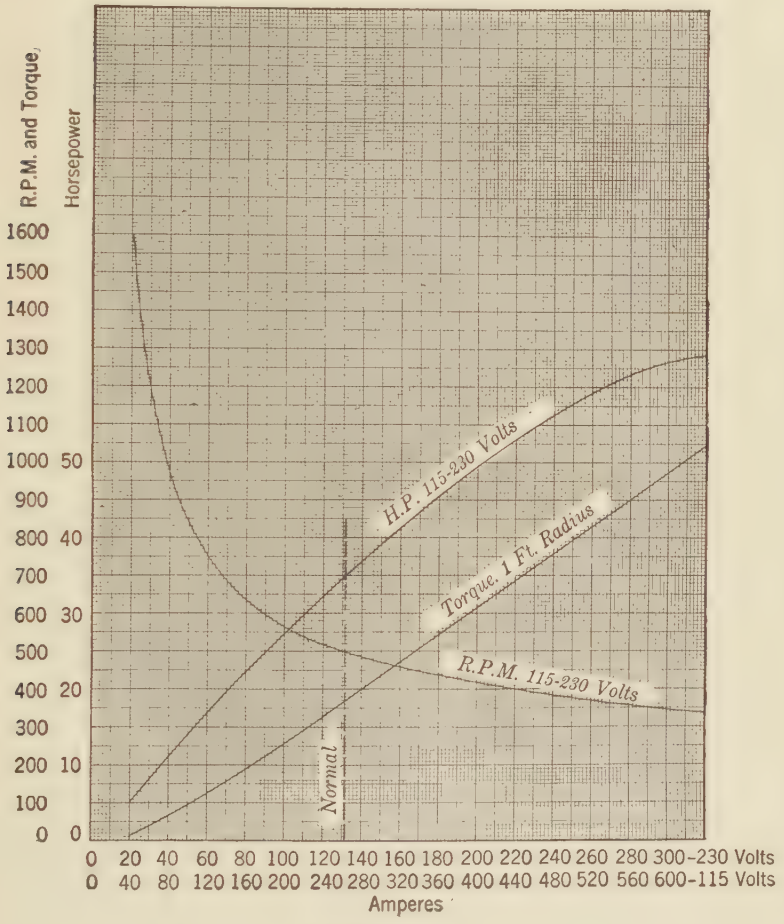


Fig. 5A.

of the motor under varying load is well illustrated by these curves. For purposes of machine design, the torque curve is the most important. With a suitable controller and resistors, a skillful operator can avoid all extreme ranges of torque, and the resistance of the bridge against movement will prevent high velocities.

It is feasible to set the overload relays to limit the current to about 150 per cent. of the normal, but this would require skillful manipulation of the controller or frequent closing of the relays. It is wise to design the machinery liberally and thus to permit more variation in the control by the operator.

The bridge-operating machinery should be proportioned for a maximum torque of about 200 per cent. of the normal torque.

The end-lifting and locking mechanism is subject to quite definite resistances. In some cases separate motors are used to drive this machinery; in others it is actuated by the main operating motor by means of clutches which are arranged to allow either train of gears to be driven by the bridge-operating motor. In the latter case it usually develops that the operating motor is larger than is required for the end machinery. The resistance, however, is limited by the function of the mechanism, so that the motor cannot exert a maximum torque, and the gear train should be proportioned from the known resistances and not from the motor torque.

8. Torque of an alternating-current motor.—Recent improvements in the construction and control of slip-ring alternating-current motors have made them available for movable-bridge operation. They are now made in the crane and mill types, the latter being the more rugged in construction. The crane type is available with enclosed and open frames, and the mill type is enclosed. The starting torque is about double the normal, and the maxiumum running torque is even greater. In some respects they are more satisfactory than direct-current motors. When the load is light, they do not acquire extremely high velocities. Under varying loads the armature speed does not vary more than about 10 per cent. from the synchronous speed in either direction. When they are overloaded, they will stall instead of running up a very high torque. The time of operation of a movable bridge is so short that with crane rating of even 15 minutes with a temperature rise of 50° C. there is not time to develop a temperature that will burn the insulation during a single operation, unless the operator fails to cut off the current at once when the motor stalls. The fact that the motor stalls is a warning to the operator and really a safeguard against the burning out of the armature.

Figure 5B shows the characteristic curves of torque of an alternating-current motor of about the same capacity as the direct-current motor represented in Fig. 5A. The curves show the torques when the controller is set on points numbered from 1 to 10. The torques are expressed in per cent. of the normal torque and the speeds in per cent. of the synchronous speed.

It will be observed that a torque 200 per cent. of the normal is likely to be realized, and the operating machinery should be proportioned for this torque.

The end machinery should be proportioned as outlined in Art. 7.

Characteristics of an Alternating-current Motor.

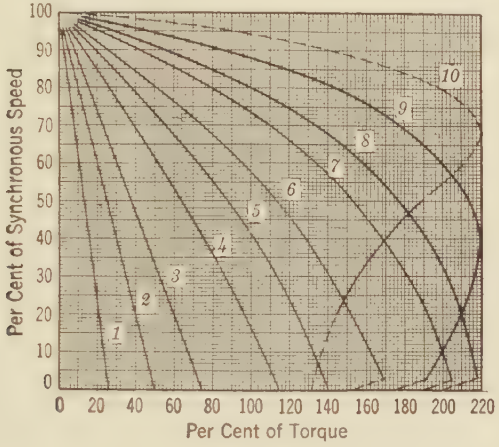


Fig. 5B.

9. Torque of a gasoline engine.—The characteristics of gasoline and oil engines are entirely different from those of electric motors. The starting torque is practically zero. The running torque does not attain a magnitude that will start a bridge from rest, even with the assistance of the fly wheel, until the engine is running at a relatively high speed. As the starting torque is *nil*, the only practicable way to start the bridge is to allow the engine to run idle until it has attained a speed at which the torque has nearly or quite its maximum value, and then to start the machinery by means of a friction clutch, which is made so that the parts can slip without injury until the bridge is accelerated.

The selection of the clutch is an important matter. No ordinary form of cut-off clutch will serve the purpose, for it would soon be worn out by the slipping and rubbing of the shoes. The best clutch for this service is a lubricated disc clutch of the general type of the Weston or the Hele-Shaw designs. The rated capacity of the clutch should never be less than twice the normal output of the engine.

Figure 5C shows the characteristic curves of torque and horsepower for a 40-h.p., 4-cylinder, 4-cycle, truck-type gasoline engine. The engine represented should be allowed to acquire a speed of at

least 1000 r.p.m. before any attempt is made to start the bridge. The clutch should then be closed slowly, or the engine will stall. It should be noted that the torque is nearly constant from 400 to 1000 r.p.m., and that it then diminishes rapidly to 1600 r.p.m., which is the practical speed limit of this engine.

In the 4-cycle engine, an explosion occurs within each cylinder only once in four piston movements, or two complete rotations of the

Characteristics of a Gasoline Engine.

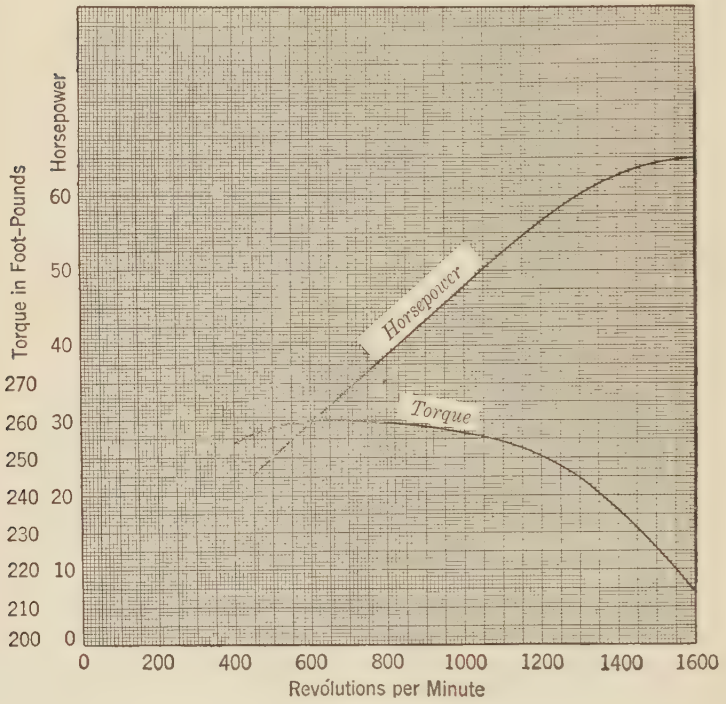


Fig. 5C.

crank shaft. The cranks are made so that the explosions in the four cylinders occur at equal angular intervals, in order to unify the torque as closely as possible. A fly wheel is depended upon to maintain uniform angular velocity and torque. The ordinary practice is to make the fly wheel so light that this result is not fully accomplished. When selecting such an engine, it is desirable to choose one with a heavy fly wheel, relatively large and long cylinders, and with a comparatively low running velocity. The latter is done to secure an engine of sturdy

construction and to reduce the gear ratio between the engine and the bridge-driving shaft, and thus simplify the transmission mechanism.

Heavy, slow-speed, 4-cycle gasoline engines formerly were made by many manufacturers, but the present tendency is to substitute heavier and cheaper oils for gasoline, with the attendant delays in starting a cold engine. The heavy engines have a higher torque and are better adapted for use on bridges, but are not available at present, and will not be until some means of quick starting becomes common.

As the fly wheels do not fully compensate for the shocks due to the explosions within the cylinders, it is necessary to make provision for the residual shock when proportioning the machinery near the engine. The clutch, reversing gear, and a few of the counter shafts and gears near the engine should be designed for excess torques. In parts of the machinery remote from the engine, the excess torque may be reduced, for the elasticity of the shafts enables them to absorb some, or all, of the shock effect, by distortions within the elastic limit.

10. Torque of a steam engine.—In a steam engine the torque is nearly constant for a given steam pressure and varies only with the pressure within the cylinders. Machine parts near the engine, like clutches and the one or two counter shafts and gears that are commonly located in the engine house, should be proportioned for about 125 per cent of the normal torque of the engine. The remainder of the mechanism may be designed for the normal torque.

11. Summation.—Table 5E gives a summary of excess torque allowances which have been successfully used by the author in the design of the machinery of many movable bridges. When these allowances are applied with good judgment, the proportions of the machinery will conform to good practice.

TABLE 5E

Motor and Engine Torques for Use in the Design of Movable-bridge Machinery

Type of Motor or Engine	Per Cent. of Normal Torque		Per Cent. of Calculated Resistance
	Operating Machinery in House, Including Transmission Machinery or Speed Reducers	Remainder of the Operating Train	End-lifting and Locking Apparatus
Alternating-current motor	200	200	100
Direct-current crane-type motor	200	200	100
Gasoline engines:			
One-cylinder, 4-cycle, about 200 to 300 r.p.m.	225	150	100
Two-cylinder, 4-cycle, about 250 to 350 r.p.m.	175	125	100
Three-cylinder, 4-cycle, about 250 to 400 r.p.m.	150	100	100
Truck or marine type, 4-cylinder, 4-cycle, about 1000 to 1400 r.p.m.	125	100	100
Steam engine	125	100	100

12. Torque for man-power.—The machinery of manually operated bridges should be proportioned for the torque produced by the maximum number of men that it is intended to use, assuming that each man pushes with an intensity of 125 lbs. on a capstan lever. The center of application of a man pushing near the end of a lever may be assumed as 9 ins. from its end.

CHAPTER VI

UNIT STRESSES

1. General.—Movable-bridge machinery operates at relatively slow speed and is heavily loaded, but it is not subject to the sudden and violent shocks that are common in high-speed mechanism and mill machinery. The loads are intermittent and are applied only for short periods of time. The direction of rotation is rarely reversed without a time interval between the reverse operations. Under such conditions the choice of unit stresses is quite a different problem from that encountered in the design of machine tools, which run continuously, or of rolling-mill machinery, which is subject to violent shocks, sudden changes in load, and, in many cases, to frequent reversal of direction.

Successful experience over a long period of time is the best criterion of good design and, fortunately, such a record is available. The Edge Moor Iron Company was organized in 1868 for the fabrication of bridges and buildings. This company was controlled by the late William Sellers, who was a noted engineer and designer and builder of machine tools. By his large experience and ability, he guided the company toward methods of machine design, as applied to movable bridges and turntables, which proved to be successful in practice. The original company was succeeded by the Edge Moor Bridge Works in 1888, and the development of methods and practice continued. In 1900 the second company became a part of the American Bridge Company. This company also included several other plants, the engineers of which had given careful study to the design of drawbridges and had developed successful practice. Among them may be mentioned the Detroit Bridge and Iron Works, the Union Bridge Company, and the Pencoyd Iron Works. The engineers of the American Bridge Company thus had the advantage of the experience of all.

With this background, the selection of unit stresses and methods of design is made with the certainty that satisfactory structures will result when good judgment is exercised in their application.

2. Unit stresses.—It is convenient to assemble the basic unit stresses, most commonly used in machine design, in tabular form for easy reference. Further illustrations of their use and modification in special cases will be found in succeeding chapters and in the Specifications in Appendix A, to which reference should be made.

	Pounds per Square Inch
Rolled, forged, and cast steel machinery supports and machine parts.	
Parts in constant tension during one operation of the span	16,000
Parts in constant compression during one operation of the span	$16{,}000 - 70\frac{l}{r}$
Parts subject to reversal of stress during one operation or subject to vibration:	
Tension	10,000
Compression	$10{,}000 - 45\frac{l}{r}$
Pedestals and wedges.	
Cast steel:	
Direct tension and tensile fiber stress	16,000
Direct compression and compressive fiber stress	$16{,}000 - 70\frac{l}{r}$
Cast iron:	
Direct tension	3,000
Tensile fiber stress due to bending	4,000
Direct compression and compressive fiber stress	$12{,}000 - 50\frac{l}{r}$
Levers and bell cranks.	
Cast or forged steel:	
Tension, compression, and fiber stress	10,000
Cast iron:	
Direct tension	3,000
Tensile fiber stress due to bending	4,000
Compression and compressive fiber stress	8,000

When the loads are eccentric, find the equivalent bending moment from:

$$M_e = \tfrac{3}{8}M_b + \tfrac{5}{8}\sqrt{M_b^2 + M_t^2},$$

in which M_b is the simple bending moment;
M_t is the simple torsional moment;
M_e is the equivalent bending moment.

Bearings.	
Maximum resultant fiber stresses:	
Cast steel:	
Tension or compression	10,000
Cast iron:	
Tension	3,000
Tensile fiber stress due to bending	4,000
Compression and compressive fiber stress	8,000
Pins, bolts, and rivets in machinery.	
Tension:	
Pins and bolts	10,000
Rivets	5,000

	Pounds per Square Inch
Bending:	
Pins and finished bolts	15,000
Bolts and rivets	10,000
Shear:	
Pins, finished bolts, and power-driven rivets	7,500
Bolts and hand-driven rivets	6,000
Bearing:	
Pins, finished bolts, and power-driven rivets	15,000
Bolts and hand-driven rivets	12,000
Large pins and similar parts that do not serve as mechanical bearings.	
Bending	24,000
Shear	12,000
Fixed bearing	24,000
Trunnions and similar mechanical joints.	
Fiber stress in bending:	
Annealed carbon-steel forgings, Class C	15,000
Heat-treated carbon-steel forgings, Class G	25,000
Shafts.	
Rolled or Class C forged shafts:	
Torsional shear from equivalent torsional moments	8,000
Fiber stress in bending from equivalent bending moments	16,000
Gear teeth.	
Fiber stress in bending, maximum at zero velocity:	
Class C forged-steel gears	22,000
Cast-steel gears	20,000
Phosphor-bronze gears	10,000
Cast-iron gears	8,000
Maximum pressures on bearings moving at slow and intermittent speeds.	
Center discs.	
On full projected area	3,500
Maximum at edge of oil hole of lenticular discs. (See Volume I, Appendix A, Art. 5)	10,000
Wedges.	
Driven and carrying dead or live load or both:	
Maximum bearing pressure	16,000
Maximum bearing pressure during motion	1,500
Trunnions.	
Steel on phosphor-bronze	2,000
Axles of balance wheels.	
Maximum loads due to overturning wind pressure:	
Steel axles in cast-iron or lined cast-steel bearings	1,500

	Pounds per Square Inch
Main-pinion shaft journals and other slow-moving shafts.	
Steel on phosphor-bronze	1,000
Steel on babbitt	800
Journals and shafts moving at ordinary speeds.	
Steel on phosphor-bronze	750
Steel on babbitt	600
Steel on cast iron	600
Cast iron on cast iron	400

For crank pins and similar joints with alternating pressures, the maximum pressures may be doubled.

Heating and seizing.

To avoid heating and seizing at high speeds the pressures shall not exceed the following:

Bearings	Materials	Pressure in Pounds per Square Inch, for Ordinary Speeds	Max. Pressures in Pounds per Square Inch, for Slow Speeds
Collars	Steel on steel	$\frac{20{,}000}{Nd}$	100
	Steel or cast iron on cast iron	$\frac{60{,}000}{Nd}$	600
	Steel on phosphor-bronze	$\frac{80{,}000}{Nd}$	750
Pivots	Steel on cast iron or phosphor-bronze	$\frac{100{,}000}{Nd}$	1500
Journals		$\frac{300{,}000}{Nd}$	As given for journals above

Acme or square threads transmitting motion.

	Nd = 140 or more	Nd less than 140
Steel screw running in a cast-iron or semi-steel nut	$\frac{262{,}500}{Nd}$	1875
Steel screw running in a phosphor-bronze nut	$\frac{350{,}000}{Nd}$	2500

In these formulae:

N = the number of revolutions per minute;

d = the diameter of the pivot or journal or the mean diameter of a collar or screw, in inches.

Maximum loads on rollers and wheels.

	Maximum Loads	
	In Motion	At Rest
Practically solid rollers:		
Cast iron	$300dl$	$450dl$
Steel castings	$500dl$	$600dl$
Hard steel	$1000dl$	$1500dl$
Tempered steel	$1500dl$	$2000dl$
Rollers for rim-bearing swing bridges: Load at each loaded point of the drum to be taken on a number of rollers equal to twice the depth of the drum divided by the diameter of the rollers.		
Iron castings	$250dl$	$400dl$
Steel castings	$400dl$	$600dl$
Balance wheels for center-bearing swing bridges. Load to be taken as that due to a wind pressure of 10 lbs. per square foot of each truss and the floor as seen in elevation.		
Iron castings	$800dl$	
Steel castings	$1200dl$	
	Recommended Maximum	Maximum Limit
Crane wheels.		
Chilled cast iron, double flange, single plate	$500dl$	$750dl$
Chilled cast iron, double flange, double plate	$800dl$	$1200dl$
Cast steel, double flanged	$700dl$	$1050dl$
Carnegie double flanged, rolled steel	$1000dl$	$1500dl$
When crane wheels run on railway rails, the elastic limit of the web of the rail under the head is usually less than the safe load on the wheel, and should be investigated.		

In these formulae:

d is the diameter of the roller or wheel, in inches.

l is the length of the cylindrical surface in contact with the track on which the roller or wheel runs.

Treads for rolling-lift bridges.

Refer to the specifications in Appendix A, Art. 719.

Wire ropes.

Refer to the specifications in Appendix A, Arts. 721 and 722.

CHAPTER VII

SHAFTS, COUPLINGS, AND KEYS

1. General.—If a shaft is subjected to a simple torsional moment only, the stress relations are expressed by the formula,

$$T = \frac{S_s J}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the shaft carries a bending moment only, the stress relations are expressed by the formula,

$$M = \frac{SI}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If a shaft is subjected to both a torsional and a bending moment, the stress relations are more involved. Several theories of these relations have been advanced by different investigators. Only a brief statement of a few of the various theories will be given.[1]

(1) *Maximum-principal-stress theory.*—On the assumption that the flexural stresses control, the Rankine theory is expressed by the formula,

$$M_e = \tfrac{1}{2}[M + \sqrt{M^2 + T^2}]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

M_e may be defined as the equivalent bending moment, which without torsion would produce a direct fiber stress equal to the maximum principal stress resulting from combined flexure and torsion.

(2) *Maximum-strain theory.*—This theory rests on the assumption that the material yields when the deformation, or unit strain, of any element in tension or compression reaches a certain amount. The theory is that advanced by Mariotte and Saint-Venant. When Poisson's ratio is taken as 0.25, this theory is expressed by the formula,

$$M_e = \tfrac{3}{8}M + \tfrac{5}{8}\sqrt{M^2 + T^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

European designers have commonly used this theory and it has been widely adopted in the United States by designers of movable-bridge machinery. It applies to hard and brittle materials, like hard steel

[1] Standard Formulas for Use in Determining the Sizes of Transmission Shafting. Report of Sub-committee No. 3 on the Standardization of Shafting. Sponsor Society, The American Society of Mechanical Engineers, July, 1925.

and cast iron, more closely than to the relatively ductile steels commonly used in transmission shafts.

(3) *Maximum-shear theory.*—This theory rests on the assumption that the maximum intensity of shear in a ductile material determines the elastic failure. It agrees closely with the results of modern tests, and is now considered to be more applicable to ordinary shafts than either of the others. The stress relations are expressed by the formula,

$$T_e = \sqrt{M^2 + T^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This theory will be adopted in the discussion of the design of ordinary steel shafts for movable bridges and is advocated for general use. The most modern tendency is toward the general adoption of this method as more satisfactory than previous practice. In many cases it will be found to require a slightly larger shaft than the older theories, but the difference is not large.

2. Materials and general proportions of shafts.—Shafts should be made from cold-rolled, cold-drawn, hot-rolled, or forged steel, whichever is the most suitable and economical for the size and purpose of the shaft. Cold-rolled and cold-drawn steel will be found to be economical for only small sizes, usually not over $2\frac{1}{2}$ to 3 ins. in diameter. When keyways are cut in shafts of these materials, the shafts are likely to become curved, particularly when the keyways are away from the ends. This is due to the eccentric relief of the internal stresses which are introduced by the cold-finishing processes.

Soft hot-rolled or forged steel is not desirable, for the friction in the bearings is greater than that of medium steel.

Line shafts should be connected by flexible couplings, designed to center the shafts connected. The use of such couplings is almost necessary when line shafts extend from the center to the ends of swing bridges. When the bridge is open, the ends droop and bend the shaft and the passage of live loads over the closed structure causes deflections in an opposite direction.

For shafts supporting their own weight only, the unsupported length should not exceed $L = 100\sqrt[3]{d^2}$, in which L is the length of shaft between bearings and d is the diameter of the shaft, both in inches. See also Arts. 14 and 15, in this chapter, for the critical speed of shafts.

When the bridge is operated by man-power, no shaft transmitting motion should be less than 2 ins. in diameter. This rule should also apply to lever-and-link systems for operating bridge and rail locks, and similar parts, but need not apply to the small shafts of indicators or limit switches on power-operated bridges.

When the diameters of the shafts are determined from angular

stiffness, they usually require no increase in size on account of keyways cut in them, but the diameters of such shafts should not be less than those determined from the limiting unit stresses, increased, when necessary, for keyways.

3. Torsional moments and deflections.—Shafts must be designed so that their strength is great enough to resist the torsional and bending stresses and also so that the torsional deflections shall not be excessive.

Let P = the force acting to rotate the shaft, in pounds;
R = the lever arm of P, in inches;
$T = PR$ = the torsional moment, in inch-pounds;
M = the bending moment, in inch-pounds;
N = the number of horse-power transmitted;
n = the number of rotations per minute;
d = the diameter of the shaft, in inches;
r = the radius of the shaft, in inches;
L = the length of the shaft, in feet;
S_s = the torsional shearing stress at the circumference, in pounds per square inch;
Use 8000 lbs. per square inch.
S = the bending fiber stress at the circumference, in pounds per square inch;
Use 16,000 lbs. per square inch.
G = the torsional modulus of elasticity = $\frac{2}{5} \times 29{,}000{,}000 = 11{,}600{,}000$;
I = the moment of inertia of a cylindrical shaft = $\frac{\pi d^4}{64}$, in inches^4;
J = the polar moment of inertia of a cylindrical shaft = $\frac{\pi d^4}{32}$, in inches^4;
θ = the angle of torsional deflection of the shaft in circular measure;
θ° = the angle of torsional deflection of the shaft in degrees.

$$T = \frac{S_s J}{r} = \frac{S_s \pi d^4}{32r} = \frac{S_s \pi d^3}{16} = 1{,}570.7964 d^3. \quad (6)$$

$$d = \sqrt[3]{\frac{16T}{S_s \pi}} = 0.08603 \sqrt[3]{T}. \quad (7)$$

$$\theta = \frac{TL}{JG} = \frac{2S_s \times 12L}{Gd} = 0.01655 \frac{L}{d}. \quad (8)$$

$$\theta = \frac{32T \times 12L}{\pi G d^4} = 0.00001054 \frac{TL}{d^4}. \quad (9)$$

$$\theta^\circ = \frac{2 \times 360 \times S_s \times 12L}{2\pi Gd} = 0.9483\frac{L}{d}. \quad . \quad . \quad . \quad . \quad (10)$$

$$\theta^\circ = \frac{32 \times 360T \times 12L}{2\pi^2 Gd^4} = 0.0006037\frac{TL}{d^4}. \quad . \quad . \quad . \quad (11)$$

For ordinary shafts a torsional deflection of 1° in a length equal to 20 times the diameter of the shaft is permissible.

In Eq. (11) let $\theta^\circ = 1$ and $L = \frac{20d}{12}$; then

$$T_1 = \frac{2 \times 1\pi^2 Gd^3}{32 \times 360 \times 20} = 993.8142d^3. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$d_1 = \sqrt[3]{\frac{32 \times 360 \times 20T}{2 \times 1\pi^2 G}} = 0.10021\sqrt[3]{T_1} \quad . \quad . \quad . \quad (13)$$

In special cases when positive drive is required, as in rocking shafts of end-wedge mechanism, the torsional deflection should be limited to 0.075° per foot of shaft.

In Eq. (11) let $\theta^\circ = 0.075$ and $L = 1$ ft. $= 12$ ins.; then

$$T_2 = \frac{2 \times 0.075\pi^2 Gd^4}{32 \times 360 \times 12} = 124.2268d^4. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$d_2 = \sqrt[4]{\frac{32 \times 360 \times 12T}{2 \times 0.075\pi^2 G}} = 0.29953\sqrt[4]{T_2} \quad . \quad . \quad . \quad . \quad (15)$$

The torsional moment, T, on a shaft transmitting N horse-power at n revolutions per minute is,

$$T_3 = \frac{33{,}000 \times 12}{2\pi} \times \frac{N}{n} = 63025\frac{N}{n}. \quad . \quad . \quad . \quad . \quad . \quad (16)$$

4. Combined bending and torsional moments.—If a shaft of hard or crystalline metal, like tempered steel or cast iron, is subjected to both bending and torsional moments, an equivalent bending moment, combining the effects of the bending and torsional moments, is found from the formula,

$$M_E = \tfrac{3}{8}K_M M + \tfrac{5}{8}\sqrt{(K_M M)^2 + (K_t T)^2}. \quad . \quad . \quad . \quad . \quad (17)$$

In this expression K_m and K_t are factors to include impact and fatigue. Then

$$M_E = \frac{SI}{r} = \frac{S\pi d^4}{64r} = \frac{S\pi d^3}{32}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$d_3 = \sqrt[3]{\frac{32M_E}{S\pi}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

in which S is the unit fiber stress in bending appropriate for the material used in the shaft.

Shafts of such materials are rarely used in movable bridges.

Most shafts in movable bridges are of ductile metal, like rolled or forged open-hearth steel. When such shafts are subjected to both bending and torsional moments, an equivalent torsional moment, combining the effects of the bending and torsional moments, is found from the formula

$$T_E = \sqrt{(K_m M)^2 + (K_t T)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

In this expression K_m and K_t are factors to include impact and fatigue. Then

$$T_E = \frac{S_s J}{r} = \frac{S_s \pi d^4}{32r} = \frac{S_s \pi d^3}{16}. \quad . \quad . \quad . \quad . \quad . \quad (21)$$

When $S_s = 8000$, $T_E = 1570.8d^3$.

$$d_4 = \sqrt[3]{\frac{16T_E}{S_s \pi}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

When $S_s = 8000$, $d_4 = 0.08603\sqrt[3]{T_E}$.

5. Horse-power transmitted by shafts.—Expressed in terms of horse-power transmitted, Eq. (6) for solid shafts becomes

$$N = \frac{S_s n d^3}{321{,}000}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

6. Very short, heavily loaded shafts.—Where shafts support heavy loads in bending over very short spans, the flexural shearing stresses become large and should be determined. The total transverse shear that a solid cylindrical shaft can support at a given shearing stress, S_s, is given by the expression,

$$V = \frac{3S_s \pi d^2}{16}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

7. Shafts subjected to combined torsion, bending, and axial tension or compression.—The shafts in movable-bridge machinery are seldom subjected to axial stresses of sufficient magnitude to require consideration in combination with torsional and bending stresses. As in other shafts, the factor determining elastic strength is the maximum intensity of shear. The maximum shear, p_t, is always found at the section where the bending moment is a maximum, and it is found from the expression

$$p_t = \frac{16}{\pi d^3}\sqrt{\left\{M + \frac{\alpha P d}{8}\right\}^2 + T^2}, \quad . \quad . \quad . \quad . \quad . \quad (25)$$

in which P is the axial load.

α is the ratio of the maximum to the average intensity of stress from axial loads only.

If the axial load is tensile, α may be taken as unity. If the axial load is compressive, α may be calculated from any suitable column formula. If it is assumed that the shaft bends in single curvature, α may be found from the formula:

$$S_c = \frac{16{,}000}{1 + \frac{l^2}{13{,}500r^2}}.$$

By substitution,

$$\alpha = \frac{16{,}000}{16{,}000 + \frac{1}{13{,}500}\left(\frac{l}{r}\right)^2} = 1 + \frac{1}{13{,}500}\left(\frac{l}{r}\right)^2 = 1 + 0.00007408\left(\frac{l}{r}\right)^2. \quad (26)$$

From this equation the following values are found:

Values of $\frac{1}{r}$	Values of α	Values of $\frac{1}{r}$	Values of α
25	1.047	125	2.157
50	1.185	150	2.667
75	1.417	175	3.269
100	1.741	200	3.964

8. Hollow shafts.—Heat-treated steel shafts more than 7 ins. in diameter and large forged-steel shafts should be made hollow.

Let d = the outside diameter of the shaft;
d_1 = the inside diameter of the shaft.

Then:

$$T = \frac{S_s\pi}{16}\frac{d^4 - {d_1}^4}{d} = 1570.8\frac{d^4 - {d_1}^4}{d}. \quad (6a)$$

$$\theta = \frac{TL}{JG} = \frac{2S_s}{G}(d^2 + {d_1}^2)\frac{12L}{d} = 0.01655(d^2 + {d_1}^2)\frac{L}{d}. \quad (8a)$$

$$\theta = \frac{32 \times T \times 12L}{\pi G(d^4 - {d_1}^4)} = 0.00001054\frac{TL}{d^4 - {d_1}^4}. \quad (9a)$$

$$\theta^\circ = \frac{360}{2\pi} \times \frac{2S_s}{G}(d^2 + {d_1}^2)\frac{12L}{d} = 0.9483(d^2 + {d_1}^2)\frac{L}{d}. \quad (10a)$$

$$\theta^\circ = \frac{360 \times 32 \times T \times 12L}{2\pi^2 G(d^4 - {d_1}^4)} = 0.0006037\frac{TL}{d^4 - {d_1}^4}. \quad (11a)$$

$$T_1 = \frac{2 \times 1 \times \pi^2 G(d^4 - {d_1}^4)}{32 \times 360 \times 20d} = 993.8142\frac{d^4 - {d_1}^4}{d}. \quad (12a)$$

$$T_2 = \frac{2 \times 0.075\pi^2 G(d^4 - d_1^4)}{32 \times 360 \times 12} = 124.2268(d^4 - d_1^4) . \quad (14a)$$

$$M_E = \frac{S\pi(d^4 - d_1^4)}{32d} = 1570.8\frac{d^4 - d_1^4}{d}. \quad . \quad . \quad . \quad . \quad . \quad (18a)$$

$$T_E = \frac{S_s\pi(d^4 - d_1^4)}{16d} = 1570.8\frac{d^4 - d_1^4}{d}. \quad . \quad . \quad . \quad . \quad . \quad (21a)$$

$$N = \frac{S_s n(d^4 - d_1^4)}{321{,}000d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23a)$$

$$V = \frac{3S_s\pi(d^4 - d_1^4)(d - d_1)}{16(d^3 - d_1^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24a)$$

$$p_t = \frac{16d}{\pi(d^4 - d_1^4)}\sqrt{\left\{M + \frac{\alpha P(d^2 + d_1^2)}{8d}\right\}^2 + T^2}. \quad . \quad . \quad (25a)$$

Reference should be made to the text and the equations bearing the same numbers, but without the suffix a, for the significance of the quantities represented by the first sides of these equations for hollow shafts.

9. Steel-shaft charts.—The following four charts, 7*A*, 7*B*, 7*C*, and 7*D*, give the diameters required for steel shafts designed to resist equivalent torsional moments, T_E, for combinations of bending moments, $K_m M$, and torsional moments, $K_t T$, calculated from the formula:

$$T_E = \sqrt{(K_m M)^2 + (K_t T)^2}. \quad \text{Eq. (20).}$$

The shaft diameters are found from the formula:

$$d = \sqrt[3]{\frac{16T_E}{S_s\pi}} = 0.08603\sqrt[3]{T_E}. \quad \text{Eq. (22).}$$

Unit stresses.—The shafts most frequently used in movable bridges are of rolled or forged structural steel which conforms to the specifications of the American Society for Testing Materials, Serial Designation: A 7–24. The maximum unit shear in shafts should not exceed about 30 per cent of the yield point in tension. The latest tests indicate that the maxium unit stress in bending should not exceed twice the shearing unit stress. For structural bridge steel the allowable shearing stress would be between 8250 and 9750 lbs. per square inch. In order to provide ample strength for steel having the minimum yield point recognized by specification A 7-24, the following unit stresses have been adopted for calculating the charts:

TABLE 7A

TORSIONAL CAPACITY OF SHAFTS

d ins.	For Strength, Torsion without Bending, $S_s = 8000$ lbs. per sq. in.		For Torsional Stiffness			
			$\theta° = 1°$ for $L = \frac{20d}{12}$. Ordinary Shafts		$\theta° = 0.075°$ per Linear Foot. Shafts for Positive Drive	
	T = in.-lbs.	$N \div n$	T = in.-lbs.	$N \div n$	T = in.-lbs.	$N \div n$
1	1,571	0.0249	994	0.0158	124	0.0020
1¼	3,068	0.0487	1,941	0.0308	303	0.0048
1½	5,301	0.0841	3,354	0.0532	629	0.0100
1¾	8,420	0.1336	5,326	0.0845	1,165	0.0185
2	12,570	0.1995	7,951	0.1262	1,988	0.0315
2¼	17,890	0.2839	11,320	0.1796	3,184	0.0505
2½	24,540	0.3894	15,528	0.2464	4,853	0.0770
2¾	32,670	0.5185	20,668	0.3280	7,105	0.1127
3	42,410	0.6731	26,833	0.4258	10,062	0.1597
3¼	53,920	0.8557	34,116	0.5414	13,859	0.2199
3½	67,350	1.069	42,610	0.6761	18,641	0.2958
3¾	82,830	1.315	52,408	0.8316	24,566	0.3898
4	100,500	1.595	63,604	1.009	31,802	0.5046
4½	143,100	2.271	90,561	1.437	50,941	0.8084
5	196,300	3.115	124,227	1.971	77,641	1.232
5½	261,300	4.147	165,346	2.624	113,675	1.804
6	339,300	5.384	214,664	3.406	160,998	2.555
6½	431,400	6.845	272,926	4.331	221,753	3.519
7	538,800	8.550	340,878	5.409	298,268	4.733
7½	662,700	10.515	419,265	6.653	393,062	6.237
8	804,200	12.757	508,833	8.074	508,833	8.074
8½	964,600	15.31	610,326	9.685	648,473	10.29
9	1,145,000	18.17	724,491	11.495	815,052	12.93
9½	1,346,700	21.37	852,072	13.520	1,011,835	16.06
10	1,570,800	24.93	993,814	15.77	1,242,268	19.71
10½	1,818,300	28.86	1,150,465	18.25	1,509,985	23.96
11	2,090,600	33.17	1,322,767	20.99	1,818,805	28.86
11½	2,388,700	37.90	1,511,467	23.99	2,172,734	34.48
12	2,714,000	43.07	1,717,310	27.25	2,575,967	40.88

	Pounds per Square Inch
Maximum shear in extreme fiber	8,000
Maximum bending stress in extreme fiber	16,000

The heavy lines on the charts are standard sizes.

10. Impact and fatigue.—The coefficients K_m and K_t, in Eq. (20), make provision for the sudden application of load, and fatigue due to repeated bending in rotating shafts. They should be applied to the calculated moments before entering the charts to find the required shaft diameter.

Movable-bridge machinery moves at relatively low velocities and without marked impact or shock. The following values of K_m and K_t should be used.

Manner of Loading	Values for	
	K_m	K_t
Stationary shafts:		
Gradually applied load	1.0	1.0
Suddenly applied load	1.5	1.5
Rotating shafts:		
Gradually applied load on shafts rotating at 10 r.p.m., or less	1.0	1.0
Gradually applied load on shafts rotating at more than 10 r.p.m.	1.5	1.0
Suddenly applied load	1.5	1.5
Suddenly applied load, heavy shocks *	2.5	2.5

* This condition should be avoided by good designing.

11. Keyways.—When standard keyways are cut in shafts at points of maximum stress, the diameters found from the charts should be increased as follows:

Shafts from Charts, Diameters in Inches	Add to Diameter when 1 Keyway is Used	Add to Diameter when 2 Keyways are Used
1 in. to 3 ins.	8 per cent.	6 per cent.
Over 3 ins.	6 per cent.	4 per cent.

12. Special unit stresses.—Special unit stresses may be required by some specifications or by the use of special steels. When the diameter of a shaft has been found from the charts on the standard basis, the diameter required for some special shear stress can be found by mul-

CAPACITY OF STEEL SHAFTS.

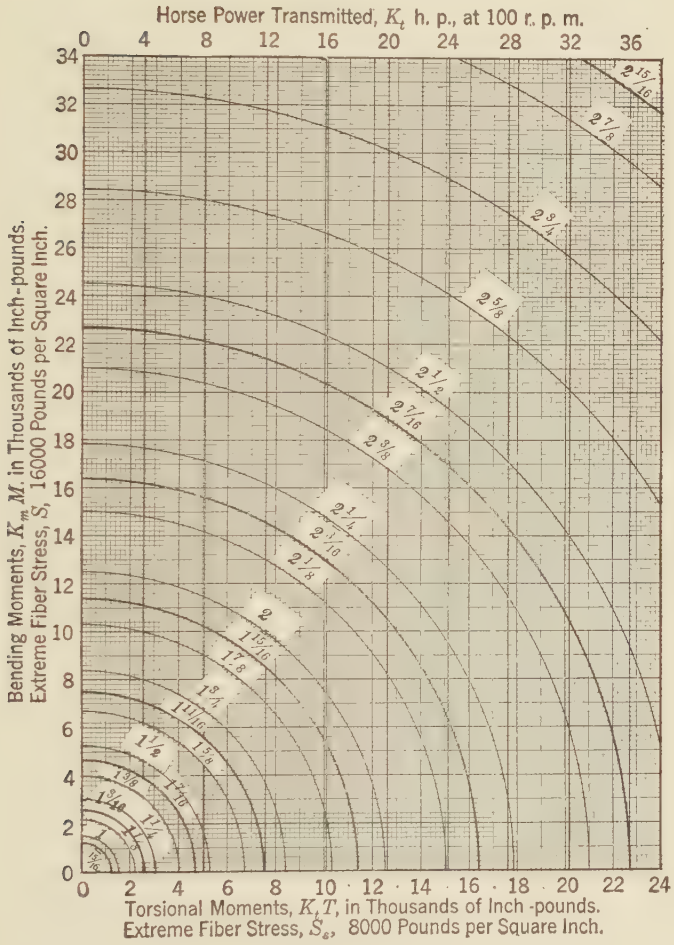


CHART 7A.

Capacity of Steel Shafts.

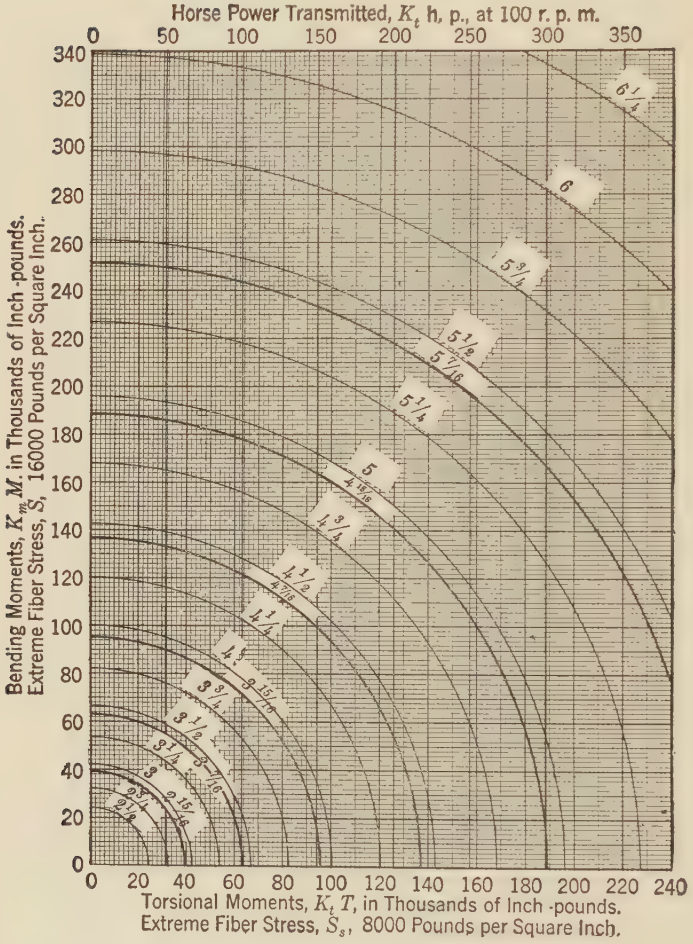


Chart 7B.

Capacity of Steel Shafts.

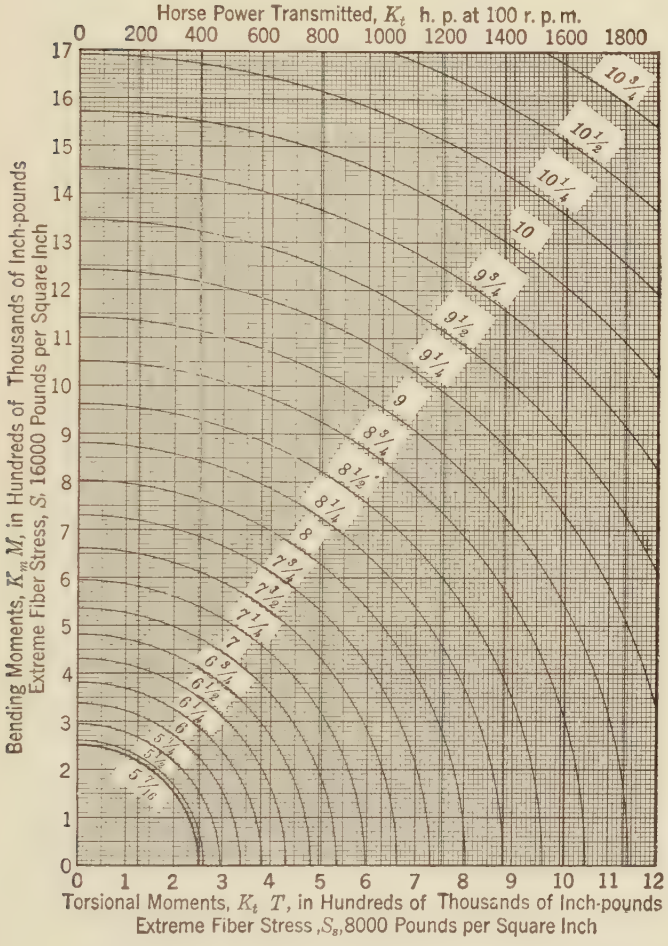


Chart 7C.

CAPACITY OF STEEL SHAFTS.

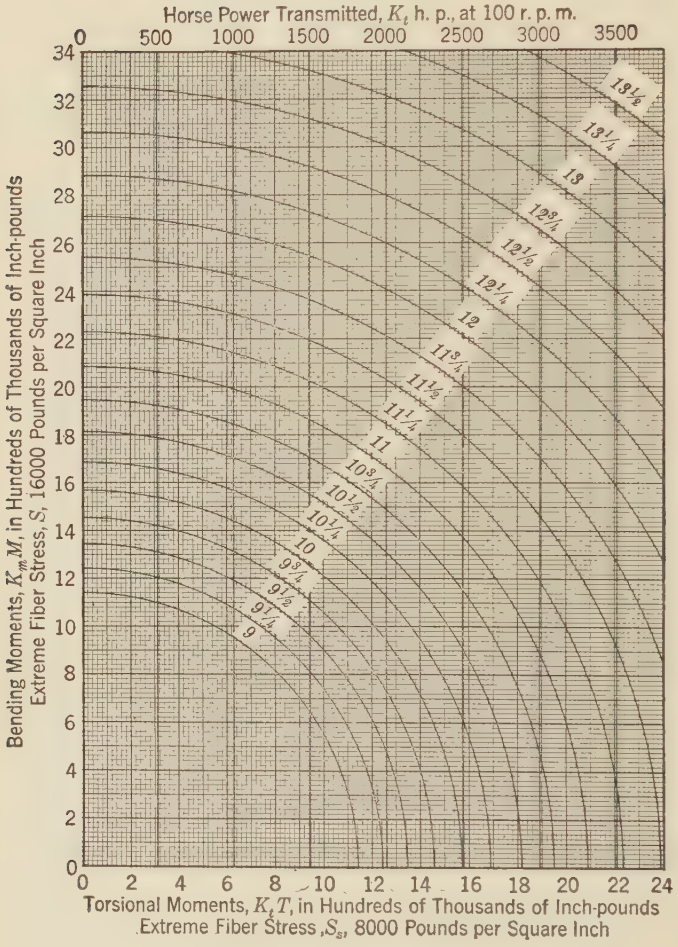


CHART 7D.

tiplying the diameter required for a shear of 8000 lbs. per square inch by the following factors.

Shear Stress in Pounds per Square Inch	Factor	Shear Stress in Pounds per Square Inch	Factor
4,000	1.26	8,000	1.00
5,000	1.17	9,000	0.96
6,000	1.10	10,000	0.93
6,500	1.07	11,000	0.90
7,000	1.05	12,000	0.87

13. Allowances for running fits of shafts.—If the diameter of a shaft were made the same as that of the inside of the bearing in which it is intended to turn, the bearing would act as a clamp and prevent rotation. The inside diameter of the bearing must be made slightly larger than that of the shaft to allow space for the lubricant and to permit the shaft to rotate freely and without undue friction. If the difference between the two diameters is made too great, the loose fit will result in unsatisfactory operation and rapid wear.

For the heavy and relatively slow-moving machinery of movable bridges, experience indicates that the allowances for running fits should be about those given in Table 7B. The allowances for each size of shaft are the differences between the diameters of the shaft and its bearing.

14. Critical speed of shafts.—It is almost impossible to balance a rotating mass so that its center of gravity coincides with its mechanical axis of rotation. When the mass begins to rotate, the center of gravity moves in a circle, of small radius, around the shaft center. A centrifugal force acts outwardly from the shaft center through the center of gravity, and rotates around the shaft center with the center of mass. The shaft is deflected and this action continues as the speed is increased until the shaft has deflected enough to establish an equilibrium between the centrifugal force and that tending to prevent the shaft from deflecting. This is the condition of maximum vibration, and the number of rotations per minute producing this equilibrium is called the critical speed of the shaft.

At the critical speed the vibrations are the maximum. They are less at both lower and higher speeds, but to attain a higher speed the shaft must pass through the critical speed where the trouble is the greatest. The critical speed is the same for both vertical and horizontal shafts.

TABLE 7B

ALLOWANCES FOR RUNNING FITS OF SHAFTS

Nominal Diameter of Shafts, Inches	Allowances for Horizontal Shafts, Inches	Allowances for Vertical Shafts, Inches	Nominal Diameter of Shafts, Inches	Allowances for Horizontal Shafts, Inches	Allowances for Vertical Shafts, Inches
$\frac{1}{2}$	0.002	0.0015	15	0.022	0.014
1	.002	.0015	16	.023	.014
$1\frac{1}{2}$	.003	.002	17	.024	.015
2	.004	.003	18	.025	.015
$2\frac{1}{2}$	.005	.003	19	.025	.016
3	.006	.004	20	.026	.016
$3\frac{1}{2}$	.007	.004	21	.027	.016
4	.008	.005	22	.027	.017
$4\frac{1}{2}$	.009	.005	23	.028	.017
5	.010	.006	24	.029	.018
$5\frac{1}{2}$	.011	.006	25	.030	.018
6	.012	.007	26	.030	.018
7	.013	.009	27	.031	.019
8	.015	.010	28	.032	.019
9	.017	.011	29	.032	.020
10	.019	.012	30	.033	.020
11	.020	.012	35	.037	.022
12	.020	.013	40	.040	.024
13	.021	.013	45	.044	.026
14	.022	.014	50	.047	.028

Allowance for horizontal shafts up to 10 ins., incl. $= 0.002 + 0.0019(d - 1)$.
Allowance for horizontal shafts over 10 ins. $= 0.019 + 0.0007(d - 10)$.
Allowance for vertical shafts up to 10 ins., incl. $= 0.0015 + 0.0012(d - 1)$.
Allowance for vertical shafts over 10 ins. $= 0.012 + 0.0004(d - 10)$.
In which d = nominal diameter of the shaft in inches.

The critical speed of a shaft with a single concentrated load is

$$N = 187.7\sqrt{\frac{1}{\Delta}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

This formula is given for inch, pound, minute units; and Δ is the deflection of the shaft at the load.

The speeds of movable-bridge machinery are seldom high enough to require attention, but it is well to be fully aware of the difficulties likely to occur when shafts are designed to run near the critical speeds.

Some of the formulae for the calculation of the critical speed of simple cases of rotating shafts, developed by Mr. S. H. Weaver,[2] are given in Tables 7C and 7D.

TABLE 7C

CRITICAL-SPEED FORMULAE

Single Concentrated Load W

Condition of Supports of the Shaft and Position of the Load					N
End	Distance to Load	Load	Distance to Support	End	
Supported	a	W	b	Supported	$387{,}000\frac{d^2}{ab}\sqrt{\frac{l}{W}}$
Supported	$\frac{l}{2}$	W	$\frac{l}{2}$	Supported	$1{,}550{,}500d^2\sqrt{\frac{1}{Wl^3}}$
Fixed	a	W	b	Fixed	$387{,}000d^2\sqrt{\frac{l^3}{Wa^3b^3}}$
Fixed	$\frac{l}{2}$	W	$\frac{l}{2}$	Fixed	$3{,}100{,}850d^2\sqrt{\frac{1}{Wl^3}}$
Fixed	a	W	b	Supported	$775{,}200\frac{d^2l}{ab}\sqrt{\frac{l}{Wa(3l+b)}}$
Fixed	$\frac{l}{2}$	W	$\frac{l}{2}$	Supported	$2{,}337{,}000\frac{d^2}{l}\sqrt{\frac{1}{Wl}}$
Fixed	l	W		Free	$387{,}000d^2\sqrt{\frac{1}{Wl^2}}$

The formulae in the table are given in inch, pound, minute units.
W = the load on the shaft, d its diameter, and l = its length.
N = the critical speed in rotations per minute.

[2] Critical Speed Calculations, A Paper by S. H. Weaver, *Jour. Am. Soc. M. E.*, June, 1910, p. 1059.

TABLE 7D

CRITICAL-SPEED FORMULAE

Total Uniform Load, W

Condition of Supports of the Shaft			N	N, Shaft Alone
End	Length	End		
Supported	l	Supported	$2{,}232{,}510d^2\sqrt{\frac{1}{Wl^3}}$	$4{,}760{,}000\frac{d}{l^2}$
Fixed	l	Fixed	$4{,}979{,}250d^2\sqrt{\frac{1}{Wl^3}}$	$10{,}616{,}740\frac{d}{l^2}$
Fixed	l	Supported	$3{,}482{,}715d^2\sqrt{\frac{1}{Wl^3}}$	$7{,}021{,}600\frac{d}{l^2}$
Fixed	l	Free	$795{,}196d^2\sqrt{\frac{1}{Wl^3}}$	$1{,}695{,}514\frac{d}{l^2}$

The formulae in the table are given in inch, pound, minute units.
W = the load on the shaft, d its diameter, and l = its length.
N = the critical speed in rotations per minute.

Mr. Weaver's paper gives other formulae for the critical speed of shafts subjected to different loads, also for the maximum deflections caused by static loads.

15. Application of Art. 14 to shafts.—The significance of the theoretical formulae given in Art. 14 is made clear by applying some of them to actual shafts.

Example 1.—Assume that a line shaft in a movable bridge is $3\frac{1}{4}$ ins. in diameter with bearings 16 ft. between centers and that it rotates at a speed of 550 r.p.m. Assume that the shaft carries no load in addition to its own weight, that jaw couplings connect the various lengths of shaft, and that the couplings are near the bearings. Each length of shaft may then be considered as simply supported at its ends. The critical speed of such a shaft is

$$N = 4{,}760{,}000\frac{3.25}{192^2} = 419.6 \text{ r.p.m.}$$

The assumed speed is 131 per cent of the critical speed; therefore the shaft would pass through its critical speed each time that the bridge

was accelerated, and much vibration might be expected. In an actual case, the vibration of the shaft was so violent that some of the bearings were broken.

There are two obvious methods of avoiding the dangerous vibrations in a shaft problem of this kind. The diameter of the shaft can be increased or the distance between the bearings can be decreased. The latter method is the more effective, because the critical speed increases directly as the diameter of the shaft, but inversely as the square of its unsupported length.

Example 2.—Assume that a shaft having the same diameter and rotating at the same speed as that in Example 1, has its bearings 8 ft. between centers, that each alternate length of shaft rests in three bearings, and that each intermediate length rests in one bearing at its middle; while its ends are supported by jaw couplings outside of and adjacent to the two end bearings of the three that support each alternate length of shaft. Each semi-length of shaft may then be considered as supported at one end, and as fixed, or continuous, at the other end. The critical speed will then become

$$N = 7{,}021{,}600\frac{3.25}{96^2} = 2476 \text{ r.p.m.}$$

The assumed operating speed of rotation of 550 r.p.m. is only 22.2 per cent of the critical speed. With this arrangement, very little vibration would occur, and the shaft would be satisfactory in operation.

Every high-speed shaft should be turned throughout its whole length, in order to produce as perfect a balance of the weight about the axis of rotation as possible. Cold-rolled and cold-drawn shafts do not require turning, for the process of manufacture secures accuracy of finish.

16. Couplings.—Figure 7A shows the three types of shaft couplings that are the most generally useful in movable-bridge machinery.

Muff couplings.—Figure 7A (*a*) is a sketch of the simplest form of muff coupling. It has the advantage of extreme simplicity and a smooth outer surface that will not catch clothing. It is not suitable in cases where parts of a shaft must be removed for repairs, unless the entire shaft can be taken from its bearings as a unit.

The usual proportions are

$$l = 3(d + \tfrac{1}{2}''). \qquad t = 0.45(d + \tfrac{1}{2}'').$$

Clutch couplings.—Figure 7A (*b*) shows a common form of clutch coupling that is well adapted for use in movable-bridge machinery.

Three jaws are desirable. They should be fitted to reduce the backlash to a minimum and still not be tight enough to prevent the shaft from deflecting slightly to conform to the elastic deformations of the superstructure. The joint in the shaft should be in the middle of the length of the jaws, in order that each piece of shaft may be held in axial alignment. The hole in the coupling may be bored of the same size throughout, as the overhanging ends of the shafts are so short that elastic deflections of the supports will not cause trouble. The square-headed setscrews shown might catch clothing, and, for safety, setscrews with a central countersunk square hole or with a straight slot across the countersunk head may be substituted.

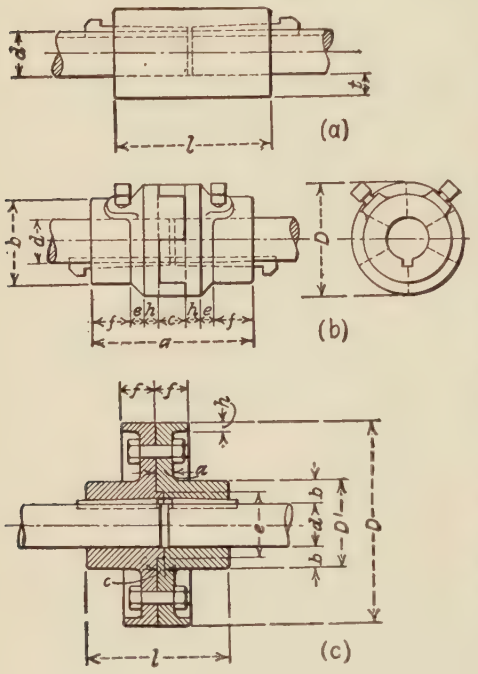


FIG. 7A.

The usual proportions are as follows:

$$a = 3\tfrac{1}{2}d \qquad f = d$$

$$b = 2d \qquad h = 0.3d$$

$$c = \tfrac{1}{2}d \qquad D = 2.85d$$

$$e = 0.2d$$

Flanged couplings.—Figure 7A (*c*) shows a common form of safety flanged coupling. The cylindrical flanges, *f*, cover the heads and nuts of the bolts so that clothing will not be caught. This type of coupling is desirable in cases where the shaft is not likely to be bent by elastic deflections of the supporting structure. It is rigid and can be made as large as necessary, and the number and size of the bolts can be varied to meet the conditions. The two parts of the coupling should overlap at the joint, at *c*, to hold the parts of the shaft in axial alignment.

Each half is secured to its shaft by keys driven from the end, thus avoiding gib-headed keys.

The usual proportions are as follows:

$$a = 0.3(d + \tfrac{1}{2}'') \qquad D' = 0.8(d + \tfrac{1}{2}'') + d$$

$$b = 0.4(d + \tfrac{1}{2}'') \qquad e = 0.4(d + \tfrac{1}{2}'') + d$$

$$c = 0.15(d + \tfrac{1}{2}'') \qquad f = 0.56d$$

$$D = 2\tfrac{1}{2}d + 6'' \qquad h = 0.07(d + 3'')$$

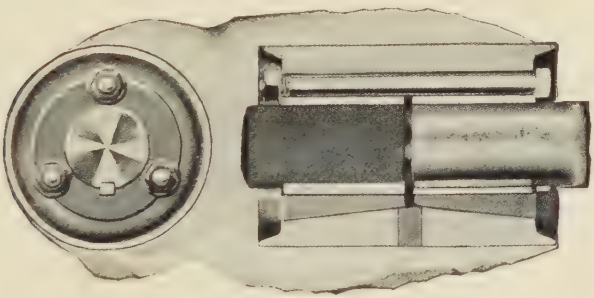
The number of bolts to be the nearest even number to

$$n = 3 + 0.5d.$$

The diameter of the bolts $= \dfrac{d}{2\sqrt{n}}$.

17. Double-cone vise coupling.—William Sellers & Co., Inc., of Philadelphia, Pa., make a line of double-cone vise couplings that have

Double-cone Vise Couplings.



William Sellers & Co., Inc.

Fig. 7B.

been satisfactory in service. They can be applied to a shaft without other special fitting than the cutting of the keyways. They are proportioned to develop the full strength of the shaft on which they are to be used.

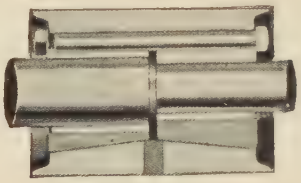
When the shafts to be axially connected are of the same diameter the coupling is in the form shown in Fig. 7B. The method of application is obvious from the details shown.

Shafts of different diameters are coupled in axial alignment by using cones of appropriate dimensions in a coupling of the size required for the larger shaft, as shown in Fig. 7C.

These couplings have no projecting parts likely to catch clothing, and can be removed when repairs of the machinery are made.

The cost of the couplings alone is greater than that of those shown in Fig. 7A, but they may prove to be the more economical in some cases.

DOUBLE-CONE VISE COUPLINGS.



William Sellers & Co., Inc.

FIG. 7C.

For example, if it becomes necessary to change existing machinery to adapt it to a new service, these couplings would be convenient and economical, for all of the work could be done by hand and on the job.

Some of the sizes and weights of this form of coupling that are likely to be required in movable-bridge machinery are given in Table 7E.

TABLE 7E

DOUBLE-CONE VISE COUPLINGS

Size of Shaft, Inches	Approximate Diameter of Coupling, Inches	Approximate Length of Coupling, Inches	Approximate Weight of Coupling, Pounds
$1\frac{15}{16}$	$5\frac{5}{8}$	8	40
$2\frac{3}{16}$	$5\frac{7}{8}$	$8\frac{1}{2}$	43
$2\frac{7}{16}$	$6\frac{9}{16}$	$9\frac{9}{16}$	59
$2\frac{11}{16}$	7	$10\frac{1}{4}$	70
$2\frac{15}{16}$	$7\frac{3}{4}$	$11\frac{3}{8}$	96
$3\frac{3}{16}$	$8\frac{1}{4}$	$12\frac{1}{8}$	112
$3\frac{7}{16}$	$9\frac{1}{16}$	$13\frac{1}{16}$	152
$3\frac{15}{16}$	10	$14\frac{5}{8}$	196
$4\frac{7}{16}$	$11\frac{3}{8}$	$16\frac{3}{8}$	295
$4\frac{15}{16}$	$12\frac{9}{16}$	$18\frac{1}{4}$	412
$5\frac{7}{16}$	$13\frac{1}{2}$	$19\frac{1}{2}$	523
$5\frac{15}{16}$	$14\frac{1}{2}$	$21\frac{1}{4}$	625

18. Keys.—The length of a key should be made such that the portion in the hub of a coupling, gear, wheel, or lever will develop the full torsional capacity of the shaft or the torsional moment that must be transmitted from the part connected to the shaft.

Let T = the torsional moment on the shaft;
r = the radius of the shaft;
s = the permissible shearing stress on the key;
B = the width of the key;
l = the required length of the key.

Then:

$$l = \frac{\frac{T}{r}}{Bs} = \frac{T}{Brs} \quad \ldots \ldots \ldots \quad (28)$$

The value of T may be taken from Table 7A; s may be assumed at 12,000 lbs. per square inch; and B may be taken from Table 7F.

For an example, find the length of a standard key that will develop the permissible simple torsional moment of a 6-in. medium-steel shaft.

T from Table 7A = 339,300 in.-lbs.;
B from Table 7F = $1\frac{1}{2}$ ins.;
r = 3 ins.;
s = 12,000 lbs. per square inch.

Then:

$$l = \frac{339{,}300}{1\frac{1}{2} \times 12{,}000 \times 3} = 6.28 \text{ ins.}$$

When one key is used, the effective length should be made $1\frac{1}{2}$ times the diameter of the shaft.

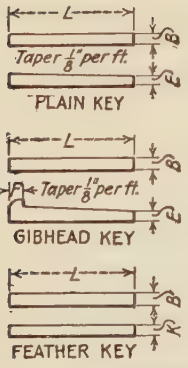
Two keys should be used when the length of the hub is limited to less than $1\frac{1}{2}$ times the diameter of the shaft. Two keys are generally used in main pinions when the shaft is 7 ins. or more in diameter. In this case the keys are fastened to the shaft with countersunk-headed screws and the pinion is forced on over the keys.

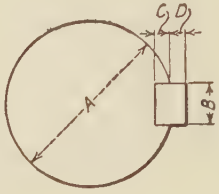
When two keys are used, the effective length, unless otherwise limited, should be made equal to the diameter of the shaft. The keys are usually placed at an angle of 120° from each other, in order to give a bearing at three points, equidistant from each other on the circumference of the shaft.

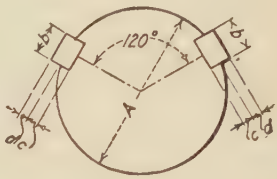
Table 7F gives the dimensions of the American Bridge Company's standard keys.

TABLE 7F

AMERICAN BRIDGE COMPANY'S STANDARD KEYS







For 1 Key						For 2 Keys					
Size of Shaft *A*, Inches	Width *B*, Inches	Depth in Shaft *C*, Inches	Depth in Hub *D*, Inches	Thickness at Small End *E*, Inches	Length of Gib Head *F*, Inches	Size of Shaft *A*, Inches	Width *b*, Inches	Depth in Shaft *c*, Inches	Depth in Hub *d*, Inches	Thickness at Small End *E*, Inches	Length of Gib Head *F*, Inches
¾ to 1 1/16	5/16	5/32	1/8	9/32		¾ to 1 1/16	5/16	5/32	1/16	5/32	
1 1/16 to 1 3/8	3/8	3/16	5/32	11/32	½	1 1/16 to 1 3/8	1/4	1/8	3/32	7/32	
1 7/16 to 1 7/8	½	7/32	3/16	13/32	5/8	1 7/16 to 1 7/8	5/16	5/32	1/8	9/32	3/8
1 15/16 to 2 3/8	5/8	1/4	7/32	15/32	11/16	1 15/16 to 2 3/8	3/8	1/8	5/32	11/32	1/2
2 7/16 to 2 7/8	3/4	9/32	1/4	17/32	13/16	2 7/16 to 2 7/8	1/2	7/32	3/16	13/32	5/8
2 15/16 to 3 3/8	7/8	5/16	9/32	19/32	1	2 15/16 to 3 3/8	5/8	1/4	7/32	15/32	11/16
3 7/16 to 3 7/8	1	11/32	5/16	21/32	1 1/16	3 7/16 to 3 7/8	5/8	9/32	1/4	17/32	13/16
3 15/16 to 4 3/8	1	3/8	11/32	23/32	1 1/16	3 15/16 to 4 3/8	3/4	5/16	9/32	19/32	7/8
4 7/16 to 4 3/4	1 1/8	3/8	3/8	25/32	1 3/16	4 7/16 to 4 3/4	13/16	5/16	5/16	19/32	7/8
4 13/16 to 5 1/4	1 1/4	13/32	7/16	27/32	1 5/16	4 13/16 to 5 1/4	7/8	11/32	5/16	21/32	1
5 5/16 to 5 3/4	1 3/8	1/2	13/32	29/32	1 7/16	5 5/16 to 5 3/4	15/16	3/8	11/32	23/32	1
5 13/16 to 6 1/4	1 1/2	17/32	1/2	1 1/32	1 5/8	5 13/16 to 6 1/4	1	3/8	11/32	23/32	1 1/16
6 5/16 to 6 3/4	1 5/8	9/16	17/32	1 3/32	1 5/8	6 5/16 to 6 3/4	1 1/8	13/32	3/8	25/32	1 1/16
6 13/16 to 7 1/4	1 3/4	5/8	19/32	1 5/32	1 13/16	6 13/16 to 7 1/4	1 3/16	7/16	13/32	27/32	1 1/4
7 5/16 to 7 3/4	1 7/8	21/32	5/8	1 3/32	1 15/16	7 5/16 to 7 3/4	1 1/4	15/32	7/16	29/32	1 3/8
7 13/16 to 8 1/4	2	11/16	21/32	1 11/32	2	7 13/16 to 8 1/4	1 3/8	1/2	15/32	31/32	1 7/16
8 5/16 to 8 3/4	2 1/8	23/32	11/16	1 13/32	2 1/8	8 5/16 to 8 3/4	1 1/2	17/32	1/2	1 1/32	1 1/16
8 13/16 to 9 1/4	2 1/4	25/32	3/4	1 15/32	2 1/4	8 13/16 to 9 1/4	1 5/16	9/16	17/32	1 3/32	1 5/8
9 5/16 to 9 3/4	2 3/8	13/16	25/32	1 17/32	2 3/8	9 5/16 to 9 3/4	1 11/16	11/16	9/16	1 5/32	1 3/4
9 13/16 to 10 1/4	2 1/2	27/32	13/16	1 21/32	2 1/2	9 13/16 to 10 1/4	1 3/4	5/8	11/16	1 7/32	1 13/16
10 5/16 to 10 3/4	2 5/8	29/32	7/8	1 23/32	2 5/8	10 5/16 to 10 3/4	1 7/8	21/32	5/8	1 9/32	1 15/16
10 13/16 to 11 1/4	2 3/4	15/16	29/32	1 27/32	2 3/4	10 13/16 to 11 1/4	1 15/16	11/16	21/32	1 11/32	1 15/16
11 5/16 to 11 3/4	2 7/8	31/32	15/16	1 29/32	2 7/8	11 5/16 to 11 3/4	2	23/32	11/16	1 13/32	2
11 13/16 to 12 1/4	3	1	31/32	1 31/32	3	11 13/16 to 12 1/4	2 1/16	3/4	23/32	1 15/32	2 1/16

The depth of the keyway in the hub should be measured at the small end of the keyway.

The taper of the keyway in the hub is ⅛ in. to 1 ft.

Feather keys should have depth *K* made so that the bearing pressure shall not exceed 24,000 lbs. per square inch.

Shafts 6 ins. and over in diameter usually should have two keys.

CHAPTER VIII

JOURNALS, BEARINGS, PIVOTS, AND COLLARS

1. Journals.—Figure 8A shows an outline of a simple cantilever journal.

Let P = the total load on the journal, in pounds;

p = the bearing pressure on a unit of projected area, in pounds per square inch;

d = the diameter of the journal, in inches;

l = the length of the journal, in inches;

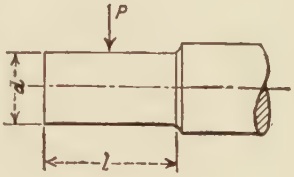


FIG. 8A.

S = the bending fiber stress at the circumference of the journal, in pounds per square inch;

M = the bending moment at the end of the journal, in inch-pounds.

Then:

$$M = \frac{Pl}{2} = \frac{2SI}{d} = \frac{\pi d^3}{32}S,$$

from which,

$$d = \sqrt[3]{\frac{16Pl}{\pi S}} = 1.7205\sqrt[3]{\frac{Pl}{S}}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If it is assumed that $S = 15{,}000$, Eq. (1) becomes

$$d = 0.06976\sqrt[3]{Pl}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If S is taken at 12,000,

$$d = 0.07515\sqrt[3]{Pl}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From these formulae d is found when P and l are known.

When investigating the proportions of a journal when both d and l are unknown, the following expressions are convenient:

$$Pl = \frac{\pi d^3}{16}S \quad \text{or} \quad \frac{Pl}{d} = \frac{\pi d^2}{16}S.$$

From this,

$$d = \sqrt{\frac{16}{\pi} \times \frac{P}{S} \times \frac{l}{d}} = 2.2568\sqrt{\frac{P}{S} \times \frac{l}{d}}. \quad . \quad . \quad . \quad . \quad (4)$$

If it is assumed that $S = 15{,}000$, Eq. (4) becomes

$$d = 0.01843\sqrt{P\frac{l}{d}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If S is taken as 12,000,

$$d = 0.02060\sqrt{P\frac{l}{d}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

When P is known, these expressions can be entered with any assumed value of $\frac{l}{d}$.

The pressure per unit of projected area on the lower half of the bearing is

$$p = \frac{P}{ld}.$$

For any assumed value of p,

$$\frac{l}{d} = \sqrt{\frac{\pi}{16} \times \frac{S}{p}} = 0.4431\sqrt{\frac{S}{p}}. \quad . \quad . \quad . \quad . \quad (7)$$

If it is assumed that $S = 15{,}000$, Eq. (7) becomes

$$\frac{l}{d} = 54.268\sqrt{\frac{1}{p}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If S is taken as 12,000,

$$\frac{l}{d} = 48.539\sqrt{\frac{1}{p}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In these equations the bending moment is considered as determining the diameter of the shaft; but many shafts are subject to both bending and torsional moments, and the diameters would be determined from the equivalent torsional moments, according to the method developed in Chapter VII, Arts. 1 to 12.

Work of friction.—In addition to the notation given above:

Let W = the work of friction per minute, in foot-pounds;

f = the coefficient of friction;

N = the number of rotations of the shaft per minute.

Then:

$$W = \frac{\pi}{12}dPfN = 0.2618dPfN. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The work of friction reduced to horse-power is

$$\text{h.p.} = \frac{\pi}{12 \times 33{,}000}dPfN = 0.00000794dPfN. \quad . \quad . \quad (11)$$

2. Location of bearings.—Bearings should be located so that the unit bearing pressures may be as nearly uniform as possible. This will

result in more nearly uniform wear of the shafts and bearings and will hold the shafts in better alignment than when the pressures and the resulting wear are uneven. The serviceable life of the machinery will also be increased.

Figure 8B shows a shaft with a gear, AB, and a pinion, CD. Assume rotation as indicated, and that AB is driven by a force at A perpendicular to the plane of the paper. The force delivered at D or C will be greater than that at A.

When delivered at D, the force A and the reaction D are in the same direction, and the resultant is equal to the sum of the two and is applied at E. For equal pressures in the bearings, one should be at F and the other at I, equidistant from E. If the second bearing is placed at G in place of I, the lengths of F and G should be inversely proportional to their distances from E, for equal unit pressures.

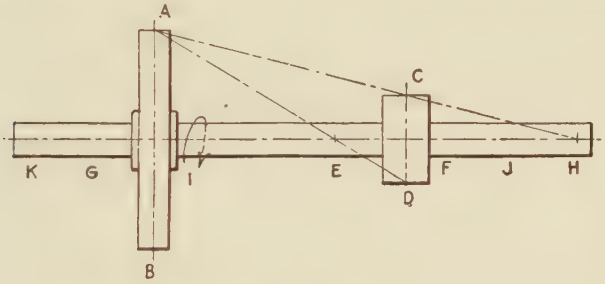


FIG. 8B.

When the force is delivered at C, force A and reaction C are in opposite directions and the resultant is the difference between them, and acts at H. For equal pressures, the two bearings should be placed symmetrically with respect to H. This is not feasible. One of the bearings should be at J, as near H as possible, and the other at K, as far from H as feasible, consistently with the stiffness and strength of the shaft, and other considerations.

These principles should be kept in mind and the mechanism laid out as nearly in the ideal manner as possible.

Figure 8C represents a gear turning on a fixed stud. Sometimes, when the clearances on one side of the gear are close, an attempt is made to reduce the bearing pressure between the hub of the gear and the stud by lengthening the hub on one side of the central plane of the gear. This results in an unequal distribution of the pressures, and unequal wear of the parts. Suppose that $CD = 2CB$; then the pressure at B will be twice what it would have been had the central

plane of the gear been midway between B and D, and $1\frac{1}{3}$ times as much as it would have been had the length of the hub been made EB, or twice BC. Arrangements similar to this should be avoided for they result in rapid and unequal wear and false alignment of the shafts.

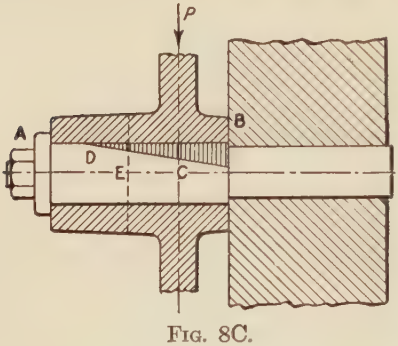


Fig. 8C.

3. Bearings.—In proportioning journals and bearings, the designer must compromise conflicting conditions. For strength with small deflections, the journal should be large and short. For the best lubrication, the bearing pressure should be small, and the bearing should be large and long. To reduce the loss work due to friction and prevent heating, the journal should be of small diameter and long.

In a new design there is scope for good judgment. The values of $\frac{l}{d}$ given below will be found to be suitable for the journals and bearings of movable-bridge machinery:

Type of bearing	Values of $\frac{l}{d}$
Rocking shafts and cranks	1
Heavy, slow-moving bearings, like trunnions and main-pinion shafts	1.5 to 1.65
Ordinary shaft bearings	1.75 to 2
High-speed bearings	2 to 3

4. Typical pedestal bearings.—The general principles of the design of ordinary pedestal bearings are best illustrated by examples, from practice, of bearings that have given good service.

Figure 8D and Table 8A give typical details of pedestal bearings with two cap and two base bolts. Figures 8E and Table 8B give the same kind of data for bearings with four cap and four base bolts.

These bearings were proportioned for cast steel and for a maximum bearing pressure of 500 lbs. per square inch on the projected area, and the pedestal bases for a maxium fiber stress of 12,000 lbs. per square inch. If the bearing pressure is equal to or less than 300 lbs. per square inch, cast iron can be used. If cast steel is used the pedestals and caps must be babbitted or lined with bronze. If of cast iron, they may be babbitted or the shaft may be allowed to run in the unlined cast iron,

PEDESTAL BEARINGS.

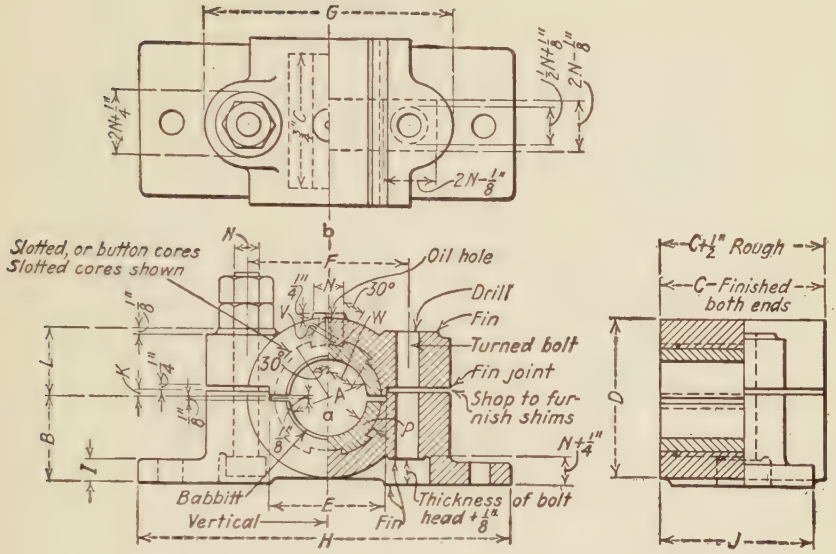


FIG. 8D.

TABLE 8A

PEDESTAL BEARINGS WITH TWO CAP AND TWO BASE BOLTS

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*N*	*P*	*V*	*W*	Weight Complete
2	$2\frac{1}{4}$	4	$4\frac{1}{4}$	$3\frac{1}{4}$	$4\frac{1}{4}$	$6\frac{3}{4}$	$12\frac{1}{2}$	1	$3\frac{1}{2}$	$\frac{1}{4}$	$1\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{3}{8}$	
$2\frac{1}{4}$	$2\frac{1}{2}$	$4\frac{1}{2}$	$4\frac{5}{8}$	$3\frac{5}{8}$	$4\frac{5}{8}$	$7\frac{1}{8}$	13	1	4	$\frac{1}{4}$	2	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{3}{8}$	
$2\frac{1}{2}$	$2\frac{3}{4}$	5	$5\frac{1}{8}$	4	5	$7\frac{1}{2}$	$13\frac{1}{4}$	$1\frac{1}{8}$	$4\frac{1}{2}$	$\frac{1}{4}$	$2\frac{1}{8}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{2}$	
$2\frac{3}{4}$	3	$5\frac{1}{2}$	$5\frac{1}{2}$	$4\frac{1}{4}$	$5\frac{3}{8}$	$8\frac{3}{8}$	$14\frac{1}{2}$	$1\frac{1}{4}$	5	$\frac{1}{4}$	$2\frac{3}{8}$	$\frac{7}{8}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{2}$	
3	$3\frac{1}{4}$	6	6	$4\frac{5}{8}$	$5\frac{7}{8}$	$9\frac{3}{8}$	$16\frac{1}{4}$	$1\frac{3}{8}$	$5\frac{1}{4}$	$\frac{5}{16}$	$2\frac{1}{2}$	1	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{5}{8}$	
$3\frac{1}{4}$	$3\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{3}{8}$	$4\frac{7}{8}$	$6\frac{1}{8}$	$9\frac{5}{8}$	$16\frac{1}{2}$	$1\frac{3}{8}$	$5\frac{3}{4}$	$\frac{5}{16}$	$2\frac{5}{8}$	1	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{5}{8}$	
$3\frac{1}{2}$	$3\frac{3}{4}$	7	7	$5\frac{1}{4}$	$6\frac{3}{8}$	$10\frac{1}{2}$	$17\frac{1}{4}$	$1\frac{1}{2}$	$6\frac{1}{4}$	$\frac{5}{16}$	$2\frac{7}{8}$	$1\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{4}$	

PEDESTAL BEARINGS.

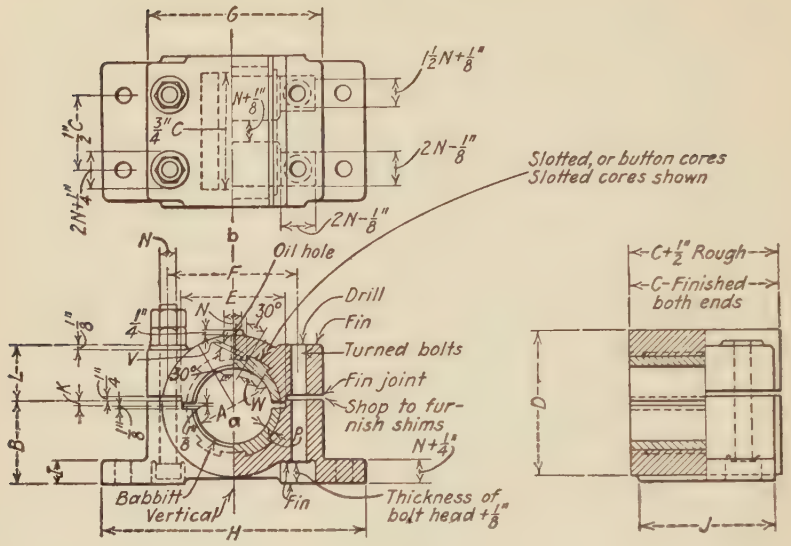


FIG. 8E.

TABLE 8B

PEDESTAL BEARINGS WITH FOUR CAP AND FOUR BASE BOLTS

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*N*	*P*	*V*	*W*	Weight Complete
2½	2¾	5	5⅛	4	5	7	12¾	1⅛	4¾	¼	2⅛	¾	5/16	3/16	½	
2¾	3	5½	5⅜	4¼	5¼	7¼	13	1⅛	5¼	¼	2¼	¾	5/16	3/16	½	63
3	3¼	6	5⅞	4⅝	5⅝	7⅝	13½	1¼	5½	5/16	2⅜	¾	5/16	3/16	5/8	68
3¼	3½	6½	6¼	4⅞	5⅞	7⅞	13¾	1¼	6	5/16	2½	¾	5/16	3/16	5/8	
3½	3¾	7	6⅞	5¼	6¼	8¼	14¼	1¼	6½	5/16	2¾	¾	⅜	¼	¾	84
3¾	4	7½	7¼	5⅝	6¾	9	15¼	1⅜	7	5/16	2⅞	⅞	⅜	¼	¾	
4	4	8	7⅝	5⅞	7	9¼	15½	1⅜	7½	⅜	3	⅞	⅜	¼	¾	137
4¼	4¼	8½	8	6⅛	7⅜	9⅞	16¼	1½	8	⅜	3¼	1	⅜	¼	¾	
4½	4½	9	8⅝	6¾	8	10⅛	17¼	1½	8½	⅜	3⅜	1	½	¼	¾	200
4¾	4¾	9½	9	7	8¼	10¾	17½	1½	9	⅜	3½	1	½	¼	¾	
5	5	10	9⅜	7⅜	8¾	11½	18¾	1⅝	9⅜	½	3⅝	1⅛	½	¼	¾	
5¼	5¼	10½	9⅞	7⅝	9	11¾	19	1⅝	9⅞	½	3⅞	1⅛	½	5/16	1	326
5½	5½	11	10½	8¼	9¾	12¾	21	1¾	10¼	½	4	1¼	5/8	5/16	1	
5¾	5¾	11½	11	8½	10	13	21¼	1¾	10¾	½	4¼	1¼	5/8	5/16	1	
6	6	12	11¼	8¾	10¼	13¼	21½	1¾	11¼	½	4⅜	1¼	5/8	5/16	1	422

provided that soft gray iron is used and that the workmanship is accurate, the bearing surfaces smooth, and the shafts in good alignment.

The dove-tailed recesses to anchor the babbitt should never be omitted when the base and cap are to be babbitted. They should stop short of the ends of both base and cap, in order to anchor the babbitt against longitudinal movement. Oil holes and grooves should always be provided. Clearance should be allowed between the cap and base, so that the cap can be adjusted to the shaft when the lining wears. These clearance spaces should be filled with thin shims and adjustment made so that the shaft runs freely.

5. Lubrication of bearings.—The cap of every common shaft bearing should be drilled for the application of oil or grease. In high-class

LUBRICATION OF HORIZONTAL BEARINGS.

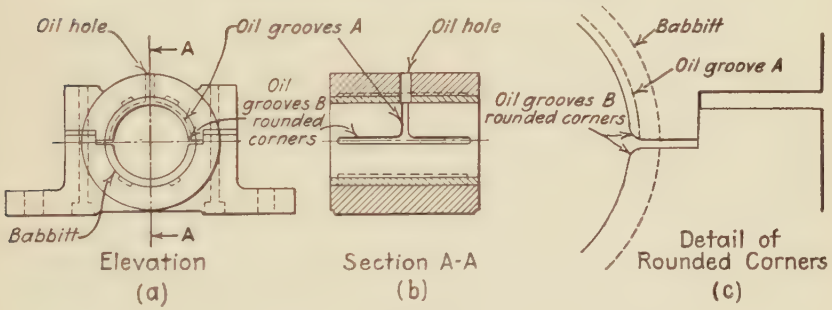


FIG. 8F.

machinery, an oil or grease cup should be fitted to each cap. Oil will usually be found to be the best for horizontal bearings, in which the shafts run at ordinary and high speeds. Grease is better in vertical bearings, and in horizontal bearings when the speed is slow and the pressures heavy.

Oil grooves of many forms have been used, some curved and expensive to cut. Experiments have proved that both oil and grease are most effectively applied to a bearing when the length of the grooves is normal to the direction of rubbing movement, and that straight grooves are better than curved.

Figure 8F shows the details of a system of oil grooves for a horizontal bearing that has been effective and that can be made cheaply. A semicircular groove, *A*, is cut in the cap to convey the lubricant from the oil hole in the cap to the distributing grooves, *B*. The outside corners of groove *A* should be slightly rounded, when in any hard lining metal, to avoid scoring the shaft. The two distributing grooves,

B, are formed by rounding the corners of the base and cap at the joint between them. The rounding should be to an adequate radius, and should not extend to the ends of the bearing, but should stop $\frac{1}{4}$ to $\frac{1}{2}$ in. back from the ends, depending upon the size of the shaft. This tends to hold the lubricant in the bearing. The liners between the base and cap should be wide enough practically to close the opening and prevent the escape of the lubricant.

LUBRICATION OF VERTICAL BEARINGS.

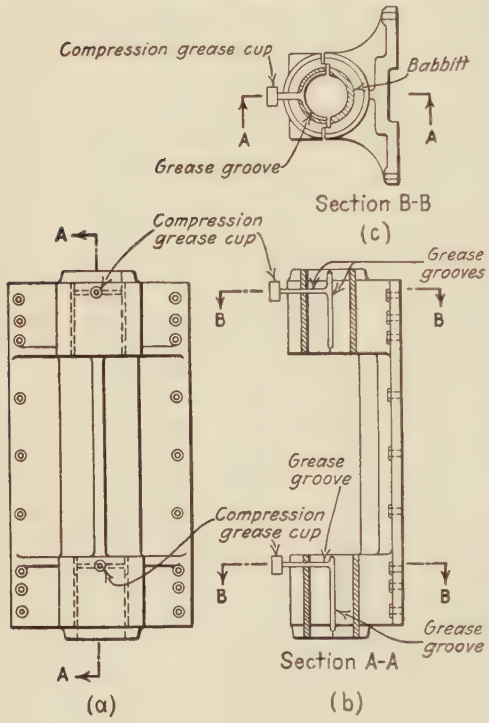


FIG. 8G.

Figure 8G shows the arrangement of similar grooves as applied to a pair of vertical bearings, like a main-pinion shaft of a swing span. The grease holes should be near the tops of the bearings.

These simple grooves are easily made and are more effective than the old systems of complicated curved grooves in the cap.

Whatever form of grooves may be adopted, it is most important that the corners adjacent to the shaft be well rounded. Sharp corners prevent the entrance of the lubricant between the rubbing surfaces and are sure to cause trouble.

6. Pivot bearings.—Figure 8H shows the simplest form of a pivot bearing. Pivot bearings, like foot-steps of shafts and simple thrust bearings, should be fitted with one, or two, bronze discs, to take the weight of, and the axial load upon, the shaft. The bronze should be Class C or Class B, depending upon the unit pressure. (Refer to the specifications in Appendix A, Arts. 804 and 805.) The simplest form of bearing, consisting of a single iron or steel casting, will serve most purposes in movable bridges. If the bearing is of soft gray cast iron, the side pressure may be taken directly by the cast iron. If of cast steel, the hole should be babbitted or lined with Class C bronze. When a

large bearing is required, it may be made in two parts with good gaskets between them, and so that one side can be removed for repairs. There should also be an annular space at the bottom to hold oil, and provision should be made for draining and cleaning without taking off the side cap.

Work of friction.—The radius of friction of a flat circular disc is $\frac{2}{3}$ of the radius of the disc or $\frac{1}{3}$ of the diameter. Using the same notation as in Art. 1, the work of friction in foot-pounds per minute is

$$W = \frac{\pi}{18} dPfN = 0.1745 dPfN. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The work of friction reduced to horse-power is

$$\text{h.p.} = \frac{\pi}{18 \times 33{,}000} dPfN = 0.00000529 dPfN. \quad . \quad . \quad (13)$$

Lubrication.—The end of the shaft, or the top of the disc, should have at least one diametral oil groove of suitable size and with well-rounded corners. In large bearings, the number of grooves should be increased until the circumferential distance between them is about 6 ins. for relatively high speed and 9 ins. for slow-moving shafts, of say, below 10 r.p.m. Vertical grooves should be cut in the inside of the bearing, with feeding grooves at the top. In large bearings, the top should be covered and the lubricant introduced by an oil cup in the side of the bearing. The vertical grooves should not extend quite to the bottom of the bearing. Vertical pivot bearings should run in a bath of oil, the entire bearing being kept full. When horizontal disc thrust bearings are used, they must be frequently oiled, or supplied by a self-feeding cup, unless they are arranged to be flooded with oil.

PIVOT BEARING.

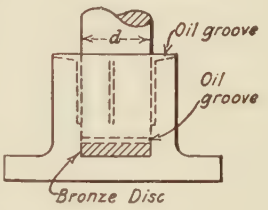


FIG. 8H.

7. Collar bearings.—Figure 8I shows a plain collar bearing on a vertical shaft under the hub of a gear wheel. When such a bearing is required, it should be of Class C bronze. A thrust bearing should be put under the hub of every gear on a vertical shaft, unless the shaft rests in a step bearing. When the design shown in Fig. 8I is used, some special device is necessary to prevent the gear from slipping off the shaft. Ordinary keys, alone, are seldom satisfactory. Thrust collars should always be used between bevel-gear hubs and the bearings, and the gears secured against slipping. Every worm should have collar, roller, or ball thrust bearings.

Work of friction.—The radius of friction of a flat collar is

$$R_f = \frac{2}{3} \times \frac{r_0^3 - r_i^3}{r_0^2 - r_i^2}. \quad \ldots \ldots \ldots \ldots \quad (14)$$

Using the notation in Art. 1, the work of friction in foot-pounds per minute is

$$W = 2\pi \times \frac{2}{3} \times \frac{1}{12} R_f P f N = \frac{\pi}{9} R_f P f N = 0.34906 R_f P f N. \quad (15)$$

The work of friction reduced to horse-power is

$$\text{h.p.} = \frac{\pi}{9 \times 33{,}000} R_f P f N = 0.0000106 R_f P f N. \quad \ldots \quad (16)$$

Lubrication.—This is a difficult class of bearings to lubricate properly. In the case shown in Fig. 8I, oil grooves should be cut in both surfaces of the collar, not over 6 or 9 ins. apart at the circumference, depending upon the speed of the shaft. An oil bath is not practicable and usually there is not space enough for an oil or grease cup. It therefore becomes necessary to oil by hand, from a can. This method requires frequent attention.

COLLAR BEARING.

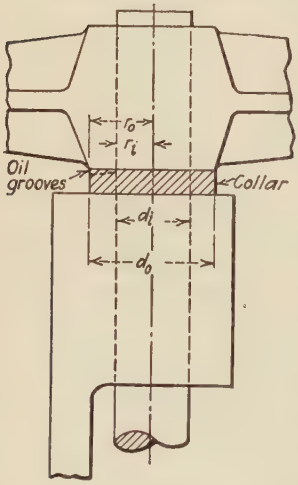


FIG. 8I.

When a collar bearing is used to take the thrust of a worm, it should run in an oil bath, so as to be constantly flooded with oil.

8. Roller bearings.—If roller bearings are used in a movable bridge, great care should be exercised in their selection. An ordinary bridge shop, with such equipment as is appropriate for movable-bridge machinery, should never be entrusted with the manufacture of roller bearings. They require a precision of workmanship that is only attainable with machine tools specially designed for this class of work.

Ball bearings are not suitable for the high pressures that occur in movable-bridge machinery, for the contact between the balls and races is practically only a slightly flattened point contact.

When roller bearings are used, only those types having solid rollers should be chosen. The rollers should be relatively short when compared with their diameters. Long rollers are likely to roll out of align-

ment, and even the best roller cages sometimes fail to hold them in line. There are some types of roller bearings having short, staggered rollers and good cages, that are desirable for this heavy class of machinery. If they cannot be obtained, two or more sets of short rollers in independent cages may be used for long journal bearings.

The rollers should never be allowed to run in contact with an ordinary steel shaft or bearing. The shaft should be fitted with a very hard steel sleeve shrunk upon it, and the bearing housing should be lined with a similar bushing, pressed in place. These sleeves and bushings should be furnished by the manufacturer of the bearings and no bridge shop should be called upon to make them. They can be fitted to the shafts and housings in any good machine shop.

The cages should be properly designed, accurately finished, and strong enough to hold the rollers in line.

Some large roller bearings have been used in movable bridges. One notable example is in the bearings of the trunnions of the Tower Bridge, in London, which is described in Volume I, Chapter IV, Art. 3.

9. General note.—The subject of bearings is one that could be expanded to almost any extent, and space is available for only a limited discussion of the simple types most needed in movable bridges. Much additional information can be found in various mechanical handbooks. The reader also is referred to a treatise entitled "Bearings," by L. P. Alford, M.E., published by the *American Machinist*, in 1911. Paper No. 1103, a part of Vol. 27, 1906, of the *Transactions of the American Society of Mechanical Engineers*, is a symposium on bearings and contains valuable data and the opinions of several engineers who are considered to be authorities on this subject.

CHAPTER IX

GEARING

1. General.—The teeth of gears for movable-bridge machinery should be shaped in accordance with an accepted, interchangeable standard requiring the least possible number of tools for cutting the teeth of the gears that run at high velocities. It also is desirable to select a form of tooth that will give good service in case the gears and shafts are not made and installed so that the pitch lines of the gears are exactly tangent to each other.

A system of interchangeable gears based upon the involute form of tooth, with an obliquity of 20°, probably meets the requirements better than any other; but some authorities now advocate an obliquity of $22\frac{1}{2}°$, on account of the greater strength of the teeth. Involute gears will run without serious difficulty, except for rapid wear, even when the pitch lines are not in exact contact. Gears with cycloidal teeth must be accurately cut and assembled with their pitch lines tangent to each other. The teeth are not as strong as those of the involute form, and more cutters are needed to develop a series of gears. They will not be considered in this discussion.

The theory of the contact of gear teeth and the development of the exact theoretical tooth forms will not be given. Reference should be made to any of the treatises on the subject.[1]

There are two treatises written from a strictly practical viewpoint that will be useful in the design and manufacture of gearing: "Practical Treatise on Gearing," by the Brown and Sharpe Manufacturing Company, and "A Treatise on Gear Wheels," by George B. Grant, published by the Grant Gear Works.

The data given in this chapter are intended for use in the actual designing of movable-bridge machinery, and not as a theoretical development of the subject.

Gears for movable-bridge machinery should be proportioned on the assumption that one tooth transmits the whole load. This is on the

[1] "The Constructor," by F. Reuleaux; "The Elements of Machine Design," by W. Cawthorne Unwin; Kent's "Mechanical Engineers' Handbook"; "Mechanical Engineers' Handbook," Editor, Lionel S. Marks.

side of safety, for in all well-made gearing a part of the load is taken by other teeth than the central one.

The minimum face of spur gears should be about $1\frac{1}{2}$ times the circular pitch. On account of the flexibility of the supports, the face of spur gears should usually be made about 2 to $2\frac{1}{2}$ times the circular pitch. They should rarely be made as wide as 3 times the pitch.

No pinion should have less than 12 teeth, and when feasible 15 or more should be used.

If 20° stub teeth are used, a pinion must have 14 or more teeth to avoid interference.

The numbers of teeth on mating gears should be incommensurate. This insures more uniform action and wear.

The face of bevel gears should never exceed 3 times the circular pitch, nor $\frac{1}{3}$ of the slant height of the pitch cone.

In most cases the gears may have teeth of the standard length, with an addendum, or module, of $\frac{1}{\pi} = 0.3183$ times the circular pitch.

When greater strength is necessary, stub teeth with an addendum of 0.25 times the circular pitch will usually fill the requirements, and this is a form of tooth that can readily be obtained. Special tooth forms should be avoided. They are expensive, and gears are not readily replaced in case of accidents.

When the machinery is not subject to shocks and is protected, so that foreign material cannot fall upon the gears, well-designed cast-iron gears give excellent service and high efficiency.

For general use, cast steel will be preferred, and small, high-speed pinions should be of forged steel.

Spur gears with a pitch-line velocity over 300 ft. per minute should have cut teeth.

Bevel gears with a pitch-line velocity of over 200 ft. per minute should have cut teeth.

2. Strength of spur-gear teeth.—The most useful and satisfactory analysis of the method of determining the strength of gear teeth is that developed by Mr. Wilfred Lewis, M. Am. Soc. M. E.[2]

Figure 9A is a diagram of a spur-gear tooth.

Let W = the load on the tooth, in pounds;
p = the circular pitch in inches;
f = the face of the tooth, in inches;

[2] "Investigation of the Strength of Gear Teeth." A paper by Wilfred Lewis. *Proc. of the Engrs. Club of Phila.*, Vol. X, p. 16, 1893.

S = the permissible fiber stress in the material, at the root of the tooth, in pounds per square inch;

y = a factor depending upon the form of the tooth.

Then will:

$$W = Spfy. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When the load is on the end of the tooth, the tooth is in an oblique position and the direction of the force is in a line ab. This force may be resolved at point b into a force W, tending to break the tooth, and a radial force in the direction of bk, tending to crush it. This latter force may be neglected.

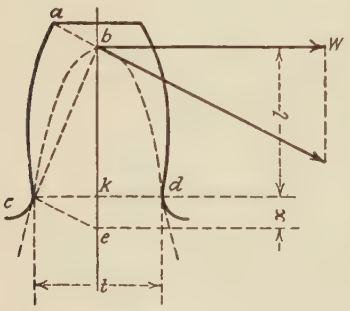


FIG. 9A.

To determine the weakest point in a tooth, advantage is taken of the fact that any parabola on the axis be and tangent to bw at the point b, encloses a beam of uniform strength. Of all the parabolas that may be thus drawn, one will be tangent to the tooth form, and the point of tangency, c, will indicate the weakest section of the tooth.

When the weakest section has been found, the strength at that point is found graphically by drawing bc and the perpendicular ce, intersecting the center line at e. The extreme fiber stress at c equals the external bending moment on the tooth divided by the moment of resistance of the section on cd. If f is the face of the tooth and the other dimensions are as given in the figure, then:

$$S = \frac{Wl}{\frac{1}{6}ft^2}, \quad \text{or} \quad W = \frac{Sft^2}{6l}.$$

By similar triangles,

$$l : \frac{t}{2} = \frac{t}{2} : x,$$

$$x = \frac{t^2}{4l}, \quad \text{or} \quad \frac{t^2}{l} = 4x.$$

Substituting,

$$W = \frac{Sf}{6} \times 4x = Sf\frac{2x}{3}.$$

To introduce the pitch into this equation, it may be written:

$$W = Spf\frac{2x}{3p}.$$

For the factor $\frac{2x}{3p}$, y may be substituted, and the equation written:

$$W = Spfy. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is the fundamental expression on which this discussion of the design of gear teeth is based.

The values of $y = \frac{2x}{3p}$ were found by laying out the various tooth forms to a large scale from which the values of x were found.

3. Strength of bevel-gear teeth.—Formula (1), for spur-gear teeth, can readily be modified so as to serve for the design of bevel-gear teeth. Figure 9B is a diagram of a pair of bevel gears.

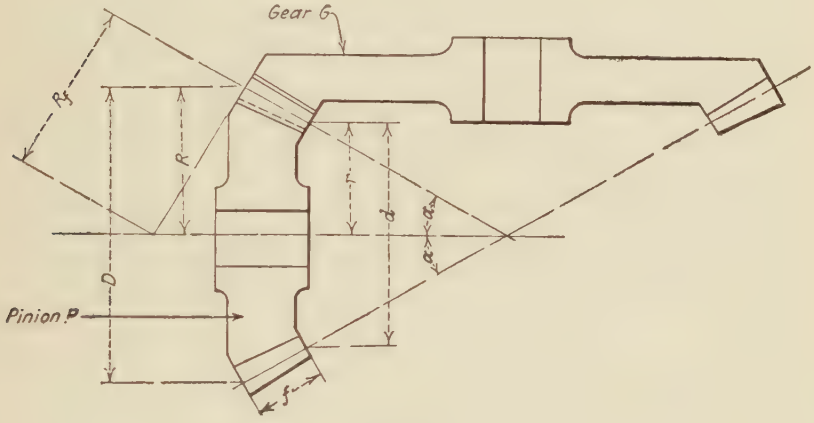


FIG. 9B.

Let P = the circular pitch on the outside pitch circle;
d = the pitch diameter at the inside end of the teeth;
D = the pitch diameter at the outside end of the teeth;
n = the number of teeth on the pinion P;
n_1 = the number of teeth on the mating gear G;
α = one-half of the apex angle of the pitch cone of the pinion;
N_1 = the *formative* number of teeth for the pinion, corresponding to the radius R_f.

$N_1 = n \sec \alpha$. But $\sec \alpha = \sqrt{1 + \tan^2 \alpha}$ and $\tan \alpha = \frac{n}{n_1}$; hence

$$N_1 = n\sqrt{1 + \left(\frac{n}{n_1}\right)^2}$$ for any pair of mating bevel gears.

The strength of the tooth is given by the formula:

$$W_1 = Spfy_1\frac{D^3 - d^3}{3D^2(D - d)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When $\frac{d}{D}$ is not less than $\frac{2}{3}$, the following simplified formula may be used without material error:

$$W_1 = Spfy_1\frac{d}{D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In formulæ (2) and (3), y_1 is the value of y corresponding to the *formative* number of teeth, N_1.

These formulae are arranged to give the permissible tooth load W_1, to be applied at the outside pitch circle, at a radius R, because this is the tooth load that it is most convenient to use in gear calculations.

4. Unit stresses.—In his original paper, Wilfred Lewis proposed unit fiber stresses to be allowed at the roots of gear teeth of cast iron and cast steel, when running at various pitch-line velocities. He used the basic unit stress for all gears running with a velocity of less than 100 ft. per minute and then reduced the units in some proportion of the velocities. At the suggestion of Mr. Carl G. Barth, he later adopted an expression which made the reductions begin at any velocity above zero and which followed a smooth curve for all velocities. This equation is

$$S = S_b\frac{600}{V + 600}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In this expression,

S_b = the basic allowable unit fiber stress at zero velocity;
V = the pitch-line velocity, in feet per minute;
S = the unit fiber stress to be used in the design of the tooth.

This formula has been quite generally used in the design of gearing, including that for movable bridges.

In 1912, Mr. Guido H. Marx reported the results of a series of tests on the actual strength of cast-iron gears.[3] In 1915, Mr. Marx and Mr. Lawrence E. Cutter gave the results of a second set of tests.[4] These tests indicated that the stresses for velocities below 100 ft. per minute were larger than had been expected, but that for relatively high velocities they were less than those contemplated by previous

[3] "The Strength of Gear Teeth," by Guido H. Marx, M. A. S. M. E., *Trans. A. S. M. E.*, Vol. 34, 1912, p. 1323.

[4] "The Strength of Gear Teeth," by Guido H. Marx, M. A. S. M. E., and Lawrence E. Cutter, M. A. S. M. E., *Trans. A. S. M. E.*, Vol. 37, 1915, p. 503.

methods of design, this difference beginning at about 300 ft. per minute pitch-line velocity.

Figure 9C shows, by curves, decimal coefficients giving the variation of the unit fiber stress with varying velocities in accordance with Eq. (4), and also the results of the Marx and of the Marx and Cutter tests on cast-iron gears, with $14\frac{1}{2}°$ involute and Fellows stub teeth.

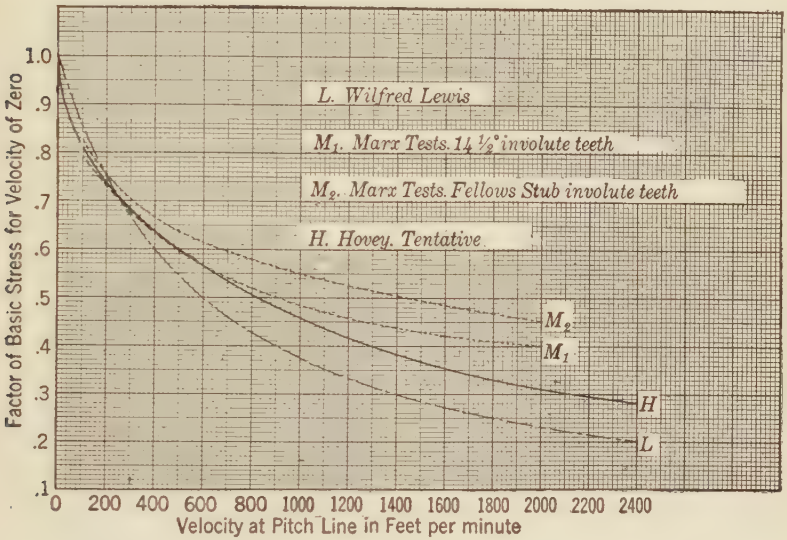


Fig. 9C.

After a study of these tests and all other available data, the author has tentatively adopted the following expression in his practice:

$$S = S_b\left[1 - \frac{1}{4200}\sqrt{6200V - V^2}\right]. \quad . \quad . \quad . \quad . \quad (5)$$

The curve marked H follows this formula. It will be noted that when V is less than 100, the allowable unit fiber stresses correspond closely to the results of the tests referred to, but that for values of V above 300 the allowable stresses are intermediate between those from Eq. (4) and the tests.

Further tests are proposed by a committee of the American Society of Mechanical Engineers, which it is hoped will give valuable new data.

Table 9A gives the values of S in accordance with Eq. (5) for forged and cast steel, bronze and cast iron. When bronze is used, it should be in accordance with the Specifications in Appendix A, Arts. 804 to 808, Class D.

TABLE 9A

VALUES OF S FROM $S = S_B\left[1 - \frac{1}{4200}\sqrt{6200V - V^2}\right]$

V	Forged Steel	Cast Steel	Bronze	Cast Iron	V	Forged Steel	Cast Steel	Bronze	Cast Iron
0	22,000	20,000	10,000	8000	500	13,160	11,960	5980	4780
25	19,950	18,120	9,060	7250	600	12,400	11,270	5640	4510
50	19,100	17,360	8,680	6940	700	11,730	10,660	5330	4260
75	18,410	16,740	8,370	6690	800	11,120	10,100	5050	4040
100	17,910	16,280	8,140	6510	900	10,560	9,600	4800	3840
125	17,440	15,850	7,920	6340	1000	10,060	9,140	4570	3660
150	17,010	15,470	7,730	6190	1100	9,600	8,720	4360	3490
175	16,630	15,120	7,560	6040	1200	9,170	8,340	4170	3330
200	16,270	14,790	7,390	5910	1300	8,780	7,980	3990	3190
225	15,930	14,480	7,240	5790	1400	8,420	7,660	3830	3060
250	15,610	14,190	7,100	5680	1500	8,090	7,360	3680	2940
275	15,320	13,920	6,960	5570	1600	7,790	7,080	3540	2830
300	15,040	13,670	6,830	5470	1800	7,260	6,600	3300	2640
350	14,510	13,190	6,590	5280	2000	6,820	6,200	3100	2480
400	14,030	12,750	6,370	5100	2200	6,460	5,880	2940	2350
450	13,580	12,340	6,170	4940	2400	6,180	5,620	2810	2250

Recommended Maximum Pitch Line Velocities in Feet per Minute

Ordinary cast spur gears	300	Ordinary cast bevel gears	200
Cut spur gears	2000	Cut bevel gears	1200

Gears in practically continuous service, subject to frequent starting, stopping, and reversal, should be designed for unit stresses two-thirds of those given in the table.

5. Values of y.—The values of y that should be used in Formulae (1), (2), and (3) are given in Table 9B, for the tooth forms indicated by the column headings.

6. Circular and diametral pitch.—The pitch surfaces of a pair of simple spur gears are parts of the surfaces of two imaginary cylinders which would roll tangent to each other at the required velocity ratio. The pitch surfaces are commonly called pitch circles, or pitch lines. The pitch circles, or lines, are the lines to which the proportions of gear teeth are referred.

Circular pitch is the length of an arc on the pitch circle, or line, from the working face of one tooth to the corresponding face of the adjacent tooth. It should be noted that when the number of teeth and the circular pitch are assumed, the pitch diameter becomes

$$D = \frac{NP}{\pi}.$$

TABLE 9B

VALUES OF y

Number of Teeth N	Involute 20° Stub. Addendum = 0.25×C.P. $y = 0.178 - \frac{1.033}{N}$	Involute 20°, Addendum = 0.3183×C.P. $y = 0.154 - \frac{0.912}{N}$	Involute $14\frac{1}{2}°$ and Cycloidal. Addendum = 0.3183×C.P. $y = 0.124 - \frac{0.684}{N}$	Radial Flanks. Addendum = 0.3183×C.P. $y = 0.075 - \frac{0.276}{N}$
12	.....	0.078	0.037	0.052
13	.....	.084	.071	.054
14	0.104	.089	.075	.055
15	.109	.093	.078	.057
16	.113	.097	.081	.058
17	.117	.100	.084	.059
18	.121	.103	.086	.060
19	.124	.106	.088	.060
20	.126	.108	.090	.061
21	.129	.111	.091	.062
23	.133	.114	.094	.063
25	.137	.117	.097	.064
27	.140	.120	.099	.065
30	.144	.124	.101	.066
34	.148	.127	.104	.067
38	.151	.130	.106	.068
43	.154	.133	.108	.069
50	.157	.136	.110	.069
60	.161	.139	.113	.070
75	.164	.142	.115	.071
100	.168	.146	.117	.072
150	.171	.148	.119	.073
300	.175	.151	.122	.074
Rack	0.178	0.154	0.124	0.075

When P is in even inches, D contains inconvenient fractions. When D is taken in even inches,

$$P = \frac{D\pi}{N}.$$

This also contains inconvenient fractions.

In order to avoid fractional diameters and to simplify the calculations, as well as to reduce the variety of circular pitches and permit the choice of a system of interchangeable gears, the diametral-pitch system was devised and has been widely used. For most purposes it is likely to supersede the circular-pitch system.

The diametral pitch is not a measurement, but a ratio. It is the number of teeth on a gear divided by the pitch diameter of the gear, in inches; or it is the number of teeth to each inch of the pitch diameter. Thus, a gear having 30 teeth and 30 ins. pitch diameter is 1 pitch, or as commonly stated, the gear is 1 D.P. The corresponding circular pitch is π, or 3.14159 ins. A gear with 30 teeth and 15 ins. pitch diameter is 2 D.P., with a circular pitch of

$$\frac{\pi}{2} = 1.57078 \text{ ins.}$$

The convenience of being able to assume commensurate numbers of teeth and pitch diameters, without inconvenient fractions, is obvious.

Fundamental relations.—The following fundamental relations are frequently needed in gear calculations:

$$\text{Pitch diameter,} \quad D = \frac{N}{\text{D.P.}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\text{Pitch diameter,} \quad D = \frac{N \times \text{C.P.}}{\pi}, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\text{Diametral pitch,} \quad \text{D.P.} = \frac{\pi}{\text{C.P.}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\text{Circular pitch,} \quad \text{C.P.} = \frac{\pi}{\text{D.P.}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\text{Number of teeth,} \quad N = \text{D} \times \text{D.P.}, \quad . \quad . \quad . \quad . \quad (10)$$

$$\text{Number of teeth,} \quad N = \frac{D\pi}{\text{C.P.}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

7. Grant's Odontograph.—Several methods have been proposed for laying out the profiles of gear teeth on drawings, patterns, or gear blanks. The simplest and most practical is that developed by Mr. George B. Grant, and given in full detail in his "Treatise on Gear Wheels."

Figure 9D is a diagram of a 12-tooth pinion meshing with a rack, which is the extreme case likely to occur in movable-bridge machinery.

Table 9C gives the pitch diameters, for one inch circular pitch, for gears with from 10 to 100 teeth.

Table 9D gives the usual proportions of gear teeth in terms of circular pitch.

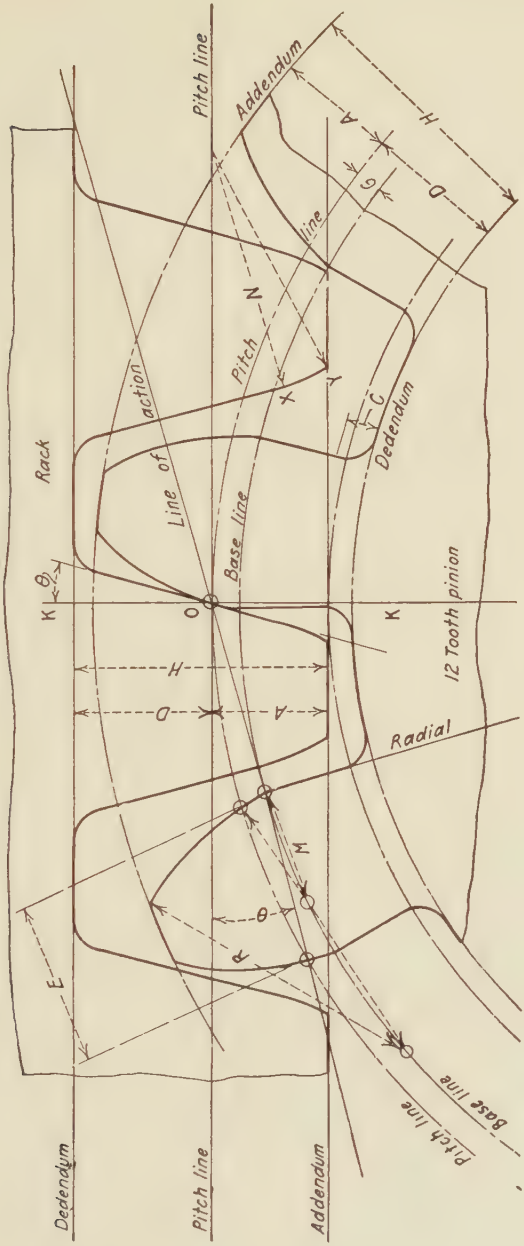

Pitch line
Addendum
Rack
Line of action
Pitch line
Base line
Dedendum
12 Tooth pinion
Radial
Base line
Pitch line
Dedendum
Pitch line
Addendum

FIG. 9D.

Table 9E gives the data for Grant's Odontograph method of drawing the outlines of $14\frac{1}{2}°$ and 20° involute teeth.

TABLE 9C

Pitch Diameters for One Inch Circular Pitch

For Any Other Pitch Multiply by that Pitch

T.	P. D.	T.	P. D.	T.	P. D.	T.	P. D.	T.	P. D.	T.	P. D.	T.	P. D.
10	3.183	23	7.320	36	11.458	49	15.598	62	19.737	75	23.878	88	28.010
11	3.501	24	7.633	37	11.778	50	15.915	63	20.052	76	24.191	89	28.330
12	3.819	25	7.956	38	12.095	51	16.233	64	20.371	77	24.510	90	28.648
13	4.137	26	8.275	39	12.415	52	16.552	65	20.691	78	24.830	91	28.965
14	4.456	27	8.593	40	12.732	53	16.870	66	21.009	79	25.145	92	29.283
15	4.774	28	8.911	41	13.051	54	17.190	67	21.327	80	25.464	93	29.602
16	5.092	29	9.230	42	13.368	55	17.507	68	21.647	81	25.783	94	29.921
17	5.410	30	9.547	43	13.686	56	17.827	69	21.963	82	26.101	95	30.239
18	5.729	31	9.866	44	14.005	57	18.144	70	22.281	83	26.420	96	30.358
19	6.047	32	10.186	45	14.324	58	18.462	71	22.602	84	26.732	97	30.875
20	6.365	33	10.503	46	14.642	59	18.781	72	22.918	85	27.056	98	37.195
21	6.684	34	10.822	47	14.961	60	19.098	73	23.235	86	27.375	99	31.518
22	7.002	35	11.141	48	15.278	61	19.417	74	23.555	87	27.693	100	31.831

TABLE 9D

Proportions of Gear Teeth

P = Circular Pitch

Part	$14\frac{1}{2}°$ Cast	$14\frac{1}{2}°$ Cut	20° Std. Cast	20° Std. Cut	20° Stub Cast	20° Stub Cut
θ	$14\frac{1}{2}°$	$14\frac{1}{2}°$	20°	20°	20°	20°
A	$0.3P$	$0.32P$	$0.3P$	$0.32P$	$0.25P$	$0.25P$
D	$0.4P$	$0.37P$	$0.4P$	$0.37P$	$0.33P$	$0.3P$
H	$0.7P$	$0.69P$	$0.7P$	$0.69P$	$0.58P$	$0.55P$
E	$0.475P$	$0.5P$	$0.475P$	$0.5P$	$0.475P$	$0.5P$
C	$0.1P$	$0.1P$	$0.1P$	$0.1P$	$0.1P$	$0.1P$
G	P. D. ÷ 60	P. D. ÷ 60	P. D. ÷ 33	P. D. ÷ 33	P. D. ÷ 33	P. D. ÷ 33

TABLE 9E

ODONTOGRAPH TABLE FOR ONE INCH CIRCULAR PITCH STANDARD INTERCHANGEABLE INVOLUTE TEETH. CENTERS ON BASE LINE

Number Teeth	$14\frac{1}{2}°$ Involute				20° Involute			
	Divide by Diameter Pitch		Multiply by Circular Pitch		Divide by Diameter Pitch		Multiply by Circular Pitch	
	Face Radius, R	Flank Radius, M	Face Radius, R	Flank Radius, M	Face Radius, R	Flank Radius, M	Face Radius, R	Flank Radius, M
10	2.28	0.69	0.73	0.22	Interference		Interference	
11	2.40	0.83	0.76	0.27	Interference		Interference	
12	2.51	0.96	0.80	0.31	Interference		Interference	
13	2.62	1.09	0.83	0.34	Interference		Interference	
14	2.72	1.22	0.87	0.39	3.08	2.23	0.98	0.71
15	2.82	1.34	0.90	0.43	3.33	2.39	1.06	0.76
16	2.92	1.46	0.93	0.47	3.46	2.54	1.10	0.81
17	3.02	1.58	0.96	0.50	3.58	2.67	1.14	0.85
18	3.12	1.69	0.99	0.54	3.71	2.80	1.18	0.89
19	3.22	1.79	1.03	0.57	3.86	2.92	1.23	0.93
20	3.32	1.89	1.06	0.60	4.02	3.05	1.28	0.97
21	3.41	1.98	1.09	0.63	4.21	3.14	1.34	1.00
22	3.49	2.06	1.11	0.66	4.43	3.27	1.41	1.04
23	3.57	2.15	1.13	0.69	4.62	3.36	1.47	1.07
24	3.64	2.24	1.16	0.71	4.84	3.46	1.54	1.10
25	3.71	2.33	1.18	0.74	5.03	3.55	1.60	1.13
26	3.78	2.42	1.20	0.77	5.21	3.64	1.66	1.16
27	3.85	2.50	1.23	0.80	5.40	3.74	1.72	1.19
28	3.92	2.59	1.25	0.82	5.59	3.80	1.78	1.21
29	3.99	2.67	1.27	0.85	5.81	3.89	1.85	1.24
30	4.06	2.76	1.29	0.88	6.00	3.96	1.91	1.26
31	4.13	2.85	1.31	0.91	6.22	4.02	1.98	1.28
32	4.20	2.93	1.34	0.93	6.41	4.08	2.04	1.30
33	4.27	3.01	1.36	0.96	6.63	4.15	2.11	1.32
34	4.33	3.09	1.38	0.99	6.82	4.21	2.17	1.34
35	4.39	3.16	1.39	1.01	7.04	4.24	2.24	1.35
36	4.45	3.23	1.41	1.03	7.22	4.24	2.30	1.35
37–40	4.20		1.34		6.28		2.00	
41–45	4.63		1.48		6.66		2.12	
46–51	5.06		1.61		7.89		2.51	
52–60	5.74		1.83		9.05		2.88	
61–70	6.52		2.07		9.61		3.36	
71–90	7.72		2.46		11.56		3.68	
91–120	9.78		3.11		14.39		4.58	
121–180	13.38		4.26		22.02		7.01	
181–360	21.62		6.88		40.52		12.90	

Method of drawing the outlines of gear teeth.—Draw the pitch line and space it for the teeth.

Draw the addendum line a distance A, taken from Table 9D, outside of the pitch line.

Draw the dedendum line a distance D inside of the pitch line.

Draw the base line a distance G inside of the pitch line.

Draw the root line a distance C outside of the dedendum line.

Take the face radius, R, from Table 9E, divide or multiply it as required by the table, set the dividers to the corrected radius, and draw in all of the faces from the pitch line to the addendum line from centers on the base line.

Take the flank radius, M, from the table, divide or multiply it as required, set the dividers to the corrected radius, and draw in all of the flanks from the pitch line to the base line.

Draw radial flanks from the base line to the root line, and round them into the dedendum line.

Stub involute teeth are drawn like standard teeth of the same obliquity, but the addendum, A, is shorter.

Draw the sides of rack teeth as straight lines inclined to the line of centers, KOK, at an angle θ, equal to the angle of obliquity of action of the teeth.

Draw the ends of the rack teeth, XY, from centers on the rack pitch line, to radii of N.

For $14\frac{1}{2}°$ standard involute teeth, N is 2.10 ins., divided by the diametral pitch, or 0.67 in. multiplied by the circular pitch.

For 20° standard involute teeth, N is 1.78 ins., divided by the diametral pitch, or 0.60 in., multiplied by the circular pitch.

For 20° stub involute teeth, it is not necessary to round the ends of the rack teeth.

8. Proportions of gear teeth.—Tables of the proportions of gear teeth, suitable for movable-bridge and similar heavy machinery, are given as follows:

Table 9F. Involute standard teeth, $14\frac{1}{2}°$ and 20° obliquity, for diametral pitch and cast teeth.

Table 9G. Involute standard teeth, $14\frac{1}{2}°$ and 20° obliquity, for diametral pitch and cut teeth.

Table 9H. Involute standard teeth, $14\frac{1}{2}°$ and 20° obliquity, for circular pitch and cast teeth.

Table 9I. Involute standard teeth, $14\frac{1}{2}°$ and 20° obliquity, for circular pitch and cut teeth.

Table 9J. Involute stub teeth, Nuttall System, 20° obliquity, for diametral pitch and cut teeth.

TABLE 9F

INVOLUTE STANDARD TEETH

$14\frac{1}{2}°$ and 20° Obliquity, Diametral Pitch, Cast Teeth

Diametral Pitch, D. P.	Circular Pitch, $\frac{\pi}{\text{D. P.}} = P$	Thickness of Tooth on Pitch Line, 0.475 P.	Total Depth of Tooth, 0.7 P.	Addendum 0.3 P.
$\frac{1}{2}$	6.2832	2.9845	4.3982	1.8850
$\frac{3}{4}$	4.1888	1.9897	2.9322	1.2566
1	3.1416	1.4923	2.1992	.9424
$1\frac{1}{4}$	2.5133	1.1938	1.7593	.7540
$1\frac{1}{2}$	2.0944	.9948	1.4661	.6283
$1\frac{3}{4}$	1.7952	.8527	1.2566	.5386
2	1.5708	.7461	1.0996	.4712
$2\frac{1}{4}$	1.3963	.6632	.9774	.4189
$2\frac{1}{2}$	1.2566	.5969	.8796	.3770
$2\frac{3}{4}$	1.1424	.5426	.7997	.3427
3	1.0472	.4974	.7330	.3142
$3\frac{1}{2}$	.8976	.4264	.6283	.2693
4	.7854	.3731	.5498	.2356
5	.6283	.2984	.4398	.1885
6	.5236	.2487	.3665	.1571
7	.4488	.2132	.3142	.1346
8	.3927	.1865	.2749	.1178
9	.3491	.1658	.2444	.1047
10	.3142	.1492	.2199	.0943
11	.2856	.1357	.1999	.0857
12	.2618	.1244	.1833	.0785
14	.2244	.1066	.1571	.0673
16	.1963	.0932	.1374	.0589
18	.1745	.0829	.1222	.0523
20	.1571	.0746	.1100	.0471
22	.1428	.0678	.1000	.0428
24	.1309	.0622	.0916	.0393
26	.1208	.0574	.0846	.0362
28	.1122	.0533	.0785	.0337
30	.1047	.0497	.0733	.0314
32	.0982	.0466	.0687	.0295
36	.0873	.0415	.0611	.0262
40	.0785	.0373	.0550	.0235
48	.0654	.0311	.0458	.0196

TABLE 9G

INVOLUTE STANDARD TEETH

$14\frac{1}{2}°$ and 20° Obliquity, Diametral Pitch, Cut Teeth

Diametral Pitch, D. P.	Circular Pitch, $\frac{\pi}{\text{D. P.}} = P$	Thickness of Tooth on Pitch Line, $0.5\,P$ $\frac{\pi}{2\text{ D. P.}}$	Total Depth of Tooth, $0.6866\,P$ $2.1571 \div \text{D. P.}$	Addendum, $0.3183\,P$ $1.0 \div \text{D. P.}$
$\frac{1}{2}$	6.2832	3.1416	4.3140	2.0000
$\frac{3}{4}$	4.1888	2.0944	2.8760	1.3333
1	3.1416	1.5708	2.1570	1.0000
$1\frac{1}{4}$	2.5133	1.2566	1.7256	.8000
$1\frac{1}{2}$	2.0944	1.0472	1.4380	.6666
$1\frac{3}{4}$	1.7952	.8976	1.2326	.5734
2	1.5708	.7854	1.0785	.5000
$2\frac{1}{4}$	1.3963	.6981	.9587	.4444
$2\frac{1}{2}$	1.2566	.6283	.8628	.4000
$2\frac{3}{4}$	1.1424	.5712	.7844	.3636
3	1.0472	.5236	.7190	.3333
$3\frac{1}{2}$	.8976	.4488	.6164	.2857
4	.7854	.3927	.5393	.2500
5	.6283	.3142	.4314	.2000
6	.5236	.2618	.3595	.1666
7	.4488	.2244	.3081	.1429
8	.3927	.1963	.2696	.1250
9	.3491	.1745	.2397	.1111
10	.3142	.1571	.2157	.1000
11	.2856	.1428	.1961	.0909
12	.2618	.1309	.1798	.0833
14	.2244	.1122	.1541	.0714
16	.1963	.0982	.1348	.0625
18	.1745	.0873	.1198	.0555
20	.1571	.0785	.1079	.0500
22	.1428	.0714	.0980	.0455
24	.1309	.0654	.0898	.0417
26	.1208	.0604	.0829	.0385
28	.1122	.0561	.0770	.0357
30	.1047	.0524	.0719	.0333
32	.0982	.0491	.0674	.0312
36	.0873	.0436	.0599	.0278
40	.0785	.0393	.0539	.0250
48	.0654	.0327	.0449	.0208

In practice the thickness of teeth is slightly reduced and the spaces between them is correspondingly increased.

The thickness of gear teeth on their pitch lines, their total depth, and addenda are direct functions of their circular or diametral pitch. The proportions for pitches not given in the tables may readily be found from the relations to pitches noted in the headings of the various columns.

TABLE 9H

INVOLUTE STANDARD TEETH

$14\frac{1}{2}°$ and 20° Obliquity, Circular Pitch, Cast Teeth

Circular Pitch, P.	Diametral Pitch, 3.1416 ÷ P.	Thickness of of Tooth on Pitch Line, 0.475 P.	Total Depth of Tooth, 0.7 P.	Addendum, 0.3 P.
$\frac{3}{4}$	4.1888	.3563	.525	.225
$\frac{7}{8}$	3.5904	.4156	.612	.262
1	3.1416	.4750	.700	.300
$1\frac{1}{4}$	2.5133	.5937	.875	.375
$1\frac{1}{2}$	2.0944	.7125	1.050	.450
$1\frac{3}{4}$	1.7952	.8312	1.225	.525
2	1.5708	.9500	1.400	.600
$2\frac{1}{4}$	1.3962	1.0687	1.575	.675
$2\frac{1}{2}$	1.2566	1.1875	1.750	.750
$2\frac{3}{4}$	1.1424	1.3062	1.925	.825
3	1.0472	1.4250	2.100	.900
$3\frac{1}{4}$	.9666	1.5437	2.275	.975
$3\frac{1}{2}$	.8976	1.6625	2.450	1.050
$3\frac{3}{4}$	.8377	1.7812	2.625	1.125
4	.7854	1.9000	2.800	1.200
$4\frac{1}{4}$	.7392	2.0187	2.975	1.275
$4\frac{1}{2}$	.6981	2.1375	3.150	1.350
$4\frac{3}{4}$	.6614	2.2562	3.325	1.425
5	.6283	2.3750	3.500	1.500
$5\frac{1}{4}$	.5984	2.4937	3.675	1.575
$5\frac{1}{2}$	.5712	2.6125	3.850	1.650
$5\frac{3}{4}$	.5463	2.7312	4.025	1.725
6	.5236	2.8500	4.200	1.800
$6\frac{1}{2}$	.4833	3.0875	4.550	1.950
7	.4488	3.3250	4.900	2.100
8	.3927	3.8000	5.600	2.400

TABLE 9I

INVOLUTE STANDARD TEETH

$14\frac{1}{2}°$ and 20° Obliquity, Circular Pitch, Cut Teeth

Circular Pitch, P.	Diametral Pitch, 3.1416 ÷ P.	Thickness of Tooth on Pitch Line, 0.5 P.	Total Depth of Tooth, 0.6866 P.	Addendum, 0.3183 P.
$\frac{3}{4}$	4.1888	.3750	.5149	.2387
$\frac{7}{8}$	3.5904	.4375	.6007	.2785
1	3.1416	.5000	.6866	.3183
$1\frac{1}{4}$	2.5133	.6250	.8583	.3979
$1\frac{1}{2}$	2.0944	.7500	1.0299	.4775
$1\frac{3}{4}$	1.7952	.8750	1.2016	.5570
2	1.5708	1.0000	1.3732	.6366
$2\frac{1}{4}$	1.3862	1.1250	1.5449	.7162
$2\frac{1}{2}$	1.2566	1.2500	1.7165	.7958
$2\frac{3}{4}$	1.1424	1.3750	1.8882	.8753
3	1.0472	1.5000	2.0598	.9550
$3\frac{1}{4}$	.9666	1.6250	2.2314	1.0346
$3\frac{1}{2}$	.8976	1.7500	2.4030	1.1142
$3\frac{3}{4}$	.8377	1.8750	2.5747	1.1937
4	.7854	2.0000	2.7464	1.2732
$4\frac{1}{4}$	.7392	2.1250	2.9180	1.3528
$4\frac{1}{2}$	.6981	2.2500	3.0897	1.4324
$4\frac{3}{4}$	.6614	2.3750	3.2614	1.5119
5	.6283	2.5000	3.4330	1.5915
$5\frac{1}{4}$	.5984	2.6250	3.6046	1.6711
$5\frac{1}{2}$	.5712	2.7500	3.7763	1.7507
$5\frac{3}{4}$	.5463	2.8750	3.9479	1.8302
6	.5236	3.0000	4.1196	1.9098
$6\frac{1}{2}$	.4833	3.2500	4.4629	2.0690
7	.4488	3.5000	4.8062	2.2281
8	.3927	4.0000	5.4928	2.5464

In practice the thickness of the teeth is slightly reduced and the spaces between them is correspondingly increased.

TABLE 9J

INVOLUTE STUB TEETH

Nuttall System, 20° Obliquity, Cut Teeth

Diametral Pitch, D. P.	Circular Pitch, $\frac{\pi}{\text{D. P.}} = P$	Thickness of Tooth on Pitch Line, $0.5P$	Total Depth of Tooth, $0.55P$	Addendum, $0.25P$
$\frac{1}{2}$	6.2832	3.1416	3.4558	1.5708
$\frac{3}{4}$	4.1888	2.0944	2.3038	1.0472
1	3.1416	1.5708	1.7279	.7854
$1\frac{1}{4}$	2.5133	1.2566	1.3823	.6283
$1\frac{1}{2}$	2.0944	1.0472	1.1519	.5236
$1\frac{3}{4}$	1.7952	.8976	.9874	.4488
2	1.5708	.7854	.8639	.3927
$2\frac{1}{4}$	1.3963	.6982	.7680	.3491
$2\frac{1}{2}$	1.2566	.6283	.6911	.3141
$2\frac{3}{4}$	1.1424	.5712	.6283	.2856
3	1.0472	.5236	.5760	.2618
$3\frac{1}{2}$	.8976	.4488	.4937	.2244
4	.7854	.3927	.4320	.1964
5	.6283	.3142	.3456	.1571
6	.5236	.2618	.2880	.1309
7	.4488	.2244	.2468	.1122
8	.3927	.1963	.2160	.0982
9	.3491	.1745	.1920	.0873
10	.3142	.1571	.1728	.0785
11	.2856	.1428	.1571	.0714
12	.2618	.1309	.1440	.0655
14	.2244	.1122	.1234	.0561
16	.1963	.0981	.1080	.0491
18	.1745	.0872	.0960	.0436
20	.1571	.0785	.0864	.0393
22	.1428	.0714	.0785	.0357
24	.1309	.0654	.0720	.0327
26	.1208	.0604	.0664	.0302
28	.1122	.0561	.0617	.0280
30	.1047	.0523	.0576	.0262
32	.0982	.0491	.0540	.0246
36	.0873	.0436	.0480	.0218
40	.0785	.0392	.0432	.0196
48	.0654	.0327	.0360	.0163

In practice the thickness of teeth is slightly reduced and the spaces between them is correspondingly increased.

9. Arms of gear wheels.—Various rules have been proposed for determining the best number of arms in gear wheels. The formula given by Reuleaux has been found to give satisfactory results. It is difficult of application; hence the curves shown in Fig. 9E have been drawn. The application then becomes very simple and is obvious from an inspection of the curves.

ARMS OF GEARS.

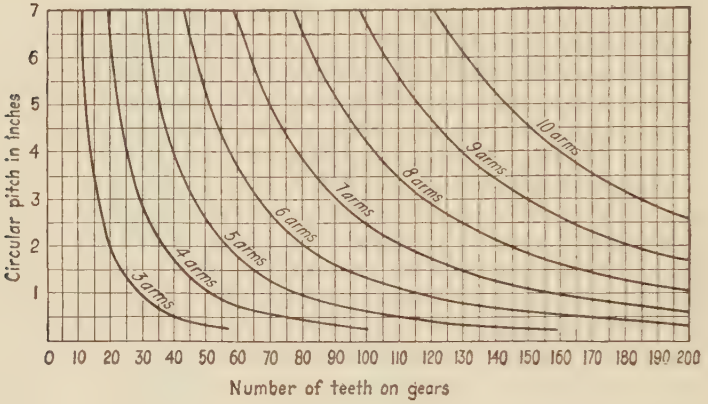


FIG. 9E.

Let N = the number of teeth on the gear;
P = the circular pitch, in inches;
A = the required number of arms.

Reuleaux's formula is

$$A = 0.56\sqrt{N} \times \sqrt[4]{P}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

An even number of arms is desirable, for cast gears are more likely to retain their circular shape during cooling when the number of arms is even than when it is odd; for the opposite arms, when an even number is used, assist in holding the shape of the castings.

When the strength of the arms of a gear is to be determined, the following assumptions and calculations should be made:

The total moment produced by the tooth load, acting with a lever arm equal to the distance from the pitch line to the hub, is considered as distributed equally over all of the arms, and the arms are to be considered as cantilevers.

The moment on one arm divided by the allowed unit fiber stress gives the section modulus for which the arm must be proportioned. If the dimensions of the arm are given, the unit fiber stress may be found.

When the dimensions are not known, they are determined by trial, the process being continued until the arms satisfy the conditions.

All keyways should be cut opposite an arm, and in some cases it is necessary to increase the thickness of the hub, to allow for the reduction in strength due to the keyways.

Let W = the tooth load;
L = the distance from the pitch line to the hub;
N = the number of arms;
S = the maximum allowable fiber stress;
Z = the section modulus.

Then

$\frac{WL}{N}$ = the moment on one arm;

$\frac{WL}{ZN}$ = the extreme fiber stress;

$\frac{WL}{SN}$ = the required section modulus.

10. Example.—Assume a cast-steel gear to have 80 involute teeth of 20° obliquity and 2-in. circular pitch, with a 5-in. face. Assume it to run with a velocity of 100 ft. per minute at the pitch line.

S from Table 9A = 16,280 lbs. per square inch.

y from Table 9B = 0.1426.

Substitute these values in Eq. (1), and

$$W = Spfy = 16{,}280 \times 2 \times 5 \times 0.1426 = 23{,}215 \text{ lbs.}$$

Figure 9E shows that 6 arms should be used. Assume the proportions of the gear to be as shown in Fig. 9F and Table 9K.

The pitch diameter is $\frac{80 \times 2}{\pi} = 50.93$ ins.

The diameter of the hub is $H = 0.085 \times 50.93 + 3.5 \times 2 = 11.33$ ins.

The lever arm of the tooth is $\frac{50.93 - 11.33}{2} = 19.8$ ins.

The moment on one arm is $\frac{23{,}215 \times 19.8}{6} = 76{,}110$ in.-lbs.

(1) Assume the arms to be of elliptical section and of the proportions given.

The section modulus is $0.0982 \times (4.75)^2 \times 2.375 = 5.262$.

The extreme fiber stress is $76{,}110 \div 5.262 = 14{,}460$ lbs. per square inch.

The area of the arm at the hub is $\pi \times 2.375 \times 1.19 = 8.88$ sq. ins.

(2) Assume that the arms are of cruciform section, of the proportions indicated.

The section modulus is $\frac{1}{6} \times (5.52)^2 \times 0.9 + \frac{1}{6} \times (0.6)^2 \times 3.12 = 4.76$.

The extreme fiber stress is $76{,}110 \div 4.76 = 15{,}990$ lbs. per square inch.

The area of the arm at the hub is $5.52 \times 0.9 + 3.12 \times 0.6 = 6.84$ sq. ins.

The maximum unit bending stress in the arms should not exceed that in the teeth.

For small gears, the elliptical form of arm is preferred.

For large gears, the cruciform arm is better and more economical.

11. Proportions of gears.—Figure 9F and Table 9K give proportions of cast-steel and cast-iron gears which are consistent with the methods of design developed in the preceding articles. The dimensions are, in general, based upon the circular pitch of the teeth in inches, and all dimensions are in inches.

The width of face was assumed as $2\frac{1}{2}$ times the pitch, $(2.5P)$.

The dimensions given are nearly the minimum for the unit stresses given in this chapter and in Chapter VI, and should never be used

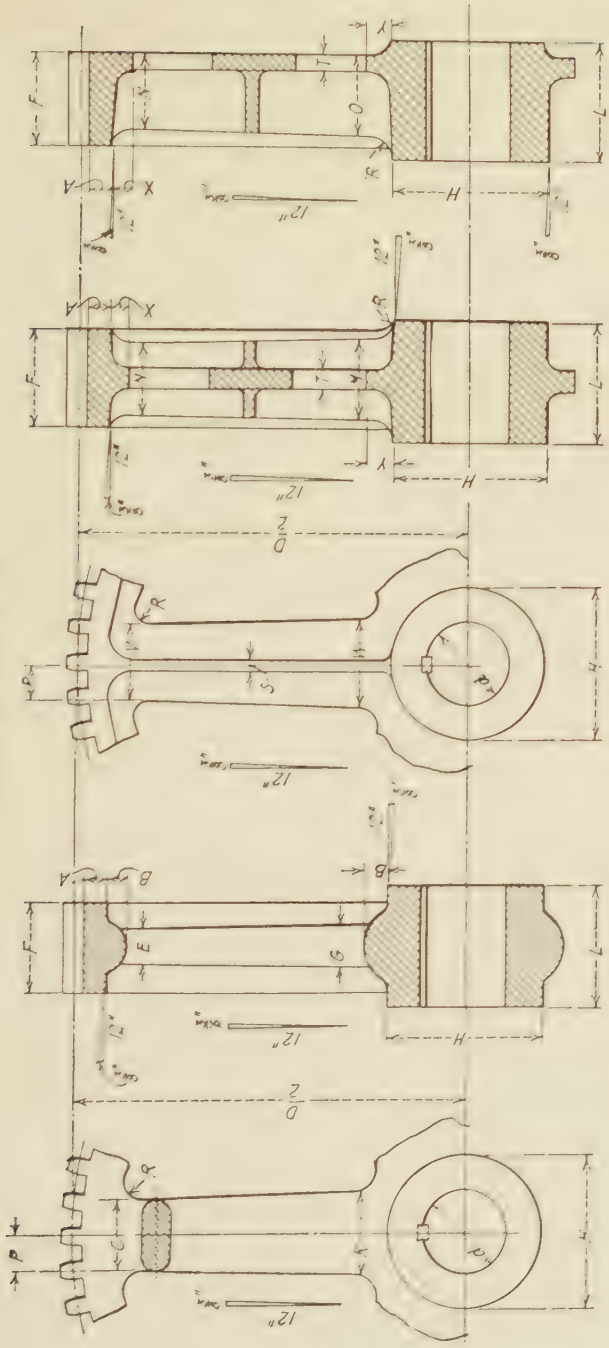
TABLE 9K

P = Circular Pitch, D = Pitch Diameter, and F = Face of Gear in Inches

Dimensions are for Gears when $F = 2.5P$. They May be Used when $F \leqq 3P$

Dimension	$14\frac{1}{2}°$ and 20° Involute Teeth. Addendum, $0.3183P$	20° Stub Involute Teeth. Addendum, $0.25P$	Dimension	$14\frac{1}{2}°$ and 20° Involute Teeth. Addendum, $0.3183P$	20° Stub Involute Teeth. Addendum, $0.25P$
A	$\frac{1}{2}P$	$\frac{1}{2}P$	X	$\frac{3}{4}P$	$\frac{3}{4}P$
B	$\frac{1}{2}P$	$\frac{1}{2}P$	Y	P	P
C	$1\frac{7}{8}P$	$2P$	V	$2\frac{1}{4}P$	$2\frac{1}{2}P$
K	$C+\frac{3}{4}''$ per foot	$C+\frac{3}{4}''$ per foot	W	$V+\frac{3}{4}''$ per foot	$V+\frac{3}{4}''$ per foot
E	$\frac{1}{2}C$	$\frac{1}{2}C$	N	$F-P$	$F-\frac{1}{2}P$
G	$E+\frac{3}{8}''$ per foot	$E+\frac{3}{8}''$ per foot	M	$N+\frac{3}{4}''$ per foot	$N+\frac{3}{4}''$ per foot
L	$1\frac{1}{4}F$ use the	$1\frac{1}{4}F$ use the	O	$N+\frac{3}{8}''$ per foot	$N+\frac{3}{8}''$ per foot
L	$1\frac{1}{4}d$ greater	$1\frac{1}{4}d$ greater	T	$0.45P$	$0.45P$
R	$\frac{1}{2}P$ min.	$\frac{1}{2}P$ min.	S	$0.3P$	$0.3P$
H	$0.085D+3.5P$	$0.09D+3.7P$	d	Largest $= \frac{H}{1.67}$	Largest $= \frac{H}{1.67}$

In special cases d may be made $\frac{3}{3}H$ but hub must then be reinforced over key seat. When $F > 3P$ the strength of the parts must be increased proportionately.



PROPORTIONS OF GEARS.

FIG. 9F.

when the face of the gear is more than 3 times the circular pitch, $(3P)$. Gears with wider faces should be designed for the actual tooth loads. The tooth load, W, is a direct function of the face, and this must not be overlooked.

The thickness of the rim, under the root of the teeth, A, and the dimensions C, E, V, and N, must all be increased for wider faces. The rate of slope of the sides of the arms may be maintained as given by the diagrams and the table of dimensions. The diameter and length of the hub may also require attention.

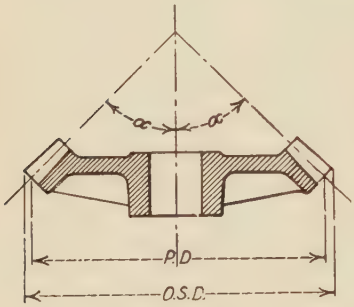


FIG. 9G.

12. Outside diameters of gears.— Table 9L shows the outside diameters of spur and bevel gears in terms of pitch diameter and circular and diametral pitch. Figure 9G shows additional functions of bevel gears that are used in the table.

Let A = the addendum;
$P.D.$ = the pitch diameter;
$C.P.$ = the circular pitch;
$D.P.$ = the diametral pitch;
α = the angle shown in Fig. 9G.

TABLE 9L

OUTSIDE DIAMETERS OF GEARS

Type of Cut Gears	Outside Diameter in Terms of Circular Pitch O. S. D.	Outside Diameter in Terms of Diametral Pitch O. S. D.
Spur gears with standard involute teeth.		
$A = 0.3183 \times$ C. P.	P. D. $+ 0.6366 \times$ C. P.	P. D. $+ \frac{2}{\text{D. P.}}$
Spur gears with stub involute teeth.		
$A = 0.25 \times$ C. P.	P. D. $+ 0.5 \times$ C. P.	P. D. $+ \frac{1.5708}{\text{D. P.}}$
Bevel gears with standard involute teeth.		
$A = 0.3183 \times$ C. P.	P. D. $+ 0.6366 \times$ C. P. $\times \cos \alpha$	P. D. $+ \frac{2 \times \cos \alpha}{\text{D. P.}}$
Bevel gears with stub involute teeth.		
$A = 0.25 \times$ C. P.	P. D. $+ 0.5 \times$ C. P. $\times \cos \alpha$	P. D. $+ \frac{1.5708 \cos \alpha}{\text{D. P.}}$

13. Thrust of gears.—Figure 9H illustrates two gears in mesh. The oblique line of action is indicated, making an angle θ with the tangent to the pitch lines at the point of contact of the teeth.

Then $W \tan \theta$ = the thrust of the gears against their shafts;
$W \sec \theta$ = the side pressure upon the shaft bearings, acting parallel to the line of action.

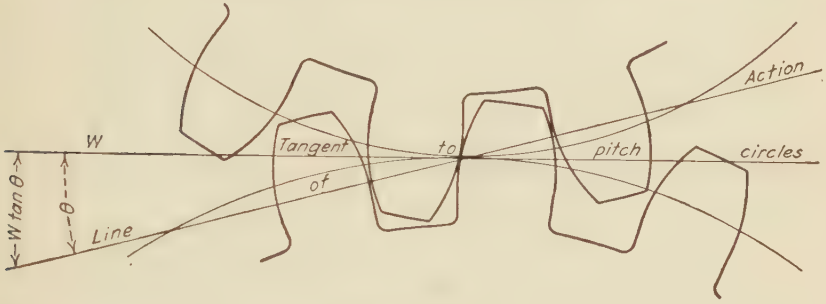


FIG. 9H.

14. Split gears.—In some cases split gears are desirable, as for example, when a broken gear is to be replaced without disturbing the remainder of the gear train. Figure 9I is a sketch of such a gear. The halves must be connected so that the splice will develop the full strength of the teeth.

SPLIT GEAR.

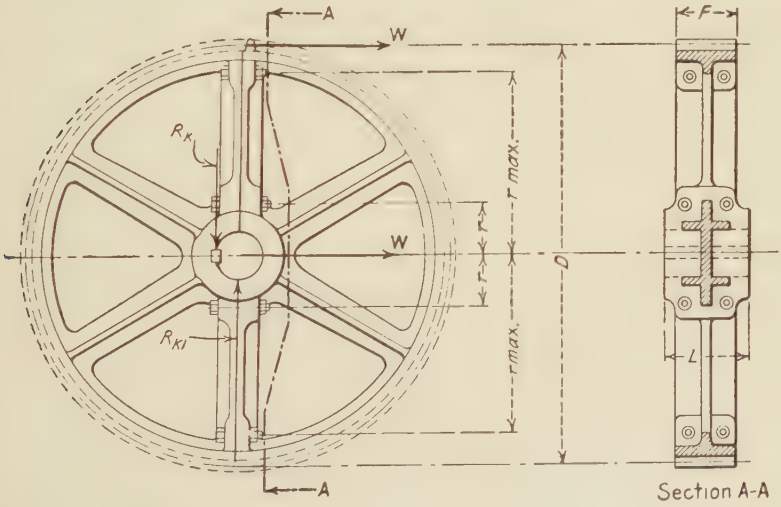


FIG. 9I.

Let D = the pitch diameter of the gear;
W = the tooth load;
N = the number of bolts in the splice;
n = the number of bolts in each row, 2 as shown;
r = the distance of any bolt from the center of the gear;
$r_{\max}$ = the distance to the outside bolts;
A = the net area at the root of the thread of one bolt;
T = the allowable unit stress, in tension, on the net section of the bolts.

The splice must be proportioned for a direct tension of W and a torsional moment of $M_T = W\frac{D}{2}$.

The strength of one bolt in tension $= AT$.

The direct tension on each bolt $= \frac{W}{N}$.

There remains $AT - \frac{W}{N} = T_M$ available to resist the tension on the outer bolts, due to M_T.

In general $M = \frac{fI}{y}$, or

$$M_{TB} = \frac{T_M \times n \times 2\Sigma r^2}{r_{\max}} = \frac{T_M}{r_{\max}} \times n \times 2\Sigma r^2. \quad . \quad . \quad (13)$$

To satisfy the requirements,

$$M_{TB} \gtrless M_T.$$

When the unit stress at the root of the threads of the outside bolt is required,

The direct tension $= \frac{W}{NA} = T_D$.

The tension due to the torsional moment is

$$T_m = \frac{M_T y}{I} = \frac{M_T r_{\max}}{2A\Sigma r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The total tensional unit stress is $T_T = T_D + T_M$.

15. Example.—Assume a cast-steel split gear with 115 standard 20° involute teeth of 3 D.P. and $5\frac{1}{2}$ ins. face.

Assume a pitch-line velocity of 1000 ft. per minute.

Let W = the tooth load = $Spfy$, then

$W = 9140 \times 1.047 \times 5.5 \times 0.146 = 7687$ lbs.

The direct tension on the splice bolts $= W = 7687$ lbs.

The torsional moment, $M_T = 7687 \times \frac{38.333}{2} = 147{,}330$ in.-lbs.

Assume 8 turned bolts $\frac{7}{8}$ in. in diameter, threads U. S. Std.

Let $r = 4\frac{3}{4}$ ins. and $r_{max} = 16\frac{1}{2}$ ins.

$A = 0.419$ sq. in.

$T = 7500$ lbs. per square inch, then

$AT = 3143$ lbs., $\frac{W}{N} = 961$ lbs., and $T_M = 3143 - 961 = 2182$ lbs.

$$\frac{T_M}{r_{max}} = \frac{2182}{16.5} = 132.2.$$

$M_{TB} = 132.2 \times 2 \times 2(16.5^2 + 4.75^2) = 155{,}900$ in.-lbs.

$155{,}900 > 147{,}330$, hence the bolts are adequate.

Find the unit stress in the outer bolts:

The direct tension $= \frac{W}{NA} = \frac{7687}{8 \times 0.419} = 2293$ lbs. per square inch.

The tension from $M_T = \frac{M_T r_{max}}{\Sigma A r^2} = \frac{147{,}330 \times 16.5}{0.419 \times 2 \times 2 \times 294.8} = 4919$ lbs. per square inch.

Total unit stress at the root of bolt threads $= 7212$ lbs. per square inch.

16. Approximate weights of spur gears.—It is often necessary to estimate the weights of spur gears before the designs have been completed. A large number of cast-steel gears were proportioned in accordance with the preceding articles and their weights calculated. From these data Tables 9M and 9N were made.

The gears were all designed for a width of face of $2\frac{1}{2}$ times the circular pitch. When the weights of gears with wider faces are required, the weights from the tables should be increased in direct proportion to the increased width of the faces. For example: gears with faces $3\frac{1}{2}$ times the circular pitch will weigh about $\frac{3.5}{2.5}$ times the weights found directly from the tables.

Case I. Table 9M.

Let P = the circular pitch;

F = the width of face;

D = the pitch diameter, all in inches.

Then:

Approximate weight in pounds $= P \times F \times D \times C$.

The values of C are to be taken from Table 9M.

Case II. Table 9N.

Let M_T = the torsional moment in inch-pounds due to the maximum permissible tooth load.

Then:

$$\text{Approximate weight in pounds} = M_T \times C_1.$$

The values of C_1 are to be taken from Table 9N.

TABLE 9M

APPROXIMATE WEIGHTS OF SPUR GEARS FROM DIMENSIONS

$$W = P \times F \times D \times C$$

VALUES OF C FOR SPUR GEARS WITH 20° STANDARD INVOLUTE TEETH

Number of Teeth	Webs or Arms	P	Webs or Arms	P		Webs or Arms	P				Webs or Arms	P
		1 in.		1½ ins.	2 ins.		2½ ins.	3 ins.	3½ ins.	4 ins.		5 ins.
20	Solid	1.39	Solid	1.40	1.40	Solid	1.41	1.41	1.41	1.40	Solid	1.40
30	Webs	1.37	Webs	1.38	1.38	Webs	1.39	1.40	1.40	1.41	Webs	1.38
40	Webs	1.47	Webs	1.47	1.47	Webs	1.47	1.47	1.48	1.48	Webs	1.55
				Arms of Cruciform Section								
60	6	1.57	6	1.57	1.53	6	1.54	1.53	1.55	1.56	8	1.65
80	6	1.64	6	1.61	1.63	8	1.74	1.74	1.74	1.75	8	1.75
100	6	1.70	8	1.84	1.85	8	1.84	1.85	1.83	1.84	9	1.91
120	6	1.77	8	1.96	1.91	8	1.94	1.91	1.90	1.91		
				Arms of Elliptical Section								
60	6	1.60	6	1.62	1.58	6	1.59	1.58	1.58	1.59	8	1.69
80	6	1.69	6	1.70	1.72	8	1.83	1.85	1.83	1.87	8	1.83
100	6	1.81	8	2.01	2.02	8	1.98	1.98	1.97	2.00	9	2.05
120	6	1.91	8	2.07	2.08	8	2.10	2.11	2.10	2.13		

VALUES OF C FOR SPUR GEARS WITH 20° STUB INVOLUTE TEETH

Number of Teeth	Webs or Arms	P	Webs or Arms	P		Webs or Arms	P				Webs or Arms	P
		1 in.		1½ ins.	2 ins.		2½ ins.	3 ins.	3½ ins.	4 ins.		5 ins.
20	Solid	1.40	Solid	1.40	1.40	Solid	1.40	1.40	1.41	1.41	Solid	1.41
30	Webs	1.39	Webs	1.40	1.40	Webs	1.41	1.41	1.40	1.39	Webs	1.44
40	Webs	1.46	Webs	1.47	1.48	Webs	1.48	1.48	1.48	1.47	Webs	1.58
				Arms of Cruciform Section								
60	6	1.57	6	1.56	1.56	6	1.56	1.56	1.56	1.56	8	1.68
80	6	1.65	6	1.64	1.64	8	1.78	1.76	1.76	1.76	8	1.78
100	6	1.73	8	1.92	1.90	8	1.88	1.86	1.86	1.86	9	1.96
120	6	1.80	8	1.96	1.96	8	1.96	1.96	1.96	1.96		
				Arms of Elliptical Section								
60	6	1.64	6	1.63	1.62	6	1.62	1.62	1.61	1.60	8	1.74
80	6	1.76	6	1.75	1.74	8	1.90	1.89	1.88	1.88	8	1.89
100	6	1.85	8	2.15	2.13	8	2.08	2.04	2.03	2.03	9	2.13
120	6	2.00	8	2.22	2.22	8	2.22	2.22	2.22	2.22		

This table was calculated for widths of face of 2½ times the circular pitch.

TABLE 9N

APPROXIMATE WEIGHTS OF SPUR GEARS FROM TORSIONAL MOMENTS

$$W = M_T \times C_1$$

VALUES OF C_1 FOR SPUR GEARS WITH 20° STANDARD INVOLUTE TEETH

Number of Teeth	Webs or Arms	P	Webs or Arms	P		Webs or Arms	P				Webs or Arms	P
		1 in.		1½ ins.	2 ins.		2½ ins.	3 ins.	3½ ins.	4 ins.		5 ins.
20	Solid	.00161	Solid	.00162	.00162	Solid	.00164	.00164	.00164	.00162	Solid	.00162
30	Webs	.00139	Webs	.00139	.00140	Webs	.00140	.00141	.00141	.00142	Webs	.00139
40	Webs	.00148	Webs	.00148	.00147	Webs	.00148	.00148	.00149	.00150	Webs	.00156
						Arms of Cruciform Section						
60	6	.00158	6	.00158	.00154	6	.00156	.00154	.00156	.00157	8	.00166
80	6	.00164	6	.00164	.00164	8	.00176	.00176	.00175	.00176	8	.00176
100	6	.00171	8	.00179	.00186	8	.00185	.00185	.00185	.00185	9	.00192
120	6	.00178	8	.00185	.00193	8	.00194	.00193	.00193	.00195		
						Arms of Elliptical Section						
60	6	.00161	6	.00163	.00159	6	.00159	.00159	.00160	.00161	8	.00170
80	6	.00171	6	.00167	.00174	8	.00185	.00186	.00185	.00189	8	.00185
100	6	.00182	8	.00195	.00203	8	.00199	.00200	.00198	.00202	9	.00207
120	6	.00193	8	.00204	.00212	8	.00214	.00213	.00218	.00225		

VALUES OF C_1 FOR SPUR GEARS WITH 20° STUB INVOLUTE TEETH

Number of Teeth	Webs or Arms	P	Webs or Arms	P		Webs or Arms	P				Webs or Arms	P
		1 in.		1½ ins.	2 ins.		2½ ins.	3 ins.	3½ ins.	4 ins.		5 ins.
20	Solid	.00140	Solid	.00140	.00140	Solid	.00140	.00140	.00140	.00141	Solid	.00140
30	Webs	.00121	Webs	.00121	.00121	Webs	.00122	.00123	.00122	.00121	Webs	.00125
40	Webs	.00127	Webs	.00128	.00128	Webs	.00128	.00128	.00128	.00128	Webs	.00138
						Arms of Cruciform Section						
60	6	.00136	6	.00136	.00135	6	.00136	.00136	.00136	.00136	8	.00146
80	6	.00143	6	.00143	.00143	8	.00148	.00153	.00153	.00153	8	.00155
100	6	.00150	8	.00157	.00165	8	.00164	.00162	.00162	.00161	9	.00171
120	6	.00156	8	.00163	.00170	8	.00170	.00170	.00170	.00170		
						Arms of Elliptical Section						
60	6	.00142	6	.00141	.00140	6	.00140	.00140	.00140	.00139	8	.00151
80	6	.00153	6	.00152	.00151	8	.00158	.00164	.00164	.00163	8	.00164
100	6	.00161	8	.00173	.00184	8	.00180	.00176	.00176	.00176	9	.00185
120	6	.00173	8	.00183	.00193	8	.00193	.00193	.00193	.00192		

This table was calculated for widths of face of 2½ times the circular pitch.

17. Worms and wheels. General.—A worm and worm wheel should always be made of materials of different hardness and crystalline structure The worm should be of the harder material. The same threads on the worm are in action at all times, while only a few of the teeth of the wheel are in contact with the worm at one time.

If the loads are small and the mechanism is not in too frequent service, the worm wheel may be made of cast iron or semi-steel, and the worm of cast, rolled, or forged steel. When the loads are heavy and the service frequent, the rim of the worm wheel should be of Class D bronze and attached to the body of the wheel as indicated in Fig. 9J (*a*). Small worm wheels may be made of solid bronze. The worm should be of cast, rolled, or forged medium steel.

Hindley worms and wheels should not be used in movable-bridge machinery. They require a high degree of accuracy in workmanship and installation that is not attainable in this class of machinery. They are expensive and difficult to replace in case of accident.

A large part of the energy required to operate a worm and wheel is consumed in friction. For this reason, cut gears give the best results, and should always be used.

The lead of a worm is the distance that the teeth of the worm wheel are advanced in one revolution of the worm.

$$\text{The tangent of the angle of the lead} = \frac{\text{lead}}{2\pi R_1}.$$

When worms and wheels are used in connection with the end and center wedge mechanism of swing spans, and in similar service, the angle of lead should be about 6°, so that the worms may not overhaul the wheels. If the angle is made too great, the shock of passing trains tends to drive the wedges backward when the toggle joints are not in perfect adjustment.

When worms and wheels are used to reduce the speed of shafts adjacent to high-speed motors, and in similar cases, the angle of lead should not be less than 20°, and a larger angle is better and more efficient.

18. Theory of worms and wheels.—Figure 9J (*b*) is a diagram of a worm, with a single thread, driving a worm wheel.

Let W = the force at the pitch line of the worm and wheel, in pounds;

R = the radius of the wheel;

R_1 = the radius of the worm;

P = the circular pitch of the teeth on the wheel;

r_1 = the radius of the inner circle of the thrust collar at the end of the worm;

r_2 = the outer radius of the collar;

F = the force acting to rotate the shaft, in pounds;
R_3 = the lever arm of F;
f = the coefficient of friction between the teeth;
f_1 = the coefficient of friction between the collar and the worm;
A = the arc of contact at the pitch circle, in inches;
F_w = the force required to move the load W, neglecting friction, in pounds;

WORM AND WHEEL.

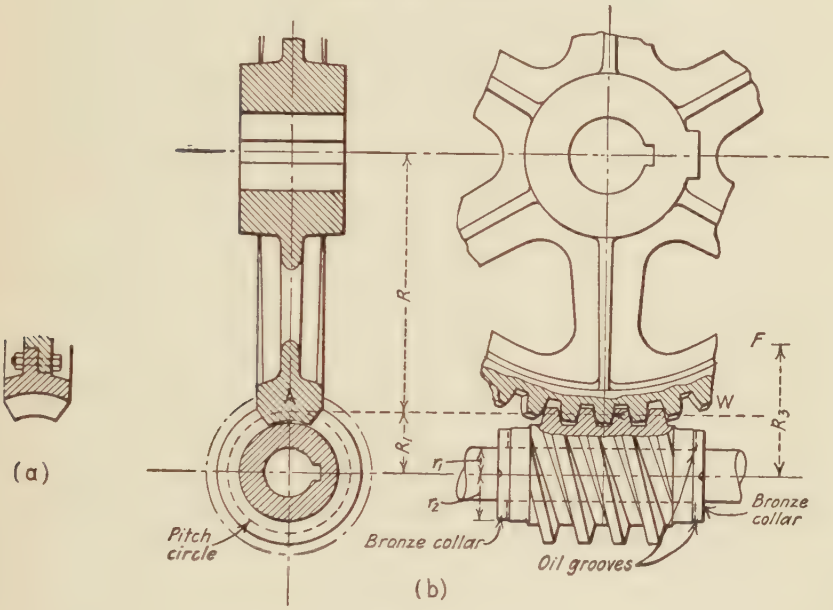


FIG. 9J.

F_t = the force required to overcome the friction between the teeth, in pounds;
F_c = the force required to overcome the collar friction, in pounds.

Then:

$$F = F_w + F_t + F_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Load W moves through a distance P during one revolution of the worm; hence,

$$WP = F_w \times 2\pi R_3, \quad \text{or} \quad F_w = \frac{WP}{2\pi R_3}, \quad . \quad . \quad . \quad (16)$$

$$F_t = \frac{fWR_1}{R_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$F_c = \frac{f_1 W\left[\frac{2}{3} \times \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right]}{R_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

These calculations are based on the assumption that the worm has a single thread. If it has a double thread so that the worm wheel moves forward two teeth for each revolution of the worm,

$$F_w = \frac{WP}{\pi R_3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

F_t and F_c would remain practically the same as before.

Efficiency.—The efficiency of the gearing is the ratio of the useful work to the total input, not including the friction of the collar. In the above formulæ, F_w is the useful work and $F_w + F_t$ is the total input. Let E = the efficiency, then

$$E = \frac{F_w}{F_w + F_t} = \frac{\frac{WP}{2\pi R_3}}{\frac{WP}{2\pi R_3} + \frac{fWR_1}{R_3}} = \frac{P}{P + 2f\pi R_1}. \quad . \quad . \quad (20)$$

If it is assumed that f is about 0.16, which is equivalent to giving f a value equal to $\frac{1}{2\pi}$, the efficiency may be expressed approximately by

$$E = \frac{P}{P + R_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

This equation shows that the efficiency is increased by increasing P, which can be done by making the lead two or three times the circular pitch of the wheel. Decreasing the radius of the worm also increases the efficiency.

Let N be the number of threads of lead, then

$$E = \frac{NP}{NP + R_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Coefficients of friction.—With good lubrication, the following coefficients of friction may be used:

	Values of f
Uncut worms and wheels	0.15 to 0.20
Cut worms and wheels	0.10
Bronze thrust collars	0.10

Lubrication.—The worm should always be below the wheel and run in a bath of oil. Solid greases should never be used, for the centrifugal force will throw off the grease and leave the contact surfaces dry. The best lubricant is castor oil. When this is not available, a good valve oil may be used. When collar thrust bearings are used, liberal oil grooves should be cut in both of their contact surfaces.

19. Working pressures on worm-wheel teeth.—When strength alone is considered, worm-wheel teeth should be designed in the same way and for the same unit stresses as $14\frac{1}{2}°$ involute teeth of spur gears. In most cases the tooth load from considerations of strength is greater than the load which will cause cutting at the contact surfaces.

The choice of an allowable bearing pressure is quite a delicate matter. In some cases cutting occurs when the conditions seem to be favorable and the pressures are relatively low. In movable-bridge machinery, the loads are intermittent and there is little danger of injurious heating. The pressures are relatively high and the velocities low. While it is rather dangerous to give rules for the working pressures, the author will give a method which he has used with no failures due to the methods of design.

It must be remembered that the pressures given are not safe for worms running at high velocities and in continuous service. Empirical rules that will cover all cases cannot be given. Almost every combination of metals, velocities, and pressures is a special problem, and the only criterion of good design is satisfactory service.

Let W = the pressure at the pitch line of the worm and wheel, in pounds;

P = the circular pitch of the teeth of the wheel, in inches;

A = the arc of apparent contact of the pitch line of the worm with the pitch line of the wheel tooth, in inches;

V = the velocity of the pitch circle of the worm, in feet per minute, and

K = a coefficient.

Then:

$$K = \frac{W}{PA}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

For various values of V, the quantity K should not be greater than is called for by the curve in Fig. 9K.

20. Mechanical screws and nuts. General.—Screws and nuts are sometimes used to actuate the ends of draw spans, and they may also be needed for other purposes. When the arrangement allows the use of worms and wheels, they are likely to be more efficient and satisfactory than screws and nuts. It is very difficult to secure good lubrication of screws. About the only way is to bore the shaft of the screw longitudinally, and drill branches to the threads and force in the grease with a pressure cup or a grease gun. Many of the older swing spans had end lifts operated by screws and roller toggles, but the modern tendency is to avoid them. However, it is well to study their characteristics.

The best type of threads for movable-bridge machinery is the form known as the Acme thread. The faces of the threads are inclined about $14\frac{1}{2}°$ with a right sectional plane. They are easier to cut than square threads, for the tools clear away the chips better. Square threads are the second choice, and buttress threads the next.

The screws should be cut on rolled- or forged-steel rounds. It is difficult to cut smooth threads on ordinary open-hearth steel. Bessemer steel with sulphur about 0.06 is much better for threads.

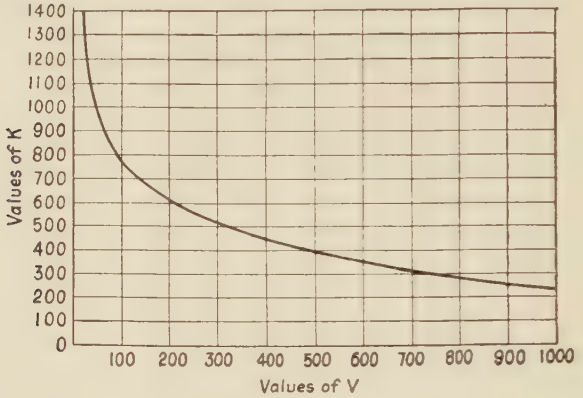


Fig. 9K.

An unlined steel nut should never be used. The metal of the threads will lock and cut. When the loads are light, a cast-iron nut would be satisfactory. When the pressures and velocities are too great for cast iron and when trunnions must be used to transmit stresses, cast-, rolled-, or forged-steel nuts should be used and lined with Class D bronze. Class C bronze can be used when the pressures are not too high.

The unit pressures should be limited to the values given in Chapter VI, Art. 2.

Square Thread.

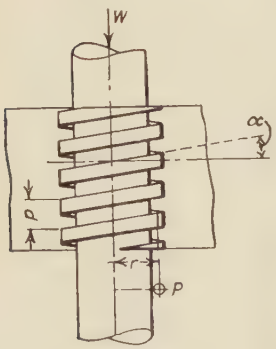


Fig. 9L.

21. Theory of screws and nuts.—Figure 9L is a diagram of a screw with a square thread running in a nut.

Let r = the mean radius of the thread;
p = the pitch, or lead, of the thread;
α = the angle of inclination of the thread;
ϕ = the angle of friction;
f = the coefficient of sliding friction;

W = the load on the screw, and

P = a tangential force acting at a distance r from the center.

Then:

$$\tan \alpha = \frac{p}{2\pi r} = 0.1592\frac{p}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

For uniform motion, neglecting the friction at the root and outside surface of the thread,

$$P = W \tan (\alpha \pm \phi). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

For motion against the direction of W, the sign of ϕ is $+$.
For motion with the direction of W, the sign of ϕ is $-$.
Substituting the values of $\tan \alpha$ and $\tan \phi$, and reducing,

$$P = W\frac{p \pm 2\pi rf}{2\pi r \mp pf}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The upper signs are for motion against, and the lower with, the direction of W.

The efficiency for motion against the direction of action of W is

$$E = \frac{\tan \alpha}{\tan (\alpha + \phi)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

For motion with the direction of action of W, the efficiency is

$$E = \frac{\tan (\alpha - \phi)}{\tan \alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

ACME THREAD.

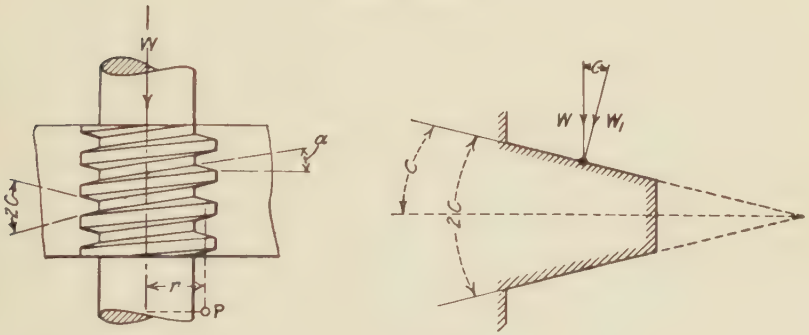


FIG. 9M.

E increases rapidly as the values of α increase from 0 up to about 20°, and becomes a maximum for $\alpha = 45° - \frac{1}{2}\phi°$.

22. Theory of Acme screws.—Figure 9M is a sketch of an Acme screw running in a nut. Use the same notation as for square threads, and let $C = \frac{1}{2}$ the angle between the faces of the threads.

For Acme threads, $C = \frac{1}{2} \times 29° = 14\frac{1}{2}°$.

The normal pressure is greater than for square threads in the proportion that W_1 is greater than W, or

$$W_1 = \frac{W}{\cos C},$$

then,

$$P = \frac{W}{\cos C}\left[\frac{p \pm 2\pi rf}{2\pi r \mp pf}\right]. \quad . \quad . \quad . \quad . \quad . \quad (29)$$

The upper signs are for motion against, and the lower with, the direction of W.

As the work of friction for threads with inclined faces increases in the proportion $W_1 = \frac{W}{\cos C}$, the same result can be obtained by increasing the coefficient of friction in the same proportion, or by making $\phi_1 = \frac{\phi}{\cos C}$. We may then write the two expressions for efficiency:

$$E = \frac{\tan \alpha}{\tan\left(\alpha + \frac{\phi}{\cos C}\right)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

This is for motion against the direction of action of W.

$$E = \frac{\tan\left(\alpha - \frac{\phi}{\cos C}\right)}{\tan \alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

This is for motion with the direction of action of W.

It should be noted that the expressions in Arts. 21 and 22 give the efficiencies of screws and nuts only. Provision must also be made for the friction due to the thrust from either the screw or the nut, depending upon the details of the assembly of the parts.

23. Friction of screws and nuts.—Mr. Albert Kingsbury made tests on square-threaded screws 1.426 ins. outside diameter and 0.33 in. pitch, running very slowly, from which he determined the coefficients of friction under loads up to 10,000 lbs. per square inch of thread contact, with good lubrication.[5] These tests gave the following results:

	Values of f
With lard oil, maximum	0.25
With lard oil, minimum	0.09
With lard oil, average	0.11
With heavy machinery oil, maximum	0.19
With heavy machinery oil, minimum	0.11
With heavy machinery oil, average	0.14

[5] *Trans. A. S. M. E.*, Vol. XVII, p. 96, 1896.

For screws used to transmit motion in draw-bridge machinery, f should be taken as 0.15, when the lubrication is good. In other cases it may be taken at 0.20, or even as high as 0.25, when the lubrication is bad.

When an end lift of a swing span is driven by screws, and in similar cases where it is important that passing loads shall not cause overhauling of the screws, the friction of screws in their nuts should not be depended upon to prevent overhauling.

CHAPTER X

LEVERS, CRANKS, AND SPRINGS

1. Levers.—Figure 10A is a sketch of a form of lever that is often required in the machinery of movable bridges. It is an ordinary bell crank, adapted to be keyed to a shaft receiving or transmitting reciprocating motion.

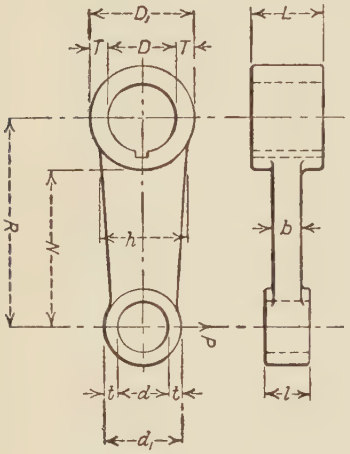


FIG. 10A.

The usual proportions are as given below.

Let D = the diameter of the shaft;
D_1 = the diameter of the hub of the lever;
L = the length of the hub, and
T = the thickness of the metal of the hub.

Then:

Minimum $L = D$ for $\frac{R}{D} \leqq 4$.

When $\frac{R}{D} > 4$, increase L to 1.5 D, maximum.

Minimum $T = \frac{5}{16}D$ for cast steel, or $\frac{3}{8}D$ for cast iron.
Maximum $T = \frac{7}{16}D$ for cast steel, or $\frac{1}{2}D$ for cast iron.

Make $\frac{L}{l} = \frac{D}{d}$ and $\frac{T}{t} = \frac{D}{d}$.

Let P = the tangential force applied at the end of the lever, in pounds;
N = the lever arm of P, in inches;
h = the width of the arm at the hub;
b = the thickness of the arm at the hub, and
S = the maximum extreme fiber stress.

Then:

$$b = \frac{6PN}{Sh^2}, \text{ or } = \frac{6M_b}{Sh^2}, \text{ in which } M_b = \text{the bending moment } PN. \quad (1)$$

When P does not act in the central plane of the lever, let a = the dis-

tance between the central plane and the line of action of P. The arm must then resist a direct bending moment of M_b and a torsional moment of $Pa = M_t$. An approximate equivalent bending moment, M_e, may be found from the formula:

$$M_e = \tfrac{3}{8}M_b + \tfrac{5}{8}\sqrt{M_b^2 + M_t^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Then:

$$b = \frac{6M_e}{Sh^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

2. Overhanging cranks.—Figure 10B is a diagram of a simple overhanging crank. The width of the arm is designated as h, as shown in Fig. 10A, which is a sketch of a lever.

The bending and torsional moments should be found and combined as in Art. 1, Eqs. (1), (2), and (3). From the results of these calculations, dimensions b and h are determined, using unit stresses as given in Chapter VI, Art. 2.

For cranks of ordinary sizes, the remaining dimensions may be determined by the method given below.[1]

Make $l = cd$ and $t = kd$, taking values of k for varying values of c from Table 10A.

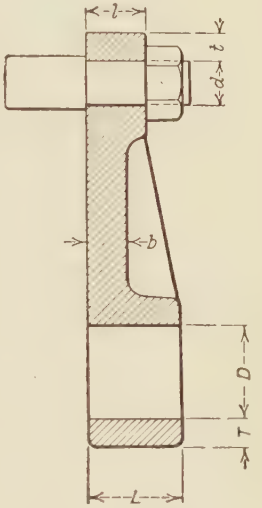


FIG. 10B.

TABLE 10A

VALUES OF K

c	0.9	1.0	1.1	1.2	1.3	1.4
k	0.62	0.50	0.41	0.35	0.30	0.26

Make $L = CD$ and $T = MD$, taking values of M for varying values of C from Table 10B.

TABLE 10B

VALUES OF M

C	0.7	0.8	0.9	1.0	1.1
M	0.44	0.40	0.37	0.34	0.31

[1] "A Manual of Machine Drawing and Design," by David A. Low and Alfred W. Bevis. Longmans, Green & Co., London, 1899.

3. Helical springs.—Cantilever and elliptical springs are seldom needed in connection with movable-bridge machinery. Helical springs, similar to the types used in the trucks of railroad cars, will serve for most purposes in this class of machinery. Fig. 10C is a typical sketch of a helical spring.

Let J = the polar moment of inertia of the bar from which the spring is made;
P = the load on the spring, in pounds;
R = the radius of the spring to the neutral axis, in inches;
d = the diameter of the bar, in inches;
b = the width of the bar, in inches, when rectangular;
h = the depth of the bar, when rectangular;
l = the length of the bar;
S = the maximum unit fiber stress allowable, in pounds per square inch of normal section;
G = the torsional modulus of elasticity;
f = the compression of the spring under the load P;
V_c = the volume of steel for a circular bar, in cubic inches;
V_r = the volume of steel for a rectangular bar, in cubic inches;
H = the free height of the spring, in inches, and
H_s = the solid height of the spring, in inches.

The relations of these quantities are given in Table 10C.

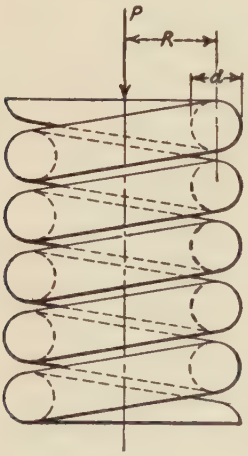


FIG. 10C.

P may be considered as either a compressive or a tensional force. When calculating f for compression springs with two flattened ends, consider one and one-half coils as inactive.

Design.—Helical springs should be designed so that the ratio $\frac{2R}{d}$ is never less than 3, and it should be 4 or more, and with an ordinary upper limit of 8. In Eqs. (16) and (17), note that the product Pf is independent of the dimensions chosen, such as d, l, and R. When designing a spring it is convenient to find the volume of steel required, from (16) or (17), and then arrange the various dimensions to use such volume under the operating conditions of the spring.

Springs made from rectangular bars require 50 per cent more material than those of equal capacity made from cylindrical sections.

TABLE 10C

FORMULAE FOR HELICAL SPRINGS

Circular Sections	Rectangular Sections
$J=\frac{\pi d^4}{32}$. . . . . (4)	$J=\frac{bh}{12}(b^2+h^2)$. . . . . (5)
$P=\frac{S\pi d^3}{16R}$. . . . . (6)	$P=\frac{Sbh\sqrt{b^2+h^2}}{6R}$. . . . . (7)
$f=\frac{2RSl}{dG}=\frac{32PR^2l}{\pi Gd^4}$. . . (8)	$f=\frac{12PR^2l}{bhG(b^2+h^2)}$. . . . . (9)
$Pf=\frac{S^2\pi d^2l}{8G}$. . . . . (10)	$Pf=\frac{S^2bhl}{3G}$. . . . . (11)
$V_c=\frac{1}{4}\pi d^2l$. . . . . (12)	$V_r=bhl$. . . . . (13)
From Eqs. (10), and (12),	From Eqs. (11), and (13),
$Pf=\frac{S^2V_c}{2G}$. . . . . (14)	$Pf=\frac{S^2V_r}{3G}$. . . . . (15)
hence,	hence,
$V_c=\frac{2GPf}{S^2}$. . . . . (16)	$V_r=\frac{3GPf}{S^2}$. . . . . (17)
$H_s=\frac{ld}{2\pi R}$ and $H=H_s+f$. (18)	$H_s=\frac{lh}{2\pi R}$ and $H=H_s+f$. (19)
From (6), (8), and (18),	From (7), (9), and (19),
$f=\frac{4S\pi R^2H_s}{Gd^2}$. . . . . (20)	$f=\frac{4S\pi R^2H_s}{Gh\sqrt{b^2+h^2}}$. . . . . (21)

Values of G.—Experiments show that in good spring practice the following values of G should be used:

Shape and Material	Values of G
Round steel spring wire under $\frac{1}{2}$ in. in diameter	10,500,000
Round steel spring bars $\frac{1}{2}$ in. and over in diameter	10,000,000
Rectangular steel spring bars	8,000,000

Unit stresses.—Safe values of S vary with diameters, $2R$ and d, as shown in Table 10D.

Manufacture.—Springs made from bars or wire under $\frac{1}{2}$ in. in diameter should be accurately coiled hot or cold, at the option of the manufacturer.

All springs made from bars $\frac{1}{2}$ in. or over in diameter must be accurately coiled when hot.

TABLE 10D

SAFE VALUES OF S_c FOR COMPRESSION SPRINGS

Value of Ratio $\frac{2R}{d}$	Values of S_c for Varying Values of d				
	d	d	d	d	d
	0 to $\frac{3}{8}$	$\frac{7}{16}$ to $\frac{3}{4}$	$\frac{13}{16}$ to $1\frac{1}{4}$	$1\frac{5}{16}$ to $1\frac{1}{2}$	$1\frac{9}{16}$ to 2
4	100,000	90,000	80,000	75,000	70,000
8	80,000	75,000	70,000	65,000	60,000

For tension springs, use values for $S_t = 70$ per cent of S_c for compression springs.

All springs must be carefully tempered, preferably in oil.

Steel.—Spring steel should be made by the open-hearth, crucible, or electric process, and should conform to the requirements given below as to chemical composition.

	Per Cent
Carbon	0.90 to 1.10
Manganese, maximum	0.50
Phosphorus, maximum	0.04
Sulphur, maximum	0.05

CHAPTER XI

SHEAVES FOR COUNTERWEIGHT ROPES

1. General.—In vertical-lift bridges and similar structures, it is undesirable to pass a large number of counterweight ropes over a single sheave. The face of the sheave becomes wide and more than one set of arms is needed to support the rim.

When made in a single piece, the sheave becomes a large cellular structure, difficult to mold and cast. The cooling stresses are serious and unavoidable, and annealing slow and uncertain. The sheave load from the ropes becomes so great that the trunnions must be of large diameter, with the resulting increase in power required to turn them in their bearings.

In most cases better results are obtained where not more than four counterweight ropes are carried by one sheave. Only one set of arms is needed. Molding and casting are simplified, and the cooling stresses are less serious. Annealing is more certain and effective. The trunnions are relatively small, reducing the power needed to turn them in their bearings.

For large structures, several sheaves can be used at each loaded point.

Loads.—The maximum stresses in the ropes should not exceed those given in Arts. 721 and 722 of the specifications in Appendix A.

The diameter of the sheaves should not be less than that called for by Art. 401. This article permits the use of sheaves smaller than should be used in standard practice. Larger sheaves permit greater working loads on the ropes, for the bending stresses are smaller.

2. Design in General.—The analysis of the stresses in sheaves loaded by ropes passing around them is so complex that in most cases only approximate methods are feasible. The principal requirements are that no part be over-stressed and that all parts be of such sections that they can be cast and cooled without shrinkage cracks.

Rims.—When a sheave is cast as a unit, the rim can be designed for stress and the parts be thick enough for casting and cooling.

Arms.—The arms may be designed for small unit stresses, in order to obtain sections large and thick enough to cast, but their size and thickness will usually be determined from experience.

Hubs.—The stresses in the hubs from the arms cause elastic distortions which increase their circumference and the inside diameter of the bore, which fits on the trunnion. This has caused serious trouble in some large sheaves. The gap between the hub and trunnion, due to the enlarged inside diameter of the hub and the compression of the trunnion, was enough to cause the inside of the hub to roll on the trunnion and loosen the keys and crush the key seats.

When the wall of the hub is thickened enough to neutralize the distortion within the limits of press fits on the trunnion, it is so thick that it becomes a reservoir of heat and adds to the danger of shrinkage cracks during cooling. When the hubs of sheaves for more than about six or eight ropes are made thin enough for cooling, the elastic distortion under load is too great to be neutralized by pressing the trunnions into the hubs. The sheaves would have to be shrunk on the trunnions, which is an expensive operation.

The hubs of sheaves for four ropes can be made of a thickness suitable for press fits on the trunnions without becoming too thick for proper cooling.

Trunnions.—The friction of the trunnions in their bearings is nearly constant for all diameters; consequently the power required to turn them increases directly with their diameters. For real economy the diameters should be reduced by the use of heat-treated steel, which permits the use of higher unit stresses in bending.

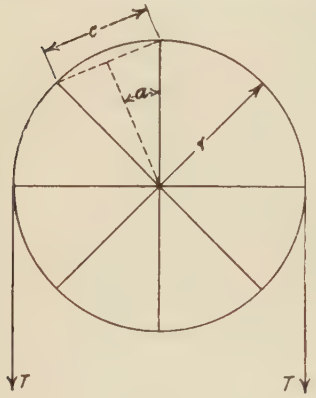


FIG. 11A.

3. Stresses.—Figure 11A is a diagram of a sheave with eight arms loaded by a rope passing 180° around its top half.

Let T = the tension in one rope;
r = the radius of the center line of the rope, and
p = the radial unit pressure, at the center line of the rope, which must be carried by the rim of the sheave.

Then:
$$p = \frac{T}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Refer to Appendix B, Article 1 for a proof of this relation.

The length of the chord between the arms is

$$c = 2r \sin \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The rim may be developed and considered as a continuous girder of equal spans with the middle half of the number of spans uniformly loaded, as shown in Fig. 11B.

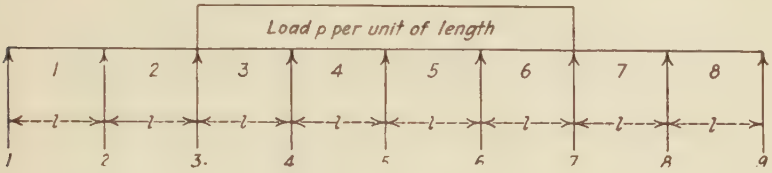


FIG. 11B.

Let l = the length of the chords joining the intersection points of the center lines of the ropes and arms;

n = the number of ropes;

$n\frac{T}{r}$ = the total load per unit of length of the chords;

M = the bending moment at any arm;

S = the shear at the right of any arm, and

S' = the shear at the left of any arm.

The maximum negative moment is at the first arm from the end of the load, as 4 and 6.

The maximum positive moment is in the first loaded span, as 3 and 6. It occurs at the point of zero shear and not at mid-span.

The maximum shear is in the end spans at the first arm from the end of the load, as at 4′ and 6.

The factors for maximum moments and shears for sheaves with 8, 10, and 12 arms are given in Table 11A. In each line of the table, the first factor is for the loaded span or the support, beginning at the left of the loaded portion of the developed rim. For maximum stresses a pair of arms must be horizontal.

TABLE 11A

MOMENTS

Number of Arms	Factors of pl^2
8 arms	$-\frac{1}{24.25}+\frac{1}{17.19}-\frac{1}{10.49}+\frac{1}{25.76}-\frac{1}{12.94}$
10 arms	$-\frac{1}{24.09}+\frac{1}{17.10}-\frac{1}{10.60}+\frac{1}{26.78}-\frac{1}{12.33}$
12 arms	$-\frac{1}{24.02}+\frac{1}{17.5}-\frac{1}{10.58}+\frac{1}{26.49}-\frac{1}{12.48}$

TABLE 11A—*Continued*

SHEARS

Number of Arms	Factors of pl	
8 arms	$+\frac{1}{2.24}+\frac{1}{1.80}$	$+\frac{1}{1.93}+\frac{1}{2.08}$
10 arms	$+\frac{1}{2.24}+\frac{1}{1.81}$	$+\frac{1}{1.95}+\frac{1}{2.05}$
12 arms	$+\frac{1}{2.24}+\frac{1}{1.81}$	$+\frac{1}{1.94}+\frac{1}{2.06}$

4. Design in Detail.—The unit stresses used in this discussion are based upon the use of open-hearth steel castings for the sheaves.

Rim.—The rim is a T-shaped section with top flanges and rope grooves added. The maximum negative moment is at an arm, and is approximately

$$-M = -\frac{1}{10.5}npl^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Refer to Figure 11C.

Assume that the rim at the bottom of the grooves and the T web are of the same thickness.

Proportion the T-shaped rim, with top side-guide flanges, so that the compressive fiber stress in the bottom of the web, taken as of the same depth as between the arms, is equal to, or less than, 11,000 lbs. per square inch.

A closely approximate analysis of the actual stresses in the sheave shows that the compressive fiber stress in the rim web where it joins the arm will then be equal to, or less than, 12,000 lbs. per square inch.

Connect the rim and its web midway between the arms by a pair of triangular brackets.

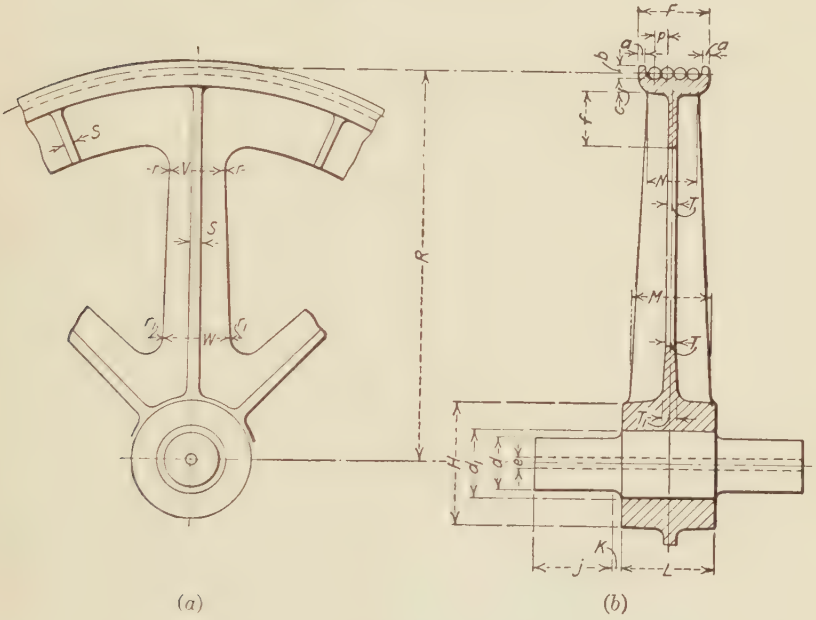
Arms.—Assume the compressive load on one arm to be

$$C = npl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the tension in the ropes, the number of ropes on the sheave, the ratio of the diameter of the sheave to the diameter of the ropes, and the number of arms are approximately in the proportions given in Table 11B, it has been found that when the smallest section of the arm,

near the rim, is proportioned for a unit stress in compression of about 5000 lbs. per square inch, the section of the arm will usually be large and thick enough for successful casting and cooling. When the proportions are widely different from those given, some other unit stress would be required to produce proper proportions in the arms. When the arms of large sheaves are made with a cruciform section, as shown in Fig. 11C, the webs in the plane of the sheave should not be less than $1\frac{3}{8}$ to $1\frac{1}{2}$ ins.

Counterweight Sheave.



(a) (b)

Fig. 11C.

thick. The thickness should increase for the larger sheaves. The web of the arms in the plane of the sheave should be of the same thickness as the web of the rim, until it becomes continuous near the hub. It should then be gradually thickened to its juncture with the hub, to allow it to retain heat during cooling. The width of arm webs should increase from rim to hub, to provide for the bending moments due to the frictional resistance of the trunnions in their bearings. The arm flanges normal to the web may be a little thinner than the web.

The fillet between the arms, near the hub, should be as near the hub as may be practicable, in order not to have a large circular web that would cool before the hub, and thus set up heavy cooling stresses.

All of the arms on and above a horizontal diameter of the sheave are in compression. The compressive unit stresses should never exceed the following:

For steel castings..... $S_c = 16{,}000 - 70\frac{l}{r}$

For iron castings..... $S_c = 12{,}000 - 50\frac{l}{r}$

Here l is the length and r is the least radius of gyration of the arm.

Hub.—The diameter of the hub at its ends should be about 1.8 times the diameter of the trunnion in the hub. Its length should not be less than 1.25 times the same diameter of the trunnion.

Trunnions.—The trunnions should be made from heat-treated carbon steel, as required by Specification A 18–21, Class G, of the American Society for Testing Materials. The unit fiber stress in this grade of steel may be 25,000 lbs. per square inch, but the designs given in Table 11B have been made for a fiber stress of 21,500 lbs. per square inch, which is based upon Art. 312 of the Specifications for Movable Bridges of 1922, of the American Railway Engineering Association, but increased proportionately to the higher yield point of the heat-treated steel. The use of high-tension steel enables the diameter of the trunnions to be made relatively small, with resulting reduction in the lost work due to trunnion friction. The unit pressure of the trunnions in their bearings will be assumed as 1500 lbs. per square inch.

Refer to Fig. 11C.

Let W = the load on one journal, $= 4T$;
p_1 = the unit pressure on the journals;
f = the maximum fiber stress in the trunnion;
d = the diameter of the trunnion in the journal;
d_1 = the diameter of the trunnion in the hub, and
j = the length of the bearing in the journal.

Then:

$$W = djp_1, \text{ or } j = \frac{W}{dp_1};$$

The bending moment in the neck of the trunnion is

$M_k = W\frac{j}{2}$. This must equal the moment of resistance of the trunnion;

$R = \frac{\pi d^3}{32}f$, or by substitution,

$$d = \left[\frac{16W^2}{\pi p_1 f}\right]^{1/4} = 1.5023\left[\frac{W^2}{p_1 f}\right]^{1/4} \quad (5)$$

The moment within the hub is

$$M_h = W\left(\frac{j}{2} + k + \frac{L}{4}\right), \text{ hence,}$$

$$d_1 = \left[\frac{32M_h}{\pi f}\right]^{1/3} = 2.1677\left[\frac{M_h}{f}\right]^{1/3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

These expressions neglect the effect due to the cooling hole of diameter e, also the torsional moment due to trunnion friction. The increase in diameter to provide for these omissions will be only a small fraction of an inch in trunnions for four-rope sheaves. Their inclusion would greatly complicate Eqs. (5) and (6). The method of providing for combined bending and torsional moments is given in Chapter VII, Art. 4. The expression for the moment of resistance of a hollow shaft is

$$R_H = \frac{\pi(d_0^4 - d_i^4)}{32d_0} f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In this, d_0 is the outside and d_i the inside diameter of the hollow shaft.

5. Material and Manufacture. —*Material.*—The material for counterweight sheaves should be open-hearth steel castings, Class B, Medium Grade, in accordance with the Standard Specifications for Steel Castings, Serial Designation, A 27–24, of the American Society for Testing Materials.

Annealing.—All cast steel sheaves should be thoroughly annealed. The annealing temperatures should be tested by the use of ball or thermo-couple calorimeters, and the heating should be continued until it is certain that all parts have been brought to the annealing temperature. The castings should then be held at this temperature several hours, to make sure that all cooling stresses are neutralized. They should then be allowed to cool in the furnace without artificial acceleration of the cooling process.

For the material specified, the annealing should be done between the temperatures of 815° C. (1499° F.), and 840° C. (1544° F.).

Press Fits.—The allowances for press fits should not be less than those given in Table 11B.

6. Designs of sheaves for four counterweight ropes.—Table 11B gives the principal dimensions of a series of counterweight sheaves, for four Monitor steel wire ropes, designed in accordance with the methods outlined in this chapter. In order to reduce the size of the trunnions and bearings and the lost work due to trunnion friction, the use of heat-treated forged-steel trunnions is assumed.

Reference should be made to Appendix B for a method of calculating the actual stresses in large counterweight sheaves and the development of the design by a more exact method.

TABLE 11B

COUNTERWEIGHT SHEAVES FOR FOUR MONITOR STEEL-WIRE ROPES WITH HEAT-TREATED TRUNNIONS OF A. S. T. M. FORGED STEEL SPECIFICATIONS, A–18–21, CLASS G

Diameter *c.–c.* of ropes, or pitch diameter		8 ft.	10 ft.	12 ft.	14 ft.
Number of ropes		4	4	4	4
Diameter of ropes		1½ ins.	1¾ ins.	2⅛ ins.	2½ ins.
Ratio of pitch diameter to rope diameter		64	68.57	67.76	67.2
Capacity of each rope. Monitor steel		24,464 lbs.	35,876 lbs.	51,440 lbs.	74,100 lbs.
Number of arms		8	8	10	12
Distance, center to center of ropes	p	1⅝ ins.	1⅞ ins.	2¼ ins.	2⅝ ins.
Face of sheave	F	8½ ins.	10 ins.	12 ins.	13½ ins.
Thickness of flanges at edge	a	⅝ in.	¾ in.	¾ in.	¾ in.
Thickness of rim	c	1⅜ ins.	1½ ins.	1⅝ ins.	1¾ ins.
Depth of flange	b	1¾ ins.	2 ins.	2¼ ins.	2½ ins.
Depth of rim web	f	8 ins.	10 ins.	10 ins.	10½ ins.
Thickness of rim web	T	1⅜ ins.	1½ ins.	1⅝ ins.	1¾ ins.
Thickness of web at hub	T_1	1¾ ins.	2 ins.	2¼ ins.	2½ ins.
Width of arms at rim	V	6¾ ins.	7½ ins.	8¼ ins.	9 ins.
Width of arms at hub web	W	8 ins.	9 ins.	10¼ ins.	11½ ins.
Radius	r	3 ins.	4 ins.	5 ins.	6 ins.
Radius	r_1	2 ins.	2 ins.	2 ins.	2 ins.
Thickness of arm flanges	S	1¼ ins.	1⅜ ins.	1½ ins.	1⅝ ins.
Thickness of rim brackets	S	1¼ ins.	1⅜ ins.	1½ ins.	1⅝ ins.
Width of arm flanges at rim	N	6½ ins.	8 ins.	9¼ ins.	10½ ins.
Width of arm flanges at hub	M	10 ins.	11 ins.	14 ins.	16½ ins.
Diameter of hub	H	15 ins.	16½ ins.	19½ ins.	23 ins.
Length of hub	L	12 ins.	13 ins.	16 ins.	18½ ins.
Bore of hub	d_1	8¼ ins.	9 ins.	10¾ ins.	12¾ ins.
Diameter of trunnions for press fit	d_1+	8.253 ins.	9.004 ins.	10.754 ins.	12.754 ins.
Diameter of trunnion in bearings	d	6½ ins.	7¾ ins.	9¼ ins.	11 ins.
Length of trunnion in bearings	j	10 ins.	12½ ins.	15 ins.	18 ins.
Fillet	k	⅝ in.	¾ in.	¾ in.	¾ in.
Cooling hole through trunnion	e	1½ ins.	2 ins.	2 ins.	2 ins.
Estimated weight of 1 sheave		5350 lbs.	7300 lbs.	11,700 lbs.	18,000 lbs.
Estimated weight of 1 trunnion		370 lbs.	550 lbs.	970 lbs.	1,480 lbs.

CHAPTER XII

WIRE ROPES

1. General.—Steel-wire ropes are a well-established and reliable material of construction for movable-bridge machinery. They are used in most forms of vertical-lift bridges and in some types of bascules. Iron, and later, steel-wire ropes have been extensively used in all kinds of hoisting machinery, including freight and passenger elevators. Nearly all modern passenger elevators carry their loads on wire ropes. In this service they prove safe and satisfactory, when properly used and maintained. They have one valuable characteristic. Long before total failure occurs, individual wires break; and this occurs on the outside of the rope where the broken wires are readily detected by inspection.

In hoisting machinery, wire ropes are subject to violent shocks and rapidly varying loads, and often to abrasion by being dragged over inclines which are not equipped with rollers. In spite of such abuse, they give long and satisfactory service. When they are used on derricks and cranes for handling materials and erecting the superstructures of bridges and buildings, they are subjected to even more unfavorable operating conditions. In order to reduce the size and weight of the blocks so that they can be handled, the sheaves are made of small diameter, often not more than about sixteen times the diameter of the ropes. This causes quite large bending stresses in the wires, but the ropes last much longer than would be possible if some of the extreme theories advanced as to the intensity of the bending stresses were correct.

If the designs of the wire ropes, their connections, and the sheaves over which they move, are properly developed, the ropes require renewal only at long intervals. If the bridges in which they are used are inspected at regular intervals, as should be done on all movable bridges, the ropes can be depended upon for long and safe service; and when broken wires are found, there is still time to make provision for the renewal of the ropes.

2. Material.—Various grades of steel are drawn into wire for use in ropes. In movable-bridge construction, it is best and most economical to select ropes made from the higher grades of steel. Counterweight

ropes should be kept as small as practicable, in order to reduce the diameter and weight of the sheaves. Operating ropes of the higher steels give better service than those made from the lower grades. Only three grades should be considered, and in most cases one of the two higher should be selected.

The physical requirements are given in detail in the specifications in Appendix A, Arts. 408 to 413. Ropes made from the three grades of steel, designated as A, B, and C, can readily be obtained from any reliable manufacturer of wire ropes.

3. Construction.—The best type of wire rope for general use in movable-bridge machinery has six strands laid around a hemp core. This is the lay commonly used in standard hoisting ropes and can be obtained for prompt delivery. The hemp core is desirable, as it forms a reservoir for the lubricant which is applied during the spinning of the ropes, and in maintenance. The initial stretch of these ropes under loads is greater than when a wire core is used. In some cases when it is necessary to limit the stretch, the six-strand type, with a core consisting of a seventh wire strand, will be preferred.

When large ropes are required for this class of service, the construction should be such that no individual wire shall be more than one-tenth of an inch in diameter. When larger sizes are used, the fiber stress, due to bending the rope around its sheave, is increased proportionately. If the bending stress is held constant, larger sheaves must be used, and when the wires are too large the sheaves become unwieldy.

The following practice is recommended:

Diameter of Ropes	Construction
Up to $1\frac{1}{2}$ ins., inclusive	6 strands of 19 wires each, 1 hemp core.
$1\frac{5}{8}$ to $2\frac{1}{8}$ ins., inclusive	6 strands of 37 wires each, 1 hemp core.
$2\frac{1}{4}$ to $2\frac{1}{2}$ ins., inclusive	6 strands of 61 wires each, 1 hemp core.

Further details of this series of wire ropes are given in the specifications in Appendix A, Art. 413.

No counterweight rope larger than $2\frac{1}{2}$ ins. in diameter should be used, and it is better to limit the diameter to 2 ins. and to use more ropes.

When the core is a wire strand, the strengths of the ropes are increased by about 10 per cent above the strengths given in the specifications, which are for ropes with hemp cores.

4. Unit stresses.—The direct tension in counterweight and operating ropes should not exceed one-fifth of the minimum specified ultimate strengths give in Art. 413, less the equivalent tension due to bending, as specified in Art. 722 of Appendix A.

The direct tension in counterweight ropes, to be used for design, should include not only the weight from the bridge, or counterweight, but also the weight of the rope and its connections and the accelerating forces from the operating machinery.

The minimum ultimate strengths given in Art. 413 of the specifications take account of the angularity of the wires in the strands and of the strands in the rope, and are the strengths that should be guaranteed by the manufacturers.

5. Bending stresses.—If a wire rope is passed over a sheave, the wires making up the rope will all be subjected to bending stresses. These stresses are independent of any direct longitudinal stresses due to the load carried by the rope. When a solid cylindrical bar is bent to an arc of a circle, the extreme fiber stress due to bending is expressed by the formula:

$$S_b = E\frac{d}{D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this S_b = the maximum extreme fiber stress due to flexure;
E = Young's modulus;
d = the diameter of the bar, and
D = the diameter of the bend.

Early writers applied this formula to the bending of wire ropes, taking the value of d as the diameter of the rope. Rankine, Reuleaux, and Unwin modified this practice by taking d as the diameter of the wires in the rope, but still assumed that E was Young's modulus for the steel in the wire, rather than the modulus of the rope as a whole.

A wire rope is an aggregate of many helical springs of long pitch laid in contact with each other, and supplied with a lubricant during manufacture and lubricated when in service. When bent around a sheave, the individual wires slip with respect to each other and the pitch of the helices is free to change, thus partially relieving the bending stresses.

Howe [1] and Hrabák [2] have called attention to the erroneous use of E for the steel in the wires and have developed a theory of bending that depends upon the value of E_r, which is Young's modulus for the rope as a whole, and not that for the steel in the wires composing the rope. Chapman [3] discusses and advocates a method involving the value of E

[1] "Determination of Stresses in Wire Rope as Applied to Modern Engineering Problems," by James F. Howe, M. A. S. M. E. *Trans. A. S. M. E.*, Vol. 40, p. 1043, 1918.

[2] "Die Drahtseile," by Josef Hrabák, 1902.

[3] "The Stress in Wire Ropes Due to Bending," by R. W. Chapman. *The Engineering Review*, 1908, p. 258.

for the steel in the wires, and compares the results of his theory with some bending tests made on a short piece of wire rope, clamped in a vise at one end. The ends of the rope must have been seized with wire to prevent untwisting. The free end of the short piece of rope was then deflected by known forces and the deflections measured. The action of the rope under such conditions would have been constrained, for the longitudinal slipping of the wires along each other would have been almost prevented by the vise and seizing. Such a constrained rope would be much stiffer than an ordinary rope, free to adjust itself when deflected.

Professor Arch. C. Elliott has given a theoretical analysis of this problem in a discussion on Electric Winding Engines.[4]

Professor Elliott's conclusions are that the bending stresses in the wires are much less than those calculated by Mr. Chapman. Professor Elliott's analysis is based upon the actual radii of curvature of the helical wires when the rope is bent around a sheave. When comparing his results with those of Mr. Chapman, he states: . . . " the fundamental difference in the views taken probably arose in this way; when a rope was bent from the straight into the arc of a circle, Mr. Chapman seemed to assume that the pitch angles remained the same; whereas, according to the theory on which the above results were based, a helix of constant pitch angle while the axis of the rope was straight, transformed, when that axis was bent, into a helix of variable pitch angle."

The bending stresses in the wires of a rope bent around a sheave, obtained by an application of Professor Elliott's theory, conform much more closely to the stresses as calculated by Howe and Hrabák than to those calculated by the method advocated by Chapman.

Mr. Howe's formula for the maximum extreme fiber stress in the wires due to flexure is

$$S_b = E_r \frac{d}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In this formula,

S_b = the maximum extreme fiber stress due to bending;
E_r = Young's modulus for the rope as a whole;
d = the diameter of the wires in the rope, and
D = the pitch diameter of the rope on the sheave.

This method of calculation is recommended for use in the design of wire ropes subjected to bending around the sheaves of movable bridges. See Specifications, Appendix A, Art. 722.

In his paper, Mr. Howe gives the stress-strain curves from several

[4] Discussion by Prof. Arch. C. Elliott, D. Sc., M. Inst. C. E., *Proceedings of South Wales Institute of Engineers*, Vol. XXVIII, 1912, pp. 426–431.

typical tests of wire ropes of different construction. They are summarized in Table 12A.

TABLE 12A

VALUES OF E_r

Rope Construction	Diameter, Inches	Value of E_r
6 strands of 7 wires each	$\frac{3}{4}$	11,922,635
6 strands of 19 wires each	$\frac{3}{4}$	9,619,200
6 strands of 37 wires each	$\frac{3}{4}$	10,154,000
8 strands of 19 wires each	$\frac{3}{4}$	5,511,250

These and many other similar tests indicate that the value of E_r may safely be taken as 12,000,000 for such ropes as are likely to be used in movable bridges. This value is greater than the usual value of E_r for ropes with 6 strands of 19 wires each; therefore the calculated bending stresses from Eq. (2) will be conservative.

6. Ultimate strength of wire ropes when bent around sheaves.—The ultimate strength of wire ropes when bent around sheaves does not follow the laws given in Art. 5 for the maximum extreme fiber stresses due to flexure. In the former case, the stresses pass beyond the elastic limit until failure occurs, while in the latter, the helical wires flex within the elastic limit.

A series of tests of the ultimate strengths of straight wire ropes, and of the same ropes bent 180° around sheaves, were made by the Bureau of Standards, of the Department of Commerce of the United States, at the request of the American Bridge Company. The results are given in detail in Technologic Paper No. 229, issued during 1923, and are summarized in Table 12B.

7. Lost work in wire ropes drawn around sheaves.—When a wire rope under tension is drawn upon and unwound from a sheave, or drum, by an additional tension applied to the rope, it is well known that the tension in the part of the rope that is unwound from the sheave is greater than that in the part that is approaching it; and that the difference in tension is greater than the tangential force at the pitch line of the rope upon the sheave that will overcome the axle friction.

In all ordinary practice, the maximum unit stress allowed in a rope is far below the elastic limit of the material. When such a stressed rope is drawn over sheave, the energy required for elastic bending of the wires as the rope passes upon the sheave is wholly restored as it leaves the sheave. The bending of the rope upon the sheave also causes relative

TABLE 12B

Ultimate Strengths of Wire Ropes when Bent over Sheaves

Diameter of Ropes, Inches	Efficiency in Per Cent. of the Strength of Straight Ropes		
	Diameters of Sheaves		
	10 Ins.	14 Ins.	18 Ins.
$\frac{5}{8}$	87.4	96.0	95.3
$\frac{3}{4}$ new	80.3	91.5	92.5
$\frac{3}{4}$ worn*	83.6	82.3	80.9
$\frac{7}{8}$	81.9	89.5	92.9
1	79.3	87.5	92.5
$1\frac{1}{4}$	75.8	81.3	84.7

* This rope had been in service for five years on a steel derrick car to lift loads from 5 to 50 tons. The car was in regular, but not constant use. The ropes passed over sheaves 14 ins. in groove diameter in tandem blocks providing 19 parallel lines. The outer wires had been worn to a depth of about one-half of their radii, and some were broken at different points along the rope.

slipping of the wires and strands with respect to each other and the core; for the outer circumference must be greater than the inner where the rope approaches and leaves the groove of the sheave. As the rope is under tension, it wraps around the sheave without slipping. The energy lost as the rope passes around the sheave is due to the friction between the wires, strands, and core as they slip with respect to each other.

The calculation of the exact relative movements of the helical wires and strands is too complicated to be practical. The only feasible method of evaluating the lost work is by experimenting with actual ropes under known tensions. Very few tests of this kind are available.

During 1914, the American Bridge Company made a series of tests on 44 arrangements of ordinary wire-rope hoisting tackle. Each test was made in triplicate. The sheaves were relatively small, for large ones are not practicable when the blocks must be kept light enough to be readily handled. For the same reason, large ropes were not used.

After deducting the effect of the axle friction, which was carefully determined, it was found that the tension in the part of the rope leaving the sheave was greater than that in the part approaching it by an amount that is closely expressed by the formula:

$$t = 0.45\frac{d}{D}T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this expression,

T = the tension in the part of the rope approaching the sheave, in pounds;

t = the increased tension necessary to overcome the resistance due to rope stiffness, in pounds;

d = the diameter of the rope, and

D = the pitch diameter of the rope upon the sheave.

In these tests, the ratio $\frac{D}{d}$ was not more than 28.4, and in most of them it was less. No tests of modern wire ropes wrapped around sheaves are available in which the ratio $\frac{D}{d}$ is as large as is required in movable-bridge practice. Refer to Appendix A, Art. 401.

An analysis has been made of the additional tension required to draw a wire rope, subjected to tension, around a sheave, assuming that the losses are due to internal friction developed in the rope from two causes:

First, the internal friction between the wires, strands, and core caused by the hoop tension developed by the tension in the spiral wires and strands;

Second, the internal friction due to the forces developed by the radial pressure of the rope against the groove of the sheave. The first is much the larger, and follows the law:

$$t = c\frac{d}{D}T.$$

The second is relatively small, and follows the law:

$$t = c\frac{d}{D^2}T.$$

The analysis is too complicated to be given in full.

Assume that a wire rope, subjected to tension, is passed around a sheave and made to move and rotate the sheave by an additional tension on one side of the sheave. The additional tension required to overcome the trunnion friction will not cause the direction of the parts of the rope to depart from true tangents to the sheave; for the moment of the additional tension is balanced by the moment of the trunnion friction. In order to overcome the stiffness of the rope at the points at which it winds upon and unwinds from the sheave, there must also be an additional tension in the part of the rope that is unwound from the sheave. For equilibrium, the moment of the tension in the part of the rope approaching the sheave must equal that of the tension in the part leaving it; but the tension in the latter is greater than that in the

former, hence, the moments being equal, the lever arms of the rope tensions must be inversely proportional to the tensions. Refer now to Figure 12A.

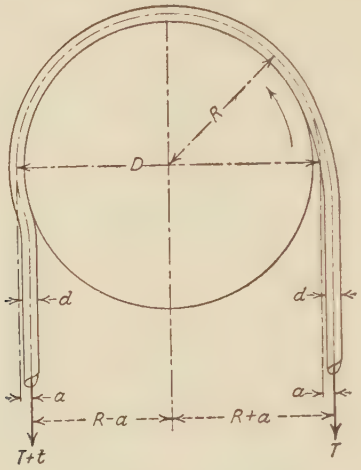


Fig. 12A.

Let T = the normal tension in the rope;
R = the pitch radius of the rope upon the sheave;
a = the distance that the rope is moved from its normal position, near its contacts with the sheave, as a result of its stiffness. This is plus on the side where it approaches the sheave and minus on the side where it leaves it.
t = the additional tension in the part of the rope leaving the sheave.

Then:

$$T(R + a) = (T + t)(R - a), \text{ from which,}$$

$$t = T\frac{R + a}{R - a} - T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equation (3) may be written:

$$t = c\frac{d}{D}T, \text{ from which,}$$

$$c = \frac{Dt}{dT}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Mr. Shortridge Hardesty, Associate Engineer to Dr. J. A. L. Waddell, recently made some tests of the force required to overcome the stiffness of wire counterweight ropes passing over large sheaves on a heavy vertical-lift bridge, and, by permission, the results are given.

The total weight of the movable span is 3,160,000 lbs., the weight being carried by 64 wire ropes $2\frac{1}{4}$ ins. in diameter, 16 at each corner. The sheaves are 15 ft. in pitch diameter.

The bridge is frequently raised about 21 ft. to allow small vessels to pass under it, and, much less frequently, it is raised to the fully open position. The parts of the counterweight ropes passing over the sheaves for small openings were clean and in good condition. The parts passing over the sheaves when the bridge is raised above the ordinary small opening, were caked solid with hard dressing, so that they appeared as solid cylinders. The tests were made in two sets, the first for the ordinary small opening, and the second for full opening.

Case I. Ordinary opening of 21 ft., ropes clean. The observed value of a, when reduced to the plane through the center of the trunnions, was $a = \pm 0.112$ in.

From Eq. (4),

$$t = 3{,}160{,}000\frac{90.112}{89.888} - 3{,}160{,}000 = 7874 \text{ lbs.}$$

From Eq. (5),

$$c = \frac{180 \times 7874}{2.25 \times 3{,}160{,}000} = 0.1993.$$

Then, for Case I, Eq. (3) becomes

$$t = 0.20\frac{d}{D}T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Case II. Full opening, ropes caked solid with hard dressing. The observed value of a, when reduced to the plane through the center of the trunnions, was $a = \pm 0.168$ in.

From Eq. (4),

$$t = 3{,}160{,}000\,\frac{90.168}{89.832} - 3{,}160{,}000 = 11{,}831 \text{ lbs.}$$

From Eq. (5),

$$c = \frac{180 \times 11{,}831}{2.25 \times 3{,}160{,}000} = 0.2995.$$

Then for Case II, Eq. (3) becomes

$$t = 0.30\frac{d}{D}T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

It is probable that Eq. (3) indicates much more than the actual tension required to overcome wire-rope stiffness when the ratio $\frac{D}{d}$ is large. The value of c of 0.45, in Eq. (3), was calculated from tests in which the ratio $\frac{D}{d}$ was below 30. The tests given above and others of a

similar character indicate that the value of c can safely be reduced by about 0.003 for each unit that the ratio $\frac{D}{d}$ exceeds 30.

The value for running trunnion friction, given as 0.08 in Chapter V, Table 5D, is too large for many cases; and when the ratio $\frac{D}{d}$ is above 60 and the trunnions move at any ordinary velocity, the stiffness of the wire ropes might be neglected and assumed as included in the allowance for trunnion friction.

8. Elongation of wire ropes.—When a new wire rope is first subjected to tension, an elongation is produced by the internal adjustment of the wires and strands around the core and the compression of the core, mainly by the hoop tension of the spiral wires and strands. Most of this initial elongation is produced by a relatively small tension in the rope. When it is necessary that the ropes be of a definite length between the end sockets, the sockets should be put on when the rope is under tension. This is particularly important when several ropes must be of the same length in order that each may take its part of a load, as in ropes for counterweights. It is recommended that the tension in the rope, when sockets are located, be not less than 2 per cent. of the guaranteed minimum ultimate strength. See Appendix A, Arts. 413 and 417. A tension in the rope of this amount will take up nearly all of the slack due to the method of manufacture.

When it is important to calculate, in advance, the probable length of a wire rope when under its working load, this can also be done with a reasonable degree of approximation. The ordinary factor of safety, with respect to the ultimate strength, should be at least 5 for ropes in movable bridges. For rough calculations, it may be assumed that the rope will elongate $\frac{1}{2}$ of 1 per cent. of its length when the tension is increased from 2 to 20 per cent. of its ultimate strength.

A set of 9 Monitor steel wire ropes, of 6 by 19 construction, 1 in. in diameter, was tested on a gauged length of 36 ins. The initial elongation under a tension of 2 per cent. of the ultimate strength averaged 2.53 per cent. for the 9 samples. When the tension was increased to 20 per cent. of the ultimate strength, the additional elongation averaged 0.413 per cent. of the gauged length. The additional elongation corresponds to a value of E_r of 8,504,000 (Young's modulus for the rope). The minimum tension, of 2 per cent. of the ultimate, was not enough to cause the stress-strain curve to be straight from this point up to a tension of 20 per cent. of the ultimate, the line corresponding to an E_r of 8,504,000 really being a chord of the stress-strain curve. When calculated from

the parts of the curves that were practically straight, the value of R_r averaged about 12,950,000.

In another case, 32 Monitor steel, 6-by-19 ropes, with wire cores and $1\frac{5}{8}$ in. in diameter, and of an average length of 145 ft., were socketed under a tension of 2 per cent. of their ultimate strength and used as temporary counterweight ropes during the reconstruction of parts of a heavy vertical-lift bridge. As they were to be in service for only a short time, the maximum tension, due to direct load combined with bending, was fixed at about one-third of the ultimate strength. After they had been in use for a few weeks, the increase in elongation, from an increase in tension from 2 to 37 per cent. of the ultimate, was found to be 0.55 per cent. of the length.

When calculating clearances, as under the counterweights of vertical-lift bridges in open position, greater allowances should be made for possible elongation. For this purpose the minimum should be about 1 per cent. of the length, if the ropes were under a tension of 2 per cent. when the sockets were placed. When no serious inconvenience would result in the design, it would be better to increase this to about 2 per cent. of the length. See Appendix A, Art. 215.

9. Lubrication of wire ropes.—Wire ropes are lubricated in the machines that spin them. One important office of the hemp cores in ordinary hoisting ropes is to receive and retain a supply of lubricant. As all parts of a rope move against each other when the rope passes around a drum or over a sheave, it should be kept thoroughly lubricated at all times. This not only diminishes the lost work of the rope as a machine element, but protects it against corrosion and materially prolongs its useful life. Adequate lubrication is so important that manufacturers of wire ropes recommend, make, and sell lubricants specially prepared for this use.

CHAPTER XIII

BRAKES

1. Design of brakes in general.—Brakes should be designed with particular attention to the following points:

(1) Ability to withstand high temperatures and to dissipate heat.

(2) Freedom from shock, when applied.

(3) Durability of the friction surfaces.

(1) All energy absorbed by a brake is transformed into heat, which must be radiated. The operating temperature of the brake depends upon the frequency of the cycle of application and the facilities for radiation. When the brake material is affected by heat, larger surfaces must be used than when it is heat-resistant.

(2) The torque of the brake should be limited, so that the stresses in the machinery due to the application of the brake will not be greater than those produced by the motor.

The brake should be applied without shock. When solenoid brakes are used, a spring applies the load more gradually than a falling weight. Manual or pedal application may be gradual and without shock.

(3) The brake material should not be sensitive to climatic changes. A lubricated brake should not be run dry, and oil should not be applied to a brake intended to be run dry. The brake should be protected from rain, snow, and dirt when the friction surface is such that it is affected by these elements.

2. Kinetic energy of moving masses.—For a mass weighing W pounds, having a motion of translation of V feet per minute, the kinetic energy in foot-pounds is

$$KE_t = \frac{WV^2}{7200g} = 0.00000431WV^2. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For a mass weighing W pounds and rotating at N revolutions per minute, its radius of gyration being r inches, the kinetic energy in foot-pounds is

$$KE_r = \frac{W}{2g}\left\{\frac{2\pi rN}{12 \times 60}\right\}^2 = 0.00000118Wr^2N^2. \quad . \quad . \quad . \quad (2)$$

The total kinetic energy in foot-pounds is

$$KE = KE_t + KE_r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In a swing draw span, or a bascule bridge the kinetic energy of the machinery is so small when compared with that of the moving bridge, (including the counterweights in a bascule), that it may be neglected.

In a vertical-lift bridge the energy stored in large sheaves, drums, and ropes should be considered.

3. Kinetic energy dissipated by brakes.—The kinetic energy being KE and the torque of the brake at a radius of 1 ft. being T, the number of revolutions of the brake wheel required to stop becomes

$$N_B = \frac{KE}{2\pi T} = 0.1591\frac{KE}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the brake wheel is rotating at N r.p.m. when the brake is applied, the time required to stop, in seconds, is

$$t = \frac{60KE}{\pi NT} = 19.1\frac{KE}{NT}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The retardation due to the friction of the machinery is neglected in (4) and (5), hence the values of N_B and t are slightly too large.

4. Heating of brakes.—The mechanical equivalent of one British Thermal Unit being 778 ft.-lbs., the total heat developed during one stop is

$$H = \frac{KE}{778}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Let S = the specific heat of iron = 0.1138;
P = the weight, in pounds, of iron heated;
R = the combined coefficient of radiation and convection = 1.8 B.t.u., (approx.), for 1° F. rise, per square foot per hour;
A = the radiating surface in square feet;
t_1 = the temperature rise for one stop, and
X = the number of stops per hour.

Then:

$$t_1 = \frac{KE}{778 \times 0.1138 \times P} = \frac{H}{0.1138 \times P}. \quad . \quad . \quad . \quad . \quad (7)$$

Assume that the brake wheel reaches a maximum temperature of 150° F., above which wood linings begin to char, and that the atmospheric temperature is 60° F.

The heat dissipated for a rise in temperature of 90° F. is $90 \times A \times R = 162A$, in B.t.u. per hour.

The permissible number of stops per hour then becomes

$$X = \frac{162A \times 0.1138 \times P}{H} = 18.43\frac{AP}{H}. \quad . \quad . \quad . \quad . \quad (8)$$

Brakes on movable bridges are operated at such long intervals that heat calculations are seldom necessary.

Brakes on turntables may be used at such short time intervals that heating should be considered.

When special heat-resisting materials are used for brake-block or brake-band linings, the critical temperature may be taken as 400° F. This applies particularly to the best kinds of asbestos-fabric and asbestos-block brake linings and cast iron.

Too much dependence should not be given to heat calculations. It is best to compare new brake designs with those in successful use as a practical check.

5. Brake blocks and linings.—(1) *Cast iron.*—The most severe brake service is on railway rolling stock. The brake shoes are usually of cast iron and the wheels either chilled cast iron or rolled steel. The rapid movement of trains is favorable to the rapid dissipation of the heat generated.

Cast iron is about the best material for block-brake shoes running on cast-iron or cast-steel brake wheels, such as are commonly used on large movable bridges. The running temperature should not exceed about 400° F.

Coefficient of friction.—The coefficient of friction varies with the rubbing velocity, pressure per square inch between the brake shoes and wheels, condition of the surfaces, and the materials used. In general, soft cast-iron shoes give a larger coefficient than those made of hard cast iron. Very extensive experiments on brake friction in railway rolling stock were made by Sir Douglas Galton, in 1878 and 1879.

If V = the velocity of rubbing, in feet per minute, and f = the coefficient of friction, the average results of Galton's 1879 experiments may be summarized as follows:

Cast-iron shoes on steel-tired wheels:

When $V \leqq 2250, f = 0.30 - 0.00006V$ . . (9)

When $V = 2250$ to $5000, f = 0.165 - 0.00002\,(V - 2250)$. (10)

These may be taken as minimum values of f.

A series of laboratory tests, made by Prof. R. A. Smart at Purdue University and reported in Proceedings of Western Railway Club in Vol. 13, for 1900–1901, gave higher friction coefficients, which, for a brake pressure of 100 lbs. per square inch, may be summarized as follows:

Soft cast-iron shoes on steel wheels:

When $V \leqq 2000, f = 0.35$ . . . . . (11)

When $V = 2000$ to $5000, f = 0.35 - 0.00004\ (V - 2000)$. (12)

Hard cast-iron shoes on chilled cast-iron wheels:

When $V \leqq 2000, f = 0.30$ . . . . . (13)

When $V = 2000$ to $5000, f = 0.30 - 0.00004\ (V - 2000)$. (14)

These may be considered maximum values of f.

These tests indicate that f diminishes when the pressure per square inch, P, increases. For a velocity, V, of 20 miles per hour, or 1760 ft. per minute, the average results were:

	Pressure in Pounds per Square Inch	
	58	144
	Values of f	
Soft cast-iron shoes on steel wheels.....	0.375	0.265
Hard cast-iron shoes on chilled cast-iron wheels...	0.340	0.255

(2) *Wood.*—Wood linings are often used in brake bands. Charring is likely to begin if the running temperature of the brake exceeds about 150° F. Such brake linings should be liberally proportioned and the wheels designed to radiate heat effectively. In ordinary cases, the web, or arms, of a brake wheel assist but little in heat dissipation, and unless they have special ribs to increase the radiating surface, the rim only should be considered as available for heat dissipation. The maximum brake pressure may be assumed at 60 lbs. per square inch.

Coefficient of friction.—Professor L. Klein, of Hanover, has given the results of extensive tests on the friction of wood on cast- and wrought-iron brake wheels. For velocities of from 180 to 3900 ft. per minute and pressures of from 7 to 140 lbs. per square inch, the values of f were found to be nearly constant. When the brake-wheel surfaces were smooth the value of f was higher than when they were rough. The minimum, average, and maximum values of f are given in Table 13A.

(3) *Asbestos.*—Manufacturers of asbestos products, like Johns-Manville and the Raybestos Company, make brake blocks and flexible brake-band linings that afford some advantages when compared with wood blocks. The running temperature may be made 400° F. with safety, and even 500° F. is possible with some asbestos products.

The maximum brake pressure may be assumed as 100 lbs. per square inch.

Coefficient of friction.—The coefficient of friction varies with the rubbing velocity and the running temperature.

TABLE 13A

COEFFICIENTS OF FRICTION, f, FOR SMOOTH, CLEAN SURFACES

Legend

⟹ Grain of wood parallel to the direction of motion.

⟹ Grain of wood at right angles to the direction of motion.

⟹ End of grain bearing on the brake wheel.

Brake wheel....	Beech	Oak			Poplar			Elm	Willow
	⟹	⟹	⟹	⟹	⟹	⟹	⟹	⟹	⟹
Cast iron.....	0.28	0.29	0.25	0.19	0.34	0.30	0.29	0.32	0.43
	0.29	0.30	0.28	0.20	0.35	0.36	0.31	0.36	0.46
	0.30	0.31	0.35	0.21	0.37	0.40	0.37	0.38	0.48
Wrought iron.	0.51	0.48	.....	.....	0.52	.....	.....	0.57	0.41
	0.54	0.51	.....	.....	0.65	.....	.....	0.60	0.63
	0.56	0.54	.....	.....	0.73	.....	.....	0.64	0.93

For ordinary temperatures, $f = 0.35$, maximum. As the temperature increases f diminishes slowly, until at 400° to 500° F. distillation of the volatile matter in the material begins and the value of f falls to about 0.20.

For any ordinary design, f may conservatively be assumed at 0.25.

(4) *Rivets for attaching brake linings.*—Copper rivets have been used for attaching brake linings in many cases. The latest experiments show that particles of copper become attached to the brake wheels when worn smooth. This results in a reduction of the coefficient of friction.

Common brass rivets do not glaze the brake wheel and should be used.

6. Types of brakes.—Brakes intended to retard, stop, and hold mechanisms operating under loads are made in a great variety of forms. There are two simple types that are particularly adapted for use on the operating machinery of movable bridges:

(1) Block brakes, in which one or more brake blocks, or shoes, are pressed against the periphery of a rotating brake wheel; (2) Band brakes, in which a flexible brake band passes around a large part of the periphery of a rotating brake wheel.

In both of these types the brakes are applied by means of a system of levers and links actuated by manual, pedal, or mechanical power.

(3) A third type, which might be termed an automatic brake, is not often used in movable-bridge machinery. Automatic brakes are made in many forms. The type particularly adapted for use on cranes, hoists, and similar devices is designed to afford full control of loads when being lowered. The frictional elements of these brakes may be shoes, blocks, multiple discs, or coils arranged in a variety of ways. Automatic brakes are often designed so that the retarding torques are directly proportional to the loads, and also so that the motors must be operated to lower the loads. In some forms of automatic brakes, as applied to the electrically operated mechanism, the retarding torques are produced by electrical resistances, as in regenerative brakes.

The first two types of brakes present no great difficulties to the designer, and are of relatively simple construction. These are the only types that will be discussed in detail.

The third type presents some of the most delicate and difficult problems of machine design, and a full treatment of this type would be out of place within the scope of this treatise. Those wishing to investigate this subject further will find many data in the transactions of the mechanical engineering societies, and in the files of the technical periodicals devoted to mechanical engineering and mechanism. The following books also contain valuable data:

Kent's "Mechanical Engineers' Handbook"; Marks' "Mechanical Engineers' Handbook"; "Machinery's Handbook"; "Machinery's Encyclopedia," article, Brakes; "Machine Design," by H. D. Hess; and "The Electrical Handling of Materials," by H. H. Broughton, Vol. II.

Most treatments of the design of brakes are confined to discussions of the design of the brakes proper, and the determination of resisting torques adequate to retard, stop, and hold loads; they do not consider the forces developed at the fulcrums and anchorages of the brake-operating mechanism. In some actual designs, these reactions were partially neglected, with the result that the parts to which anchorages were made did not possess the necessary rigidity to insure smooth operation of the brakes without vibration of the structural parts.

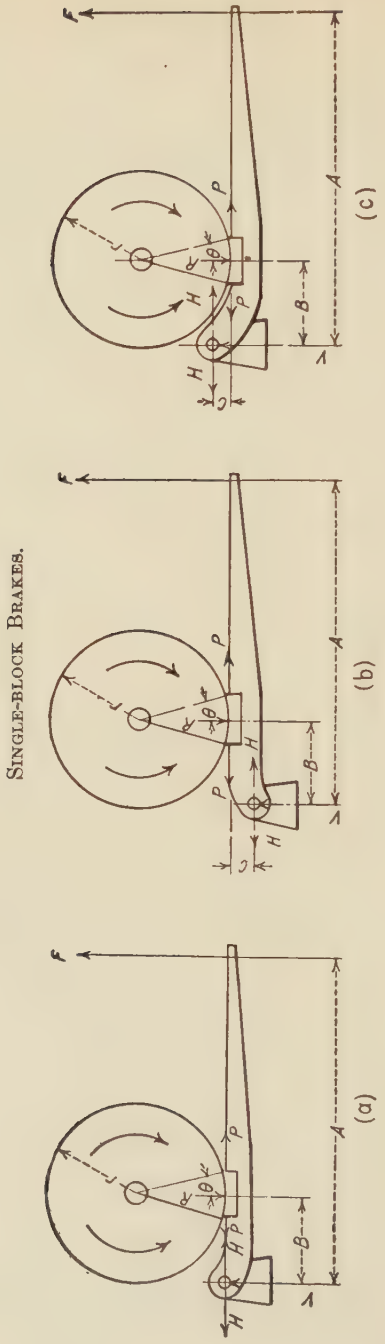
In this discussion of the two types adapted for use on movable bridges, expressions for the reactions, as well as for the braking torques, will be given.

7. Single-block brakes.—Three arrangements of single-block brakes are shown in Fig. 13A (*a*), (*b*), and (*c*).

Let F = force applied to lever, in pounds;

A = distance from point of application of F to fulcrum, in inches;

B = distance from center of brake-block to fulcrum, in inches;



Single-block Brakes.

Fig. 13A.

R = reaction between brake block and wheel, in pounds;
f = coefficient of friction between brake block and wheel;
P = tangential frictional resistance, $= R \times f$, in pounds;
r = radius of brake wheel, in inches;
T = torsional moment on brake-wheel shaft, in inch-pounds, and
C = distance from fulcrum to line of action of P, in inches.

Note that in Fig. 13A (a), $C = 0$.

Figure 13A (a). *For rotation in either direction.*

$$R = \frac{FA}{B}. \quad P = Rf. \quad F = \frac{RB}{A} = \frac{PB}{fA}. \quad . \quad . \quad . \quad . \quad (15)$$

$$T = Rfr = \frac{FA}{B} fr. \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Horizontal reaction at fulcrum,

$$H = \pm P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Vertical reaction at fulcrum,

$$V = \frac{F(A - B)}{B}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Figure 13A (b). *For clockwise rotation.*

$$R = \frac{FA + PC}{B}. \quad P = Rf. \quad \therefore R = \frac{FA + RfC}{B} \text{ or } R = \frac{FA}{B - fC}. \quad (19)$$

$$F = \frac{R(B - fC)}{A}, \quad \text{but} \quad R = \frac{P}{f}. \quad \therefore F = \frac{PC\left(\frac{B}{C} - f\right)}{fA}. \quad . \quad (20)$$

If $\frac{B}{C}$ is made less than f the brake becomes a self-acting clamp which will wreck the mechanism.

$$T = Rfr = \frac{FA}{B - fC} fr. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Horizontal reaction at fulcrum,

$$H = P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Vertical reaction at fulcrum,

$$V = \frac{F(A - B) + PC}{B}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Figure 13A (b). *For counter-clockwise rotation.*

$$R = \frac{FA - PC}{B}. \quad P = Rf. \quad \therefore R = \frac{FA - RfC}{B} \text{ or } R = \frac{FA}{B + fC}. \quad (24)$$

$$F = \frac{R(B + fC)}{A}, \quad \text{but} \quad R = \frac{P}{f}. \quad \therefore F = \frac{PC\left(\frac{B}{C} + f\right)}{fA}. \quad . \quad (25)$$

$$T = Rfr = \frac{FA}{B + fC} fr. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Horizontal reaction at fulcrum,

$$H = P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Vertical reaction at fulcrum,

$$V = \frac{F(A - B) - PC}{B}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Figure 13A (*c*). *For clockwise rotation* the formulae are the same as for Fig. 13A (*b*) for counter-clockwise rotation; and for counter-clockwise rotation they are the same as for Fig. (*b*) for clockwise rotation.

In Fig. 13A (*c*) for counter-clockwise rotation $\frac{B}{C}$ must be greater than f or the brake would become a self-acting clamp.

8. Double-block brakes.—A typical arrangement of a double-block brake is shown in Fig. 13B, (*a*), (*b*), and (*c*). A fixed frame, TU, is attached to the superstructure of the bridge. Brake-block levers OS and PQ, are anchored to the frame at points S and Q. Lever MN is articulated to lever OS at O, and to link NP at N. Link NP is attached to lever PQ at P. The brake is operated by a force F, acting at point M. Any movement of point M upward applies the brake and a movement downward releases it. Link NP is made so that it can be lengthened or shortened in order to adjust the position of the brake block with respect to the brake-wheel, W. Lever NM is shown as extending horizontally to the right of point N. It might extend vertically upward or horizontally to the left of N, as indicated by broken lines in Fig. (*a*).

Let the main lever, NM, extend horizontally from N to M;
force F be applied vertically upward;
the brake-wheel, W, rotate in a clockwise direction;
the various dimensions, forces, and reactions be as indicated in the figures, and as given in Art. 7, so far as they apply to this form of brake, and
t = the tension in link NP.

Refer to Fig. 13B (*a*) and (*b*), and take moments about the point O, then,

$$Fa = tb \cos (\beta + \theta);$$

hence

$$t = \frac{Fa}{b \cos (\beta + \theta)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

DOUBLE-BLOCK BRAKE.

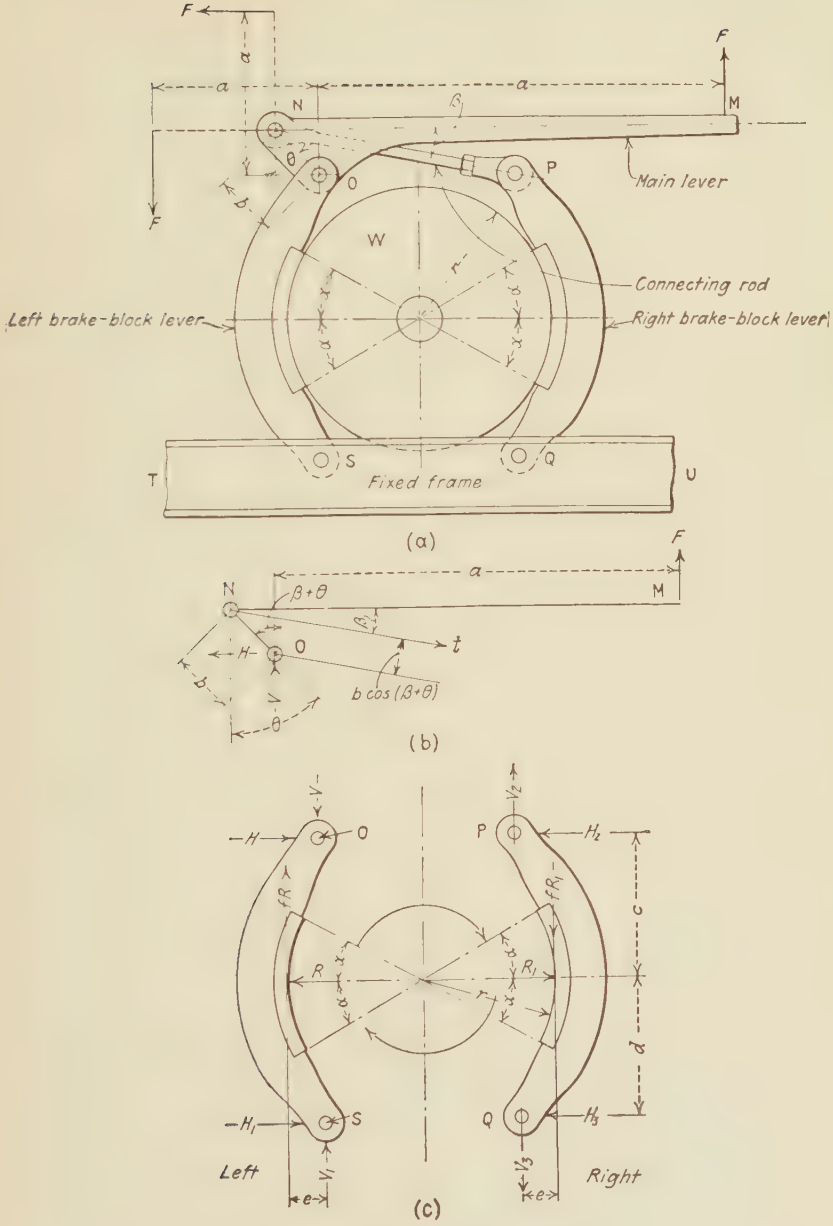


FIG. 13B.

$$H = t\cos\beta = \frac{Fa\cos\beta}{b\cos(\beta+\theta)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

$$V = t\sin\beta - F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$V = F\left\{\frac{a\sin\beta}{b\cos(\beta+\theta)} - 1.\right\} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Let $\frac{Fa\sin\beta}{b\cos(\beta+\theta)} = B$ and $\frac{Fa\cos\beta}{b\cos(\beta+\theta)} = C$, then

$$H = C \quad \text{and} \quad V = B - F. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Refer now to Fig. 13B (*c*). If f is the coefficient of friction of the blocks against the wheel, the vertical forces at the point of contact will be fR and fR_1, and they will act in the directions shown. If moments are taken about S,

$$H(c+d) + fRe = Rd,$$

and

$$R = H\frac{c+d}{d-fe} = C\frac{c+d}{d-fe}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Taking moments about O,

$$H_1(c+d) = Rc + fRe,$$

and

$$H_1 = R\frac{c+fe}{c+d} = C\frac{c+fe}{d-fe}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$V_1 = V - fR = B - F - fC\frac{c+d}{d-fe}. \quad . \quad . \quad . \quad (36)$$

For the right brake-block lever, PQ, $V_2 = B$ and $H_2 = C$.

The reactions R_1, H_3, and V_3 are found in a manner similar to that which was used in determining R_1, H_1, and V_1.

The total torsional moment, at the shaft of the brake wheel produced by the force F, acting through the lever system, is

$$T = f(R + R_1)r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

If the operating lever, NM, extends to the left of point N or vertically upward from it, and the force F acts as indicated by the broken lines in Fig. 13B (*a*), all of the reactions can be found in a precisely similar manner to that which was used for the lever extending to the right of point N.

Table 13B gives the magnitude and direction of the various forces and reactions for each of the three positions of lever NM and the corresponding directions of force F.

TABLE 13B

FORCES ON BRAKE-BLOCK LEVERS AND TORSIONAL BRAKING MOMENTS

Position of main lever and direction of applied force, F	Lever Right, Force Up				Lever Left, Force Down				Lever Vertical, Force Left			
Rotation of wheel	Clockwise		Counter-clockwise		Clockwise		Counter-clockwise		Clockwise		Counter-clockwise	
Position of brake-shoe lever	Left	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left	Right
Intensity and direction of vertical force at top pin	$A\downarrow$	$B\uparrow$	$A\downarrow$	$B\uparrow$	$O\downarrow$	$B\uparrow$	$O\downarrow$	$B\uparrow$	$B\downarrow$	$B\uparrow$	$B\downarrow$	$B\uparrow$
Intensity and direction of horizontal force at top pin	$C\rightarrow$	$C\leftarrow$	$C\rightarrow$	$C\leftarrow$	$C\rightarrow$	$C\leftarrow$	$C\rightarrow$	$C\leftarrow$	$S\rightarrow$	$C\leftarrow$	$S\rightarrow$	$C\leftarrow$
Pressure against brake shoe	D	E	E	D	D	E	E	D	T	E	U	D
Intensity and direction of vertical force at brake shoe	$fD\uparrow$	$fE\downarrow$	$fE\downarrow$	$fD\uparrow$	$fD\uparrow$	$fE\downarrow$	$fE\downarrow$	$fD\uparrow$	$fT\uparrow$	$fE\downarrow$	$fU\downarrow$	$fD\uparrow$
Intensity and direction of vertical force at bottom pin	$G\uparrow$	$I\downarrow$	$J\uparrow$	$K\downarrow$	$P\uparrow$	$I\downarrow$	$Q\uparrow$	$K\downarrow$	$W\uparrow$	$I\downarrow$	$X\uparrow$	$K\downarrow$
Intensity and direction of horizontal force at bottom pin	$L\rightarrow$	$M\leftarrow$	$M\rightarrow$	$L\leftarrow$	$L\rightarrow$	$M\leftarrow$	$M\rightarrow$	$L\leftarrow$	$Y\rightarrow$	$M\leftarrow$	$Z\rightarrow$	$L\leftarrow$
Total torsional braking moment for both brake-shoe levers	N		N		N		N		Δ		Θ	

$A = B - F$ $\quad B = \dfrac{Fa\sin\beta}{b\cos(\beta+\theta)}$ $\quad C = \dfrac{Fa\cos\beta}{b\cos(\beta+\theta)}$ $\quad D = C\left(\dfrac{c+d}{d-fe}\right)$

$E = C\left(\dfrac{c+fe}{d+fe}\right)$ $\quad G = B - fD - F$ $\quad I = B - fE$ $\quad J = B + fE - F$

$K = B + fD$ $\quad L = C\left(\dfrac{c+fe}{d-fe}\right)$ $\quad M = C\left(\dfrac{c-fe}{d+fe}\right)$ $\quad N = fr(D+E)$

$O = B + F$ $\quad P = B + F - fD$ $\quad Q = B + F + fE$ $\quad S = C - F$

$T = S\left(\dfrac{c+d}{d-fe}\right)$ $\quad U = S\left(\dfrac{c+d}{d+fe}\right)$ $\quad W = B - fT$ $\quad X = B + fU$

$Y = S\left(\dfrac{c+fe}{d-fe}\right)$ $\quad Z = S\left(\dfrac{c-fe}{d+fe}\right)$ $\quad \Delta = fr(T+E)$ $\quad \Theta = fr(U+D)$

In the discussion of both single- and double-block brakes, it has been assumed that the unit pressures between the curved surfaces of the brake blocks and the wheels were uniform and parallel to the middle element, indicated by R in the figures. When the block subtends a small central angle this is nearly true. When the central angle becomes larger it is more probable that the radial unit pressures are uniform.

Double-block brakes, Figs. 13B, (*a*) *and* (*c*).

Let W_B = the width of the face of the brake wheel in contact with the brake blocks;
α = one-half of the central angle subtended by one brake block;
p = the unit radial pressure of the left block against the brake wheel;
p_1 = the unit radial pressure of the right block against the brake wheel;
T_B = the torsional moment of the brake; and
N = number of rotations of the brake wheel per minute.

Then:

$$p = \frac{R}{2W_B r \sin \alpha} \quad \text{and} \quad p_1 = \frac{R_1}{2W_B r \sin \alpha}. \quad . \quad . \quad . \quad (38)$$

$$T_B = 2fW_B r^2 \alpha (p + p_1), \text{ in which } \alpha \text{ is in radians.} \quad . \quad (39)$$

When α is expressed in degrees this becomes

$$T_B = 2fW_B r^2 \frac{\pi}{180} \alpha^\circ (p + p_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Substitute the values of p and p_1 from (38)

$$T_B = fr\frac{\pi}{180}\alpha^\circ \frac{R + R_1}{\sin \alpha} = 0.01745fr\alpha^\circ \frac{R + R_1}{\sin \alpha}. \quad . \quad . \quad (41)$$

The velocity of the surface of the brake wheel, in feet per minute, is

$$V_B = 2\pi \frac{r}{12} N = \tfrac{1}{6} \pi r N. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

In practice, the following limits should be observed:

$$p \text{ or } p_1 \leqq 60. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

$$pV_B \text{ or } p_1 V_B \leqq 60{,}000. \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

9. Simple band brakes.—Figure 13C (*a*) and (*b*) shows sketches of two arrangements of simple band brakes.

Let F = force applied to lever, in pounds;
A = distance from point of application of F to fulcrum, in inches;
B = lever arm of stress in T_1 to fulcrum, in inches;

λ = angle of T_1 (or T_2) with long arm of lever, in degrees;
θ = angle of T_2 (or T_1) with long arm of lever, in degrees;
δ = angle made by short with long arm of lever, in degrees;
T_1 = tension in band on the tight side, in pounds;
T_2 = tension in band on the loose side, in pounds;
P = tangential force of rim of wheel, in pounds, $= T_1 - T_2$;
f = coefficient of friction between the band and wheel;
α = angle of wrap of band around wheel, in degrees;
r = radius of wheel, in inches;
T = torsional moment on brake-wheel shaft, in inch-pounds.

SIMPLE BAND BRAKES.

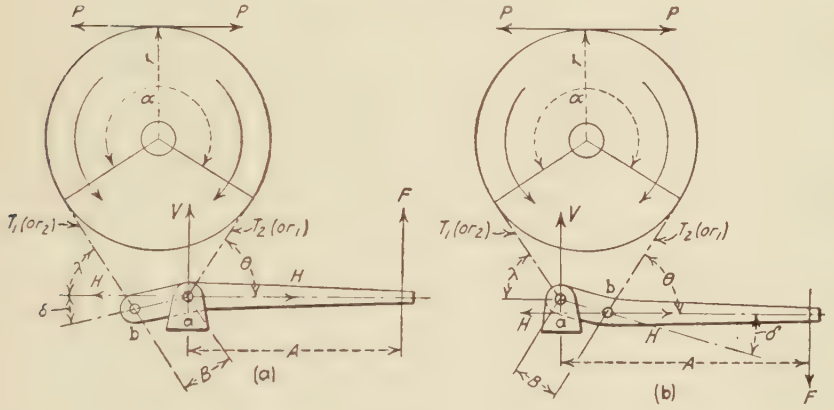


FIG. 13C.

The fundamental equation expressing the relation between T_1 and T_2 is

$$\frac{T_1}{T_2} = e^{f\alpha_1},$$

e being the base of Napierian logarithms and α_1 the angle of wrap in radians. Passing to common logarithms and degrees, this becomes

$$\frac{T_1}{T_2} = 10^{0.0076f\alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

If

$$m = 0.0076f\alpha, \frac{T_1}{T_2} = 10^m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$$T = Pr \quad \text{or} \quad P = \frac{T}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Figure 13C (*a*). *Clockwise rotation.* T_1 *at b.*

From (46)

$$T_1 = T_2 \times 10^m.$$

Also

$$T_1 - T_2 = P. \quad \therefore P = T_2(10^m - 1) \quad \text{or} \quad T_2 = \frac{P}{10^m - 1} \Bigg\}. \quad (48)$$

and

$$T_1 = \frac{10^m}{10^m - 1}P.$$

$$FA = T_1B. \qquad \therefore F = T_1\frac{B}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Tension in

$$ba = T_1 \cos(\lambda + \delta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

Horizontal force at

$$a = H = T_1 \cos\lambda - T_2 \cos\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

Vertical force at

$$a = V = F\left[\frac{A}{B}\sin\lambda + 1\right] + T_2 \sin\theta. \quad . \quad . \quad . \quad . \quad (52)$$

Figure 13C (*a*). *Counter-clockwise rotation,* T_1 *at a.*

Eqs. (48) are unchanged.

$$FA = T_2B. \qquad \therefore F = T_2\frac{B}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Tension in

$$ba = T_2 \cos(\lambda + \delta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

Horizontal force at

$$a = H = T_1 \cos\theta - T_2 \cos\lambda. \quad . \quad . \quad . \quad . \quad (55)$$

Vertical force at

$$a = V = F\left[\frac{A}{B}\sin\lambda + 1\right] + T_1 \sin\theta. \quad . \quad . \quad (56)$$

Figure 13C (*b*). *Clockwise rotation,* T_1 *at a.*

Eqs. (48) are unchanged.

$$FA = T_2B. \qquad \therefore F = T_2\frac{B}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Tension in

$$ab = T_2 \cos(\theta + \delta) + F \sin\delta. \quad . \quad . \quad . \quad . \quad (58)$$

Horizontal force at

$$a = H = T_1 \cos\lambda - T_2 \cos\theta. \quad . \quad . \quad . \quad . \quad (59)$$

Vertical force at

$$a = V = F\left[\frac{A}{B}\sin\theta - 1\right] + T_1 \sin\lambda. \quad . \quad (60)$$

Figure 13C (*b*). *Counter-clockwise rotation,* T_1 *at b.*

Eqs. (48) are unchanged.

$$FA = T_1 B. \qquad \therefore F = T_1 \frac{B}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Tension in

$$ab = T_1 \cos(\theta + \delta) + F \sin \delta. \quad . \quad . \quad . \quad . \quad (61a)$$

Horizontal force at

$$a = H = T_1 \cos \theta - T_2 \cos \lambda. \quad . \quad . \quad . \quad . \quad (62)$$

Vertical force at

$$a = V = F\left[\frac{A}{B}\sin \theta - 1\right] + T_2 \sin \lambda. \quad . \quad (63)$$

Arrangements shown in Fig. 13C (*a*) and (*b*) may be used for wheels rotating in either direction.

Figure 13C (*a*) is the more efficient for counter-clockwise rotation.

Figure 13C (*b*) is the more efficient for clockwise rotation.

10. Compound and differential band brakes.—Figure 13D (*a*) and (*b*) shows sketches of the general arrangement of a compound and a differential band brake. The compound brake shown in Fig. 13D (*a*) is the most generally useful of the four types discussed in this and the preceding article. When the proportions are properly chosen, this type of brake is reversible and the braking effect can be made the same for rotation of the brake drum in either direction.

COMPOUND AND DIFFERENTIAL BAND BRAKES.

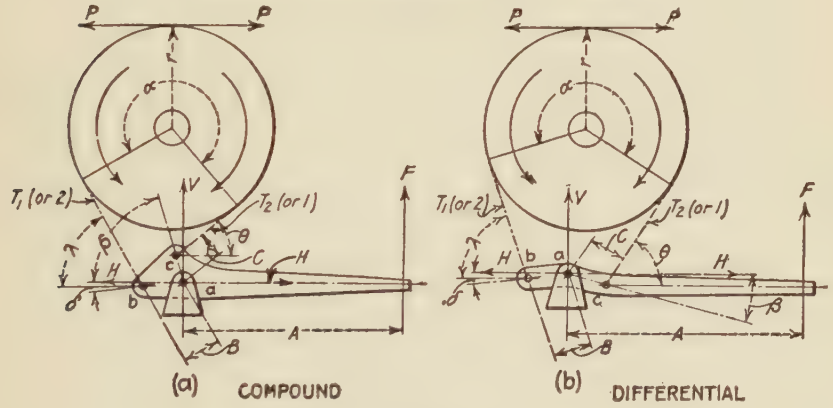


FIG. 13D.

Let notation be the same as for simple band brakes and let:

C = lever arm of stress T_2 (clockwise rotation) about the fulcrum, and

β = angle made by lever C with the horizontal.

The fundamental equations (45), (46), (47), and (48) are the same as for simple band brakes.

Figure 13D (*a*). *Clockwise rotation, T_1 at b.*

$$FA = T_1B + T_2C. \quad \therefore F = \frac{T_1B + T_2C}{A}. \quad . \quad . \quad . \quad (64)$$

Tension in

$$ba = T_1 \cos (\lambda + \delta).$$

Tension in

$$ca = T_2 \cos (\beta + \theta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Horizontal force at

$$a = H = T_1 \cos \lambda - T_2 \cos \theta. \quad . \quad . \quad . \quad . \quad (66)$$

Vertical force at

$$a = V = F\left[\frac{A \sin \lambda}{B} + 1\right] + T_2 \sin \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

Figure 13D (*a*). *Counter-clockwise rotation, T_1 at c.*

$$FA = T_2B + T_1C. \quad \therefore F = \frac{T_2B + T_1C}{} \quad . \quad . \quad . \quad (68)$$

Tension in

$$ba = T_2 \cos (\lambda + \delta).$$

Tension in

$$ca = T_1 \cos (\beta + \theta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

Horizontal force at

$$a = H = T_1 \cos \theta - T_2 \cos \lambda. \quad . \quad . \quad . \quad . \quad (70)$$

Vertical force at

$$a = V = F\left[\frac{A \sin \lambda}{B} + 1\right] + T_1 \sin \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

The arrangement in Fig. 13D (*a*) may be used for wheels rotating in either direction. It is frequently desirable to obtain the same braking effect, for a given applied force, for clockwise and counter-clockwise rotation. The torsional braking moment

$$T = Pr = (T_1 - T_2)r = T_1r\left(\frac{10^m - 1}{10^m}\right).$$

It is therefore a function of T_1. From (64) and (46),

$$T_1 = \frac{10^mFA}{10^mB + C}.$$

From (68) and (46),

$$T_1 = \frac{10^mFA}{B + 10^mC}.$$

These two value of T_1 are the same if $B = C$. Making $B = C$ therefore makes the brake equally effective for both directions of rotation.

Figure 13D (*b*). *Clockwise rotation, T_1 at b.*

$$FA = T_1B - T_2C. \quad \therefore F = \frac{T_1B - T_2C}{A}. \quad . \quad . \quad . \quad . \quad . \quad (72)$$

Tension in

$$ba = T_1 \cos(\lambda + \delta) \quad \text{and in} \quad ac = T_2 \cos(\theta + \beta) + F \sin \beta. \quad (73)$$

Horizontal force at

$$a = H = T_1 \cos \lambda - T_2 \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Vertical force at

$$a = V = F\left[\frac{A \sin \lambda}{B} + 1\right] + T_2 \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

Note from (72) and (48) that whenever $C > 10^m B$ no braking effect would be produced, for any movement in the direction of F would loosen the brake band.

Figure 13D (*b*). *Counter-clockwise rotation, T_1 at c.*

$$FA = T_2B - T_1C. \quad \therefore F = \frac{T_2B - T_1C}{A}. \quad . \quad . \quad . \quad . \quad . \quad (76)$$

Tension in

$$ba = T_2 \cos(\lambda + \delta) \quad \text{and in} \quad ac = T_1 \cos(\theta + \beta) + F \sin \beta. \quad (77)$$

Horizontal force at

$$a = H = T_1 \cos \theta - T_2 \cos \lambda. \quad . \quad . \quad . \quad . \quad . \quad (78)$$

Vertical force at

$$a = V = F\left[\frac{A \sin \lambda}{B} + 1\right] + T_1 \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad (79)$$

Note from (76) and (48) that whenever $10^m C > B$ the brake becomes a clamp which would wreck the mechanism, for the action of F and T_1 become cumulative.

The arrangement in Fig. 13D (*b*) should never be used unless the wheel always rotates in the same direction and even in this case is not an efficient mechanism, for the required movement of the lever at F is relatively large.

11. Friction of bands.—Figure 13E is a chart giving the values of the function

$$\frac{T_1}{T_2} = 10^{0.0076/\alpha} \text{ of Eq. (45).}$$

The value of $\frac{T_1}{T_2}$ increases rapidly with the angle of contact between the brake band and the wheel. The details should be arranged to take advantage of this relationship and to make the angle of contact as large as practicable. In some unusual cases the angle of contact is increased by passing one end of the brake band through a slot in the other end.

The brake band will usually be a bar of thin steel lined with some heat-resisting fabric with an asbestos base. Large brake bands are usually lined with blocks of suitable wood, or with special heat-resisting blocks.

Care should be taken not to make the band too thick. The increased stiffness of a thick band rapidly reduces the efficiency of the brake.

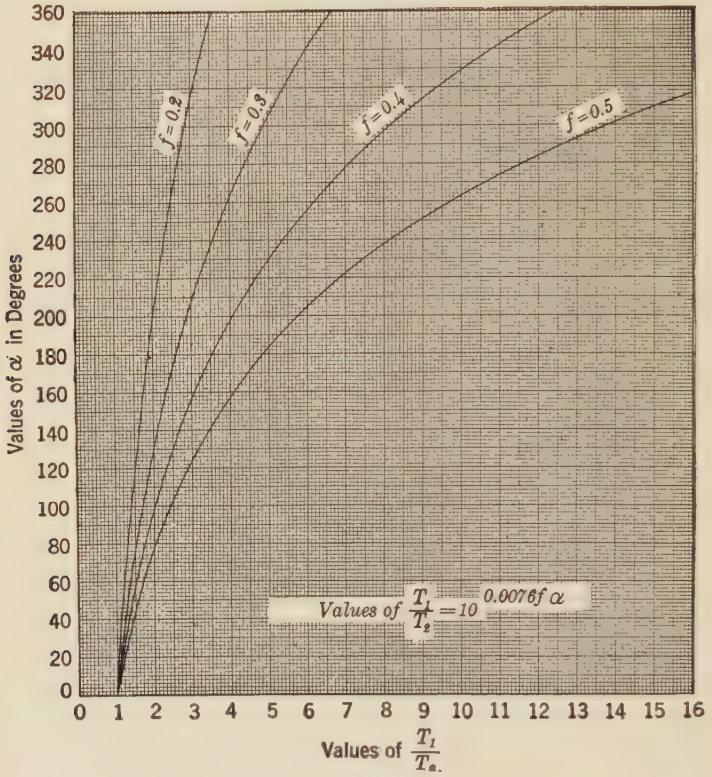


Fig. 13E.

Band brakes should be operated by hand levers. This insures better control than when a pedal is used.

A man can pull up to about 50 or 60 lbs. on a hand lever, but can apply his full weight to a pedal, with the attendant danger of applying the brake too suddenly and causing unnecessary stresses and shocks in the mechanism.

12. Band or block brakes with V-shaped groove in the brake wheel. —Refer to Fig. 13F.

Let D = the mean diameter of the bearing surface of the brake band, or block, in a grooved wheel;

p = the normal unit pressure on an element of a flat-faced wheel, of diameter D;

p_1 = the normal unit pressure on one side of the V-groove.

$$ab = \frac{p}{2}. \qquad ac = p_1.$$

$$\sin \alpha = \frac{\frac{p}{2}}{p_1} = \frac{p}{2p_1}, \quad \text{or} \quad p_1 = \frac{p}{2 \sin \alpha}.$$

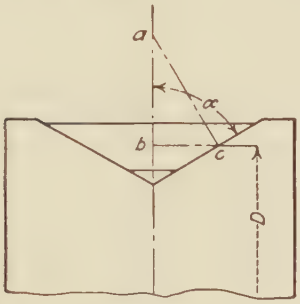


FIG. 13F.

p_1 occurs on each side of the V-groove, hence

$$2p_1 = \frac{p}{\sin \alpha}.$$

The unit tangential braking force for a flat-faced wheel is,

$P = pf$, and for a V-groove,

$$P_1 = 2p_1 f = \frac{pf}{\sin \alpha}. \quad \frac{P_1}{P} = \frac{1}{\sin \alpha} = \operatorname{cosec} \alpha,$$

hence,

$$P_1 = P \operatorname{cosec} \alpha. \qquad \ldots\ldots\ldots \quad (80)$$

Band brakes. Working limits.

Let W_B = the width of the band in contact with the brake wheel;

p_1 = the maximum radial unit pressure between the band and the brake wheel at the tight end of the band.

Then:

N = the number of rotations of the brake wheel per minute.

$$p_1 = \frac{T_1}{W_B r}. \qquad \ldots\ldots\ldots \quad (81)$$

The velocity of the surface of the brake wheel, in feet per minute, is

$$V_B = 2\pi \frac{r}{12}(N) = \tfrac{1}{6}\pi r(N). \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

In practice, the following limits should be observed:

$$p_1 \lessapprox 60. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

$$p_1 V_B \lessapprox 60{,}000. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

13. Comparison of the efficiency of brakes.—In order to make a rough comparison of the efficiency of the different kinds of brakes, the following assumptions have been made:

For all brakes, $f = 0.3$.

For single-block brakes, $A = 5B$, $C = B$.

For double-block brakes $\dfrac{a}{b\cos(\beta + \theta)} = 5$, $\beta = 10°$, $c = d = 4e$.

For all band brakes, $A = 5B$, $C = B$, $\alpha = 260°$.

The tangential retarding force, P, exerted under these conditions at the circumference of the brake wheel for each unit of force, F, applied on the brake lever, is recorded in Table 13C.

TABLE 13C

Design of Brake		Rotation of Brake Wheel: Clockwise Values of P	Rotation of Brake Wheel: Counter-Clockwise Values of P
Single-block brakes	Fig. 13A (*a*)	1.50	1.50
	Fig. 13A (*b*)	2.14	1.15
	Fig. 13A (*c*)	1.15	2.14
* Double-block brakes, main lever, Fig. 13B.	right	5.94	5.94
	left	5.94	5.94
	vertical	5.30	5.38
Simple band brakes	Fig. 13C (*a*)	3.73	14.60
	Fig. 13C (*b*)	14.60	3.73
† Compound band brake	Fig. 13D (*a*)	2.97	2.97
Differential band brake	Fig. 13D (*b*)	5.00	Clamps

* Of the brakes analyzed, this is the most expensive.

† This is the cheapest effective brake. While the value of P, as shown in the table, is not high, it can readily be increased by making B and C small with respect to A. Thus, making $A = 10B = 10C$, $P = 5.94F$.

CHAPTER XIV

HYDRAULIC MACHINERY

1. General.—Hydraulic machinery is sometimes used for the operation of movable bridges, and hydraulic jacks are often required in connection with the erection and adjustment of movable-bridge superstructures. It will be of interest to refer to a few of the general principles of design and some of the structural details of hydraulic mechanism.

The stresses in hydraulic machinery can be determined with a greater degree of accuracy than those in some of the more common types of mechanism. For this reason, larger unit stresses are permissible, but the designs must be developed with great care. Cast iron should be avoided in most cases. The various parts should be of cast or forged steel and the size and shape of the parts will usually determine which is the more desirable. Carefully rounded fillets should be used at all points where there is a change of section. Holes in tension elements should be avoided wherever it is possible to do so. If holes must be used, the castings should be reinforced by thickening the metal around them. Concentrations of stress around holes is likely to cause failure unless adequate precautions are taken in the design.

Cast-steel cylinders and similar parts may be stressed to from 16,000 to 18,000 lbs. per square inch, if the maximum stresses are fully determined. Forged steel may be stressed a little higher in special cases; but these high unit stresses are only appropriate for large parts which would become uneconomical and unwieldy if lower unit stresses were adopted. In the case of parts of reasonable size, stresses of from 10,000 to 14,000 lbs. per square inch will be found to be practicable.

Attention should be given to the stresses developed during the cooling of large castings. The thinner material required by the larger unit stresses may result in castings that are actually stronger than those made thicker but with the resulting increased cooling stresses. It is particularly important that the designs be made so that it is practicable to make sound castings in the foundry.

All castings should be thoroughly annealed.

2. Stresses in hydraulic cylinders.—If a cylinder with closed ends is subjected to internal pressure, the pressure tends to burst it along a

longitudinal element of the cylinder, to rupture it on a circumferential element, and to buckle the closed ends outwardly. It is required to find the stresses resulting from the internal pressure.

(*1*) *Cylinders with thin walls.*

Let p = the pressure per unit of area within the cylinder;
d = the inside diameter;
t = the thickness of the wall of the cylinder, and
T = a tensional unit stress.

Considering a unit of length of the cylinder, the tangential tensional unit stress along an element of the cylinder is

$$T_t = \frac{pd}{2t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The total pressure on each cylinder head is

$$P_h = \frac{p\pi d^2}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Considering any circumferential element of the cylinder, the longitudinal tensional unit stress is

$$T_l = \frac{pd}{4t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(*2*) *Cylinders with thick walls.*

Let p = the pressure per unit of area within the cylinder;
r_1 = the inside radius of the cylinder;
r_2 = the outside radius of the cylinder, and
T = the tangential unit tensional stress at the inside of the wall of the cylinder, then

$$T = \frac{1}{3}\frac{r_1^2 + 4r_2^2}{r_2^2 - r_1^2}p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This formula is convenient for the investigation of the strength of a known cylinder which is subjected to known internal pressures.

In the case of a cylinder for a particular purpose, the internal pressure and the inner radius are known, and the usual problem is to determine an outside radius such that T shall not exceed a given value. Equation (4), when solved for r_2, becomes

$$r_2 = r_1\sqrt{\frac{3T + p}{3T - 4p}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Equations (4) and (5) are Clavarino's formulae, which in this form apply to thick cylinders with inside, but no outside, pressures. The

tangential tensional stress, T, is a maximum at the inside surface of the cylinder.

If the effect of elastic distortions on unit stress be neglected, and if it is assumed that the stresses are distributed uniformly across the sections, it will be noted that the tensional unit stress on any circumferential element of a cylinder is only one-half of that tending to burst it along a longitudinal element. On this basis, the resultant combined unit stress on an element of the cylinder is

$$\sqrt{1^2 + 0.5^2} = \sqrt{1.25} = 1.118,$$

in terms of the tangential bursting tension.

3. Spheres with thick walls.—Let the nomenclature be the same as for cylinders with thick walls, then

$$T = \frac{1}{3}\frac{2r_2^3 + r_1^3}{r_2^3 - r_1^3}p. \quad \ldots \quad (6)$$

$$r_2 = r_1\sqrt[3]{\frac{3T + p}{3T - 2p}}. \quad \ldots \quad (7)$$

Equation (6) is suitable for the investigation of a known sphere and Eq. (7) for the design of a sphere under assumed conditions.

4. Heads of cylinders.—If the cylinder heads are hemispherical, Eqs. (6) and (7) apply, but such heads are seldom practicable. Cylinder heads for high pressures should not be made flat, but should approach a hemispherical form. Good results may be expected when the inside radius of the curved head is about three-fourths of the inside diameter of the cylinder. The design of such a cylinder head offers considerable difficulty from a theoretical point of view; and it is best accomplished by approximations. If it is considered as a circular flat plate with its edges supported, or supported and fixed, it can be proportioned in accordance with formulae which are given in Appendix C, Table 11, Art. 4. If, a crowned head is nearly a hemisphere, Eqs. (6) and (7) apply. If the curvature is intermediate between flat and hemispherical, the thickness will have a corresponding intermediate value. It will usually be found that, in a cylinder head with ordinary curvature, the thickness of the metal should be from one-third to one-half greater than that of the walls of the cylinder.

The details of the cylinder heads should receive careful attention and any point of weakness should be avoided. No abrupt changes in section should be made. All such junctures should be carefully rounded with liberal fillets or by curves of long radius. If the cylinder head is perforated for the introduction of an inlet pipe, or for some other reason,

there should be a liberal flange or pad around the hole to resist stresses localized by the hole.

5. Hydraulic stuffing box.—Figure 14A (*a*) is a sketch of an ordinary hydraulic stuffing box and gland. This type of stuffing box has the advantage of extreme simplicity, and the packing is accessible for renewals and repairs.

The packing for ordinary pressures may be of common hemp. For higher pressures, square-braided flax or hemp is desirable. Various manufacturers specialize in forms of packing for which they make claims of superiority. Much valuable advice can be had from them. The

HYDRAULIC STUFFING BOX.

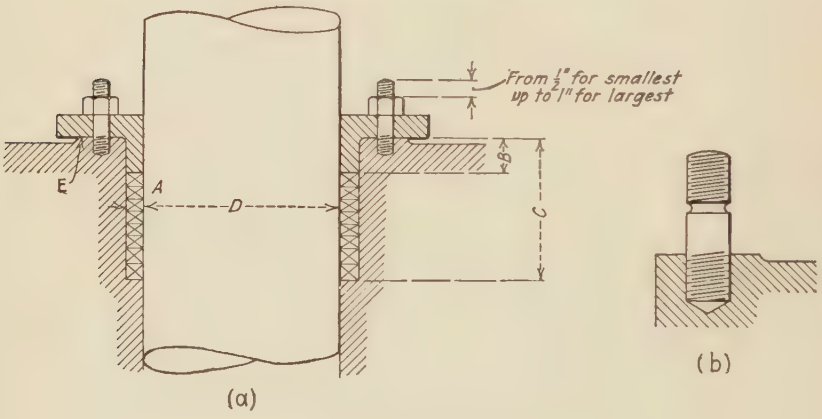


FIG. 14A.

material should be firm, readily applied, should not leak, and should hold its shape so as not to cause undue pressure against the plunger, which would result in excessive friction.

The gland bolts should be proportioned so that the tensile stress resulting from the hydraulic pressure upon the annular area of the packing will not exceed 5000 lbs. per square inch of net section of the bolts. Bolts with fine threads are better than those with coarse threads. The loss of effective section is less and they are not so likely to work loose.

On large rams, the stud bolts, Fig. 14A (*b*), should be grooved below the outer thread, so that if they break the fracture will be outside of the screw extending into the cylinder. They can then be readily removed and renewed.

Only sufficient packing should be inserted to permit the gland to

close on the end of the cylinder at E, and at the same time make a tight joint.

Table 14A gives the proper dimensions, in inches, for the various parts.

TABLE 14A

DIMENSIONS OF HYDRAULIC STUFFING BOXES, INCHES

	Below		Below		Below		Below		Below		Below	
D	1	$1\frac{1}{2}$	$1\frac{1}{2}$	3	3	5	5	8	8	12	12	20	20	24
A	$\frac{1}{4}$		$\frac{3}{8}$		$\frac{1}{2}$		$\frac{5}{8}$		$\frac{3}{4}$		$\frac{7}{8}$		1	
B	$\frac{3}{8}$		$\frac{1}{2}$		$\frac{3}{4}$		1		$1\frac{1}{4}$		$1\frac{1}{2}$		$1\frac{3}{4}$	
C	2		3		4		5		6		7		8	

6. Hydraulic gland with cup packing.—Figure 14B (*a*), (*b*), and (*c*), shows a type of hydraulic gland with leather cup packing. This type of packing is suitable for hydraulic presses and machines that are operated at frequent intervals, and may be used for pressures up to 6000 lbs. per square inch. Leather cup packing should operate with less loss by friction than the solid, square flax packing described in Art. 5.

Refer to Fig. 14B (*a*) and (*b*). The fluid in the cylinder in which the plunger moves enters the spaces between the bronze ring, C, and the leather cup, A, and as the fluid is under pressure, the circumferential flanges of A are pressed against the wall of the recess in the cylinder outside of the gland, and against the outer surface of the plunger, thus making fluid-tight contacts. The portion of ring C between the flanges of cup A must be made of such shape as to give proper support to the cup without danger of cutting the leather. The follower ring, B, must also properly support the leather, and the total height of ring C, leather cup A, and follower B must be made so that the leather will not be unduly compressed between the rings.

While ring flanges of the form shown in (*b*) are often successfully used, in some cases, if there is too much clearance between the rings and the plunger, the leather is drawn in between ring B and the plunger at points $X\,X$ and gradually cut, and the tightness of the packing is destroyed. Figure 14B, (*c*) shows a modification of the arrangement

HYDRAULIC GLAND CUP PACKING.

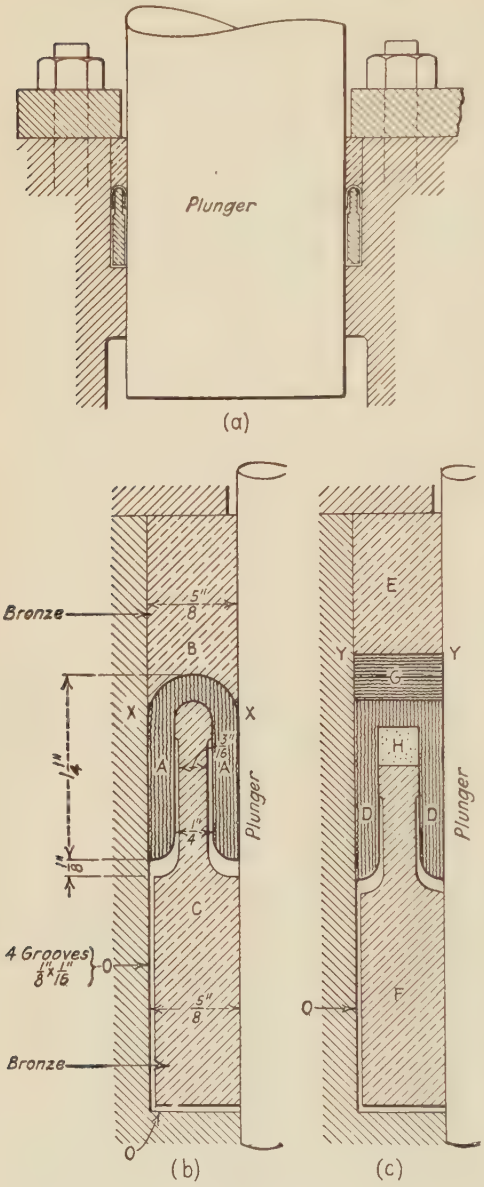


FIG. 14B.

designed to overcome these difficulties. The sharp edges of the flange of ring *B* at *X X* are replaced by square corners, *YY*, on the end of ring *E*. Two leather washer-rings, *G*, are inserted to form a cushion for cup *D*. One or more rings of braided flax, *H*, are inserted between the specially shaped flange of ring *F* and cup *D*. This detail is superior to that shown in Fig. (*b*). It is strongly advocated by E. F. Houghton & Co., a firm which makes a specialty of packings and similar supplies for hydraulic machinery.[1]

All workmanship should be accurate and all surfaces should be smoothly finished.

The clearances between the plunger and the rings should be kept accurate within rather narrow limits. If they are too small the friction becomes too great or the plunger may bind; if they are too large the leather cup will be damaged by extruding between the metal surfaces at the corners *X X* and *YY*. Table 14B gives the clearances that are recommended by E. F. Houghton & Co.

In order that the fluid within the cylinder may

[1] "Hydraulic Engineering," by the Houghton Research Staff, Philadelphia, 1926.

flow freely and act properly on the leather cup packing, grooves should be cut in the bottom and on the outside of the bronze ring C, as indicated in the figures at points O.

TABLE 14B

SUITABLE ALLOWANCES FOR SLIDING FIT BETWEEN PLUNGER OR CYLINDER AND BRONZE PACKING RINGS FOR PRESSURES UP TO 8000 LBS. PER SQUARE INCH

Plunger Diameter, Inches	Allowance, Inches	Plunger Diameter, Inches	Allowance, Inches
1	0.002	16	0.008
2	0.003	18	0.009
3	0.004	20	0.009
4	0.004	22	0.010
5	0.005	24	0.010
6	0.005	28	0.011
7	0.006	32	0.012
8	0.006	36	0.013
9	0.006	40	0.014
10	0.007	44	0.015
12	0.007	48	0.016
14	0.008		

For pressures of 2000 lbs. per square inch and under, 30 per cent may be added to these allowances.

For pressures between 2000 lbs. per square inch and 8000 lbs. per square inch, 10 per cent. may be added for each 2000 lbs. below 8000 lbs. per square inch.

Phosphor-bronze of Grade D, as specified in Appendix A, Arts. 803 to 808, is suitable for rings B and C in Fig. (b) or E and F in Fig. (c).

The packing cups and washers should be of the best quality of mineral-tanned leather, made for use in packings, and should be specially pressed to the form required to fit the rings with which they are to be used. Poor and ordinary grades of packing leathers should be avoided. For hard, continuous service, the best grade of leather will prove to be the most economical and will require the least maintenance.

7. Friction of hydraulic plunger packing.—A study of the results of numerous tests indicates that there is no simple relation between the loss due to the friction of hydraulic packings and the pressures within the cylinders, or between this loss and the diameters of the plungers. It appears that the percentage of loss in such combinations diminishes by some varying function of the increase in the unit pressures within the

cylinders and by a different varying function of the increase in the diameters of the plungers.

It is probable that the loss from the friction of well-made leather cup packings is less than that of equally well-made solid, square flax packings; but even in this case the tests are partially contradictory. The skill of the workman who packs a gland may influence the resulting frictional loss nearly as much as the design of the gland and the quality of the materials used in the packing.

Let F = the total frictional loss, in pounds;
p = the unit pressure within the cylinder, in pounds per square inch;
d = the diameter of the plunger, in inches, and
k = an experimental coefficient.

The frictional loss may then be approximately expressed by

$$F = kpd. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The coefficient k is not a constant, but varies quite widely with several circumstances, including the workmanship. The best that can be done is to give a few approximate values of k, which are shown in Table 14C.

TABLE 14C

APPROXIMATE VALUES OF k

Design and Condition of Packing	Approximate Values of k
Cup leather packing, soft and well made	0.04
Cup leather packing, ordinary	0.06 to 0.08
Cup leather packing, hard and stiffly tanned	0.12 to 0.16
Solid, square flax, hemp, or cotton packing, loose or braided, dipped in hot tallow, in contact with smooth plungers of ordinary dimensions; if the glands are not drawn down to tightly	0.075
Hard packing with unfavorable conditions	0.30

For large jacks, presses, and accumulators, for high pressures and with well-packed glands, the frictional loss may be taken as about $\frac{1}{2}$ of 1 per cent. of the total load on the plunger.

The results of several tests of the actual losses from the friction of packings, and the corresponding values of k and the percentages of frictional loss are given in Table 14D.

TABLE 14D

FRICTION OF HYDRAULIC PACKING

Type of Machine	Kind of Packing	Pressure per Square Inch, p	Diameter of Plunger, Inches, d	Load on Plunger, Pounds, L	Friction Loss, Pounds, F	$\frac{F}{pd}$ k	Friction Loss in Per Cent. $\frac{100F}{L}$	Authority
Test plunger	New leather cup	188	4	2,362	108.6	0.145	4.6	John Hick
	Worn leather cup	188	4	2,362	50.3	0.067	2.13	
	New leather cup	446	4	5,604	112.1	0.063	2.0	
	Worn leather cup	446	4	5,604	70.0	0.039	1.25	
	New leather cup	673	4	8,453	131.1	0.049	1.55	
	Worn leather cup	673	4	8,456	80.3	0.030	0.95	
	New leather cup	5865	4	73,700	788.6	0.034	1.07	
	Worn leather cup	5865	4	73,700	464.4	0.020	0.63	
	New leather cup	1882	8	94,580	435.0	0.029	0.46	
	Worn leather cup	1882	8	94,580	397.3	0.026	0.42	
	New leather cup	6375	8	320,400	1,602	0.031	0.50	
	Worn leather cup	6375	8	320,400	1,057.4	0.021	0.33	
	Leather cup	106.83	6	3,022	33.5	0.057	1.11	Henry Flad
	Leather cup	201.79	6	5,703	54	0.045	0.95	
	Leather cup	273	6	7,719	71	0.044	0.92	
	Leather cup	106.83	9	6,795	75	0.078	1.10	
	Leather cup	201.79	9	12,835	130	0.072	1.01	
	Leather cup	273	9	17,368	164.5	0.067	0.95	
Hydraulic jack	Leather cup	500	$3\frac{1}{4}$	4,148	319	0.196	7.69	Robert A. Cummings
	Leather cup	1000	$3\frac{1}{4}$	8,296	427	0.132	5.15	
	Leather cup	2000	$3\frac{1}{4}$	16,592	672	0.103	4.05	
	Leather cup	3000	$3\frac{1}{4}$	24,888	896	0.092	3.60	
	Leather cup	3200	$3\frac{1}{4}$	26,546	1,029	0.099	3.88	
Accumulator	Square flax	1600	$4\frac{5}{16}$	23,370	219	0.032	0.937	Edwin Graves
		320	11	30,410	475	0.135	1.562	
Pencoyd intensifier	Square hemp	475 to 1450	8 and $14\frac{1}{4}$	72,885 to 75,744	2,859	2 packings 0.155	3.775	Walter Ferris
Athens testing machine	Square flax	600	$51\frac{3}{4}$	1,223,400	4,000	0.129	0.327	Charles Macdonald
Phoenix testing machine	Square flax	800	64.1	2,400,000	12,000	0.234	0.50	James E. Howard

8. Packing for hydraulic pipe-joints.—Figure 14C gives a longitudinal section and a sectional elevation of a type of packing that is suitable for pressure pipe lines for pressures up to 4000 lbs. per square inch.

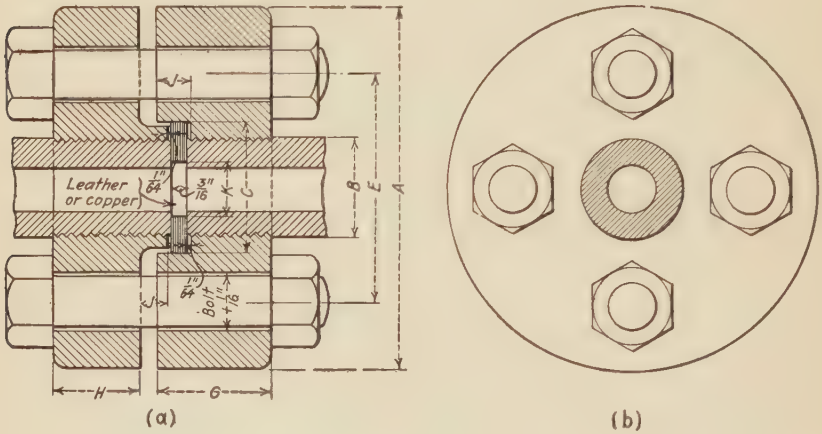


FIG. 14C.

When leather packing washers are used, they should be $\frac{3}{16}$ in. thick. If of copper, they should not be more than $\frac{1}{32}$ in. thick. The dimension marked C should be $\frac{1}{32}$ in. smaller on the male flange.

Dimensions of flanges and packings for pipes from $\frac{3}{4}$ to 8 ins. in diameter are given in Table 14E. When these dimensions are used, flanges subject to pressures up to 1500 lbs. per square inch may be of cast iron. For pressures from 1500 to 4000 lbs. per square inch, steel castings are necessary.

9. Packings for high-pressure steam and air pipe lines.—Figure 14D is a sketch of a type of flange and gasket that is suitable for high-pressure steam and air pipe lines. The workmanship and installation are simple. The ends of the pipes are threaded with cylindrical threads and the ends are rounded where they bear against the gaskets. The flanges are connected by turned bolts that fit the holes tightly.

Gasket A is of copper $\frac{1}{16}$ in. thick. The bolts are drawn up until the rounded ends of the pipes are pressed deeply enough into the copper to make a tight joint.

10. Cylinder-head packing.—Figure 14E is a sketch of a type of cylinder-head packing that is satisfactory under high pressures. It has been successfully used in the hydraulic cylinders of large testing machines.

TABLE 14E

DIMENSIONS OF FLANGES FOR HIGH PRESSURES, INCHES

Size of Pipe	Bolts		A	B	C	E	Iron		Steel		J	K	
	Number	Size					G	H	G	H		Extra Strong	Double Extra Strong
$\frac{3}{4}$	4	$\frac{1}{2}$	$3\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{5}{16}$	$2\frac{7}{16}$	$1\frac{7}{8}$	$1\frac{1}{2}$	$1\frac{3}{8}$	1	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{2}$
1	4	$\frac{3}{4}$	5	$1\frac{5}{16}$	$1\frac{5}{8}$	$3\frac{1}{2}$	2	$1\frac{5}{8}$	$1\frac{1}{2}$	$1\frac{1}{8}$	$\frac{3}{8}$	1	$\frac{11}{16}$
$1\frac{1}{4}$	4	$\frac{3}{4}$	$5\frac{1}{2}$	$1\frac{21}{32}$	2	4	$2\frac{1}{4}$	$1\frac{7}{8}$	$1\frac{1}{2}$	$1\frac{1}{8}$	$\frac{3}{8}$	$1\frac{5}{16}$	1
$1\frac{1}{2}$	4	$\frac{7}{8}$	$6\frac{1}{2}$	$1\frac{29}{32}$	$2\frac{1}{4}$	$4\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{16}$
2	4	1	$7\frac{1}{2}$	$2\frac{3}{8}$	$2\frac{3}{4}$	$5\frac{1}{2}$	$2\frac{3}{4}$	$2\frac{3}{8}$	$1\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$	2	$1\frac{5}{8}$
$2\frac{1}{2}$	4	1	8	$2\frac{7}{8}$	$3\frac{1}{4}$	6	$2\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{8}$	$2\frac{3}{8}$	$1\frac{7}{8}$
3	6	1	9	$3\frac{1}{2}$	$4\frac{1}{8}$	7	$2\frac{15}{16}$	$2\frac{9}{16}$	$1\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{8}$	$2\frac{15}{16}$	$2\frac{3}{8}$
$3\frac{1}{2}$	8	$1\frac{1}{8}$	$10\frac{1}{2}$	4	$4\frac{5}{8}$	$8\frac{1}{4}$	3	$2\frac{5}{8}$	$1\frac{7}{8}$	$1\frac{1}{2}$	$\frac{3}{8}$	$3\frac{3}{8}$	$2\frac{13}{16}$
4	8	$1\frac{1}{8}$	$10\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{8}$	$8\frac{1}{4}$	$3\frac{1}{16}$	$2\frac{11}{16}$	$1\frac{7}{8}$	$1\frac{1}{2}$	$\frac{3}{8}$	$3\frac{7}{8}$	$3\frac{1}{4}$
$4\frac{1}{2}$	8	$1\frac{1}{4}$	$12\frac{1}{2}$	5	$5\frac{3}{4}$	10	$3\frac{1}{8}$	$2\frac{3}{4}$	2	$1\frac{5}{8}$	$\frac{3}{8}$	$4\frac{1}{4}$	$3\frac{3}{8}$
5	8	$1\frac{1}{4}$	$12\frac{1}{2}$	$5\frac{9}{16}$	$6\frac{1}{4}$	10	$3\frac{1}{4}$	$2\frac{7}{8}$	2	$1\frac{5}{8}$	$\frac{3}{8}$	$4\frac{7}{8}$	$4\frac{3}{16}$
6	8	$1\frac{3}{8}$	$14\frac{1}{2}$	$6\frac{5}{8}$	$7\frac{3}{8}$	$11\frac{3}{4}$	$3\frac{3}{8}$	3	$2\frac{1}{4}$	$1\frac{7}{8}$	$\frac{3}{8}$	$5\frac{3}{4}$	5
8	12	$1\frac{3}{8}$	$17\frac{1}{2}$	$8\frac{5}{8}$	$9\frac{3}{8}$	$14\frac{3}{4}$	$3\frac{1}{2}$	$3\frac{1}{8}$	$2\frac{1}{4}$	$1\frac{7}{8}$	$\frac{3}{8}$	$7\frac{1}{2}$	7

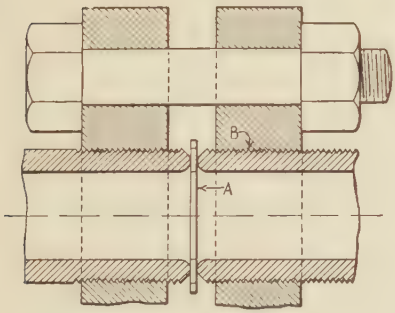


FIG. 14D.

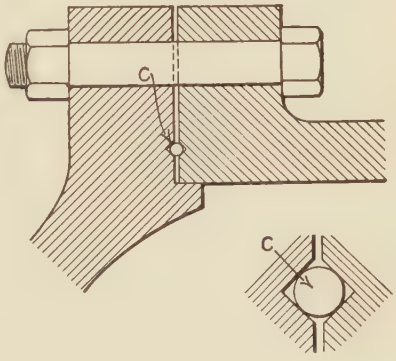


FIG. 14E.

V-shaped circular grooves are cut opposite each other in the cylinder and head. Gasket *C* is a copper wire, bent to a circle, electrically welded, and annealed. The wire is from $\frac{1}{4}$ to $\frac{1}{2}$ in. in diameter, and the V-grooves are cut so that the bolts are drawn up tight, compressing the wire to a tight joint. There should still be some clearance between the end of the cylinder and the head after the bolts are tightened.

11. Loss of hydraulic pressure in straight pipes.—A formula which is widely used in determining the frictional loss of water flowing through pipes was developed by Gardner S. Williams and Allen Hazen.[2] This formula gives results in substantial accord with the latest and best tests. In its usual form for use in water works engineering it is

$$v = cr^{0.63}s^{0.54}0.001^{-0.04}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In this expression,

v = the velocity of the flow through the pipe, in feet per second:
c = a coefficient varying with the roughness of the interior surface of the pipe;
r = the hydraulic radius of the pipe, in feet;
s = the hydraulic slope, and
$0.001^{-0.04}$ = a reduction factor, which is a constant.

This expression is not in a convenient form for use in the design of hydraulic machinery. In addition to the notations enumerated, let d = the inside diameter of the pipe, in inches, and

p_s = the loss of pressure in a straight pipe 1 ft. long, in pounds per square inch.

Substituting these values, evaluating the constant $0.001^{-0.04}$, passing from the loss of head expressed as the hydraulic slope to the loss of pressure expressed in pounds per square inch, p_s, and solving for p_s,

$$p_s = \frac{24}{c^{1.85}} \times \frac{v^{1.85}}{d^{1.17}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Table 14F gives values of the factor $\frac{24}{c^{1.85}}$ which will be found con-

TABLE 14F

VALUES OF $\frac{24}{c^{1.85}}$

Condition of the Inside Surface of the Pipe	c	$\frac{24}{c^{1.85}}$
Very smooth and straight	140	0.0026
Brass, tin, and drawn copper	130	0.0029
Smooth, new steel or iron	120	0.0034
Ordinary steel or iron	100	0.0048
Old steel or iron	80	0.0072
Very rough	60	0.0123

[2] "Hydraulic Tables," by Gardner S. Williams, M. A. S. C. E., and Allen Hazen, M. A. S. C. E., John Wiley & Sons, Inc., New York.

venient when designing hydraulic machinery. The value of this factor when $c = 100$ is probably the most generally useful.

12. Loss of hydraulic pressure in 90° bends.—Many tests have been made of the loss of hydraulic pressure caused by bends in pipe. Some of the tests and the rules and formulae based up them are quite contradictory. An investigation of hydraulic curve resistance made by Leland Rella Balch [3] affords a basis for formulae which appear to be quite generally applicable to designing. As is usual in such cases, the formulae were developed with particular reference to the needs of the engineers of water works, and are not in convenient form for use in the design of hydraulic machinery. They have been transformed so that they take the forms given below.

Let v = the velocity of flow in a pipe around a 90° bend, in feet per second;
R = the radius of the bend at the center line of the pipe;
d = the inside diameter of the pipe, and
p_b = the loss in fluid pressure for one 90° bend, in pounds per square inch, then:

For ratios of $\frac{R}{d}$ from 0 to 5,

$$p_b = 0.0042\left(\frac{R}{d}\right)^{-0.43} v^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

For ratios of $\frac{R}{d}$ from 5 to 20,

$$p_b = 0.0005\left(\frac{R}{d}\right)^{0.924} v^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

These expressions give the total loss from resistance due to curvature and also include the friction within the pipe for flow around one bend of 90°.

The values of the coefficients of v^2 for bends with various values of the ratio $\frac{R}{d}$ are given in Table 14G.

13. Loss of hydraulic pressure in pipe fittings.—Various contradictory rules for the loss of head caused by the flow of fluids through pipe fittings have been given by different investigators. One of the most convenient and reliable methods was proposed by Dean E. Foster.[4]

[3] "Investigation of Hydraulic Curve Resistance Experiments with Three Inch Pipe," by Leland Rella Balch, C. E. *Bulletin of the University of Wisconsin*, No. 578, issued during 1913.

[4] "Resistance of Fittings to Flow through Pipe Lines," by Dean E. Foster, M. A. S. M. E., *Trans. A. S. M. E.* Vol. 42, p. 647, 1920.

TABLE 14G

VALUES OF THE COEFFICIENTS OF v^2

$\frac{R}{d}$	$0.0042\left(\frac{R}{d}\right)^{-0.43}$	$0.0005\left(\frac{R}{d}\right)^{0.924}$
1	0.0042	
2	0.0031	
3	0.0026	
4	0.0023	
5	0.0021	0.0022
6	.....	0.0026
7	.....	0.0030
8	.....	0.0035
9	.....	0.0038
10	.....	0.0042

Foster quotes a formula by Konrad Meier, M. A. S. M. E., which is

$$H_f = 1.38r\frac{v^{1.86}}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In this expression,

H_f = pressure loss per fitting, in feet,
r = a factor of resistance the value of which depends upon the shape of the fitting, and
v = the velocity of the fluid, in feet per second.

Equation (13) is not in convenient form for use in determining loss of pressure per square inch, which is most frequently needed in design.

In addition to the notations given,

let p_f = the loss of pressure from flow through a fitting, in pounds per square inch.

Using the same exponent of v that was adopted in the preceding articles, and transforming Eq. (13), we have

$$p_f = 0.0093rv^{1.85}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For convenience when making calculations, the values of r and $0.0093r$ are given in Table 14H.

14. Loss of hydraulic pressure at entrance orifices.—A further source of loss of hydraulic pressure in a system of pipes and fittings is the resistance to flow caused by the shape of the entrance orifice. The ideal shape of an entrance orifice is the *vena contracta*, the approximate pro-

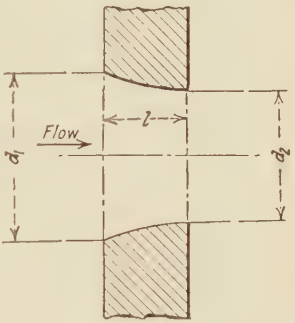
TABLE 14H

VALUES OF r and $0.0093r$ FOR VARIOUS SCREW FITTINGS

Type of Fittings	r	$0.0093r$
Gate valve	0.25	0.00233
Long-sweep elbow or on run of standard tee	0.33	0.00307
Medium-sweep elbow or on run of tee reduced in size by $\frac{1}{4}$ in.	0.42	0.00391
Standard elbow or on run of tee reduced in size by $\frac{1}{2}$ in	0.67	0.00623
Angle valve	0.90	0.00837
Close return bend	1.00	0.00930
Tee through side outlet	1.33	0.01237
Globe valve	2.00	0.01860

portions of which are shown in Fig. 14F. If the orifice has these proportions there is practically no loss of hydraulic pressure. A plain cylindrical orifice with square entrance corner offers a resistance about equivalent to that of a straight pipe having a length of 20 times the internal diameter of the pipe.

While it is not always practicable to make the entrance orifice of the shape and proportions of the *vena contracta*, the entrance should always be well rounded. This will greatly reduce the loss in pressure below that which would be caused by a square entrance corner.



$d_1 = 1$
$d_2 = 0.7854\,d_1$
$l = 0.498\,d_1$

FIG. 14F.

15. Limits of the applicability of Arts. 11 to 14.—The formulae given in Arts. 11 to 14 were all developed for water flowing under pressure. They apply almost equally well to the flow of any other non-viscous fluid.

The formulae given should not be applied if the internal diameter of the pipe is less than $\frac{1}{8}$ in.

A discussion of the flow of water and non-viscous fluids through pipes of very small internal diameter, and of the laws of the flow of viscous fluids, are not included in the scope of this treatise.

APPENDICES

APPENDIX A

SPECIFICATIONS FOR MOVABLE BRIDGES

BY

O. E. HOVEY

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1925

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1. GENERAL DESIGN

101. Definition of terms.—The term *Owner* refers to the Company, Public Authority, or other purchaser, party to the contract. The term *Engineer* refers to the Engineer of the Owner, or his subordinate in authority. The term *Inspector* refers to the inspector, or inspectors, representing the Owner. The term *Contractor* refers to the manufacturer or fabricator, party to the contract. The term *Erector* refers to the erecting contractor, party to the contract.

102. Specifications.—The superstructure shall be designed in accordance with the Specifications of the American Society of Civil Engineers,[1] except as supplemented herein.

103. Types.—Movable bridges shall preferably be of one of the following types:

A. Swing Bridges.

B. Single-leaf Bascule Bridges.

C. Vertical-lift Bridges.

104. Swing bridges.—The following types of girders and trusses are recommended for swing bridges:

(1) Plate girders for spans up to 150 ft. between centers of end supports.

(2) Riveted trusses from 150 to 300 ft. between centers of end supports.

(3) Pin-connected trusses may be used for spans longer than 300 ft.

105. Center-bearing.—Movable bridges shall preferably be swing bridges of the disc, center-bearing type, when local conditions admit of this construction. Center bearings shall be designed so that the entire dead load of the bridge is carried on a center pivot when the bridge is swinging. When the bridge is closed the trusses, or center girders, shall rest on fixed supports so that they are in contact under dead load and thus directly carry the center reactions due to live loads. The center pivot shall be made adjustable in height, preferably by shims between the center girders and the upper pivot casting.

106. Rim-bearing.—The load of rim-bearing bridges shall be distributed equally among the points of bearing upon the drum. The bearing points shall be equally spaced around the circumference of the drum. Rigid struts shall connect the drum to a center pivot, firmly anchored to the pier. A strut shall be attached to the drum at each bearing point and at intermediate points when required. No fewer than 8 struts shall be used in any case. The drum shall be designed so that the load is properly distributed to the rollers. For calculating stresses in the drum, the loads shall be assumed to be distributed equally to all rollers. The span length shall be taken as the developed length of the drum between adjacent points receiving load from the superstructure. This part of the drum shall be considered as a girder fixed at both ends. The drum shall be designed in accordance with the requirements for plate girders. The roller-track shall be deep and strong enough to distribute the load from the rollers to the masonry.

107. Combined-bearing.—If a combined rim-and-center-bearing turntable is used in a swing bridge a definite portion of the load shall be carried to the center, preferably by radial girders rigidly attached to the drum and to the center.

108. Shear over center.—If rim-bearing swing bridges have a center panel, this panel shall not be designed to carry shear past the center. The web members of such a panel shall, however, be strong enough to make the bridge secure against longitudinal wind pressure and vibration when it is open.

109. Bascule and vertical-lift bridges.—The general principles governing the design of swing bridges are also applicable to bascule and vertical-lift bridges, as well as to other types of movable bridges.

[1] *Railway Bridges:* Paper No. 1518, *Trans. Am. Soc. C. E.*, Vol. LXXXVI, p. 471, 1923.

Highway Bridges: Paper No. 1550, *Trans. Am. Soc. C. E.*, Vol. LXXXVII, p. 1273, 1924.

These specifications also are issued in pamphlet form, and may be obtained from the American Society of Civil Engineers, 33 West 39th St., New York, N. Y.

When two bascule leaves meet at the center of the channel, effective shear locks shall be provided so that the leaves shall deflect together.

110. Counterweights.—In bascule and vertical-lift bridges the counterweights shall be sufficient practically to balance the bridge in any position, except that there shall be a positive reaction at all supports, for all conditions of loading, when the bridge is closed.

111. Auxiliary power.—Bridges equipped with mechanical power shall be provided with auxiliary power. In many cases this may be manually operated. In large and heavy structures it may become necessary to provide auxiliary mechanical power.

112. Signals.—Bridges carrying railroads shall have an automatic signal at each shore end, and these signals shall be arranged so that the bridge cannot be opened without setting each signal at the danger position; and so that the signals cannot be set to indicate a clear track until the bridge is closed and the rails are in their proper position. Signals and interlocking shall not be supplied by the contractor for the superstructure.

113. Navigation signal lights.—All movable bridges shall be provided with navigation signal lights in accordance with the Regulations for Lighting Bridges of the Department of Commerce of the United States; and provision shall be made for access to such lights.

Navigation signals shall not be supplied by the contractor for the superstructure, unless definitely provided for in the contract.

2. DETAILS OF DESIGN

201. Drum girders.—Drum girders of rim-bearing bridges shall be provided with effective stiffeners, with fillers, on both sides of the web at all points of concentrated loads. Intermediate stiffeners, with fillers, shall be provided at distances apart not exceeding 2 ft. Whenever the drum is more than 5 ft. deep, alternate intermediate stiffeners may extend only one-half of the depth of the drum from the lower flange, unless the web requires stiffening, in which case all stiffeners shall extend the full depth of the drum.

The thickness of the outstanding legs of stiffener angles shall not be less than one-twelfth of their width.

All stiffeners shall be fitted tight against the lower flange angles. If concentrated loads are delivered to the top of the drum, the stiffeners at these points shall be filled tight against both flanges.

202. Center pivots.—Center pivots shall be designed so that the discs may be taken out and replaced, when the bridge is closed, without lifting the center of the bridge more than one inch.

The top and bottom discs shall have circular, central lugs not over one-half of an inch high, fitting in corresponding recesses in the upper pivot and the lower pedestal, to insure the centering of the discs. The discs shall not be keyed or doweled to the pivots or pedestals.

An oil ring attached to the lower pedestal shall enclose the discs and extend up around the lower end of the upper pivot, so that the discs will run in a bath of oil.

The oil ring shall be made in sections, which shall be securely bolted together and to the pedestal, with oil-tight joints.

203. Center discs.—The center pivot shall, preferably, be provided with two

discs. The lower disc shall be of steel, with its top surface a concave segment of a sphere. The upper disc shall be of phosphor-bronze, with its bottom surface a convex segment of a sphere, the radius of which shall be a little less than the radius of the concave top surface of the lower disc.

204. Rack and track.—The rack and track shall be made in sections, preferably not less than 6 ft. long. The track shall be deep enough to insure good distribution of the balance wheel, or roller, loads to the masonry. If a cast lower track is used and the loads are light, as in center-bearing bridges, the rack and track segments shall preferably be cast in one piece. In rim-bearing bridges the rack shall be cast separately from the track, so that the parts may be removed easily for repairs or renewals. In rim-bearing bridges the track shall be deep enough to permit the replacement of rollers without removing the rack. The lower track shall be anchored to the masonry by bolts not less than $1\frac{1}{2}$ ins. in diameter and extending at least 12 ins. into the masonry and set in Portland cement mortar or grout. The lower track of hand-operated center-bearing bridges shall have an ample number of anchor bolts so that the mortar or grout in which they are set shall not be crushed by the tractive force developed when turning the bridge. If center-bearing bridges are operated by mechanical power the lower track shall be anchored down by bolts, and the tractive force developed by the machinery shall be taken by lugs extending from the bottom of the track downward into the masonry and set in cement mortar, grout, or concrete.

In small bridges, steel rails may be used for the track.

205. Pinions.—If the tractive force on the rack is too large for one pinion two or more pinions shall be used. If two pinions are used they shall be placed at opposite ends of a diameter of the rack. If four pinions are used they may be placed in pairs at opposite ends of a diameter of the rack.

206. Equalizers.—If more than one pinion is driven by one motor or engine, effective equalizers shall be provided so that each pinion shall deliver its part of the tractive force to the rack.

207. Brackets and bearings.—The brackets that support the main pinions, which gear with the rack, shall be made of ample strength for the maximum pinion load. They shall preferably be built up from plates and shapes and in a single piece supporting both bearings of the pinion shaft. They shall be thoroughly braced and rigidly attached to the drum or the superstructure.

The pinion-shaft bearings shall preferably be cast in one piece. They shall be provided with shims, so that their position can be adjusted in order that the pinions may mesh accurately with the rack. The bearings shall be provided with caps, so that the pinion shaft can be taken out without removing the bearings or the bracket.

208. Balance wheels and bearings.—In center-bearing swing bridges not less than four balance wheels, running on a circular track, shall be provided to balance the bridge and carry the wind pressure to the track when the bridge is turning. The balance-wheel bearings shall be adjustable for height, preferably by shims between the structure and the tops of the bearings.

209. End-lifting apparatus.—All swing bridges shall be provided with an effective end-lifting and latching apparatus. End-lifting wedges and automatic bridge latches are preferred.

The mechanism for driving the wedges shall be arranged so that the amount of end lift can easily be adjusted, and so that the mechanism cannot lock in case of an over-drive.

The latches shall be arranged so that they will lock the bridge automatically

when in closed position; but only when the speed is so slow that the shock will do no damage.

The end-rail lifting, or rail-lock, apparatus shall preferably be actuated by the same mechanism that gives motion to the end-lifting and latching apparatus.

210. Center wedges.—All center-bearing swing bridges shall be provided with adjustable center wedges. The center and end wedges shall preferably be operated by the same mechanism. The center wedges shall be adjusted so that when the end wedges are fully driven the center wedges shall just take a bearing, but shall not carry any part of the dead load.

211. Centering and locking devices.—Bascule and vertical-lift bridges shall be provided with self-centering and seating devices at the free ends. There shall be a locking device for each truss, to force down and lock the free end.

212. Air buffers.—Power-operated skew swing bridges, bascule, and vertical-lift bridges shall be provided with effective air buffers. An air buffer shall be used on the free end of single-track bascule bridges and one at each end of single-track vertical-lift bridges. For multiple-track bridges, twice the number shall be used.

213. Segmental and track girders.—The flanges of the segmental and track girders of rolling-lift bascule bridges shall be symmetrical about the central plane of the webs. The central planes of the webs of the segmental girders shall coincide with the corresponding central planes of the webs of the track girders. That part of the outstanding leg of a flange angle which is beyond the outside face of the vertical leg shall not be considered as bearing area. The width of contact between the segmental girder and the back of the tread plates shall be equal to the corresponding width of contact on the track girder.

214. Treads.—The tread plates shall be of ample thickness properly to distribute the rolling load, concentrated at the line of rolling contact, to the webs of the track and segmental girders.

The moment of inertia of the tread on the supporting track girder shall not be less than

$$I_t = \frac{I_g b}{8l - 5b},$$

in which I_t = the moment of inertia of the tread, in inches^4;

I_g = the moment of inertia of the girder supporting the tread, in inches^4;

b = the length, in inches, of the portion of the tread required to transmit the concentrated rolling load to the web of the girder by solid contact with it, assisted by the rivets attaching the vertical parts of the girder flange to the web, and

l = the length of the girder between its supports, in inches.

The track on the rolling segment shall have the same moment of inertia as that on the supporting girder.

If the rollers have a full circumference they shall be rolled, forged, or cast as a single piece.

215. Counterweights.—Counterweights shall preferably be of concrete supported in a steel box or by a steel frame. They shall be made adjustable, so that variation in the weight of the moving span may easily be provided for. Usually this shall be done by adding or taking off concrete or cast-iron blocks in properly located pockets. The pockets shall be provided with drain holes not less than 6 ins. in diameter. The pockets shall be covered. The cover, its fastenings, and frame

shall be of metal. If the counterweights of bascule and vertical-lift bridges are located above the tracks or the floor of the bridge approaches, ample clearance between the counterweights and the tracks shall be provided when the bridge is in its fully open position. In calculating this clearance, counterweight ropes shall be assumed to stretch 2 per cent. of their calculated length.

216. Concrete.—Concrete for counterweights shall preferably be mixed in the proportion of 1 part of Portland cement, 3 parts of sand, and 6 parts of broken stone, or gravel, by volume. The unit of volume shall be a barrel of Portland cement containing 4 cubic feet and weighing 376 lbs., net. Steel punchings, or scrap metal, may be used when necessary to increase the weight. If clean steel punchings are used, and placed in layers and grouted with a mortar composed of 1 part of Portland cement and 2 parts of sand, the maximum available total weight per cubic foot shall not be taken as more than 315 lbs. Such a mixture shall not be used as a counterweight which revolves about a horizontal axis, unless retained in place by a surrounding steel box, or by walls of reinforced concrete.

3. OPERATING MACHINERY

DETAILS OF DESIGN

301. Location.—The operating machinery shall be located so as to be easily erected, adjusted, and taken apart, and shall be accessible for lubrication, inspection and cleaning. Crowding of parts shall be avoided. On bascule bridges the machinery shall preferably be on the stationary part.

302. Design.—The machinery shall be designed and constructed in a substantial manner. All complicated and flimsy contrivances shall be avoided. Particular attention shall be given to the design of the structural parts supporting the machinery. These parts must be made so that all stresses are provided for; and vibration avoided, so far as feasible. The attachments shall be made so that after all parts are set up, aligned, and adjusted, all machinery shall be firmly held in its correct position.

303. Bearings.—Journal bearings shall be proportioned to resist the stresses to which they are subjected without exceeding the permissible fiber and bearing stresses; and also so as to prevent heating and seizing.

304. The bearings of shafts shall be placed as near the points of loading as practicable.

In cases of line shafts running at ordinary speeds, cast-iron bearings without linings may be used. If the stresses are such that it is necessary to use bearings of cast steel they shall be lined with babbitt. If the bearing pressure is too great for babbitt, bronze linings shall be used.

305. Linings.—Bearing linings shall be provided with an effective device to prevent them from rotating in the bearings. For linings of heavy bearings, like main trunnions, the force tending to produce rotation shall be taken as one-eighth of the load on the bearings and as acting at the outer circumference of the lining. The inside longitudinal edges of split linings, as in a bearing and its cap, shall be rounded except for a distance of $\frac{3}{8}$ of an inch from each end.

306. Adjustment.—Provision shall be made for the adjustment in height of all important bearings, and also for the adjustment of their caps, by shims or some other effective means.

Provision shall be made for the field alignment of all shafts that cannot be fitted up in the shop. Usually the best method is to punch small holes in the structural supports at the shop, and to enlarge them by reaming after the machinery has been aligned in the field. In some cases the holes should be omitted and drilled in the structural supports after aligning and clamping the machine parts.

307. Bolts and nuts.—All bolts and nuts shall preferably have U. S. Standard threads. All nuts and exposed bolt heads shall be hexagonal. Each nut that bears on an inclined surface shall be provided with a suitable beveled washer. Bolt heads that are countersunk in a casting shall preferably be square. Nuts subject to vibration and frequent changes of load shall be locked, or check nuts may be used for this purpose.

308. Shafts.—Shafts shall be made from cold-rolled, cold-drawn, hot-rolled, or forged steel, whichever is the most economical. Line shafts shall be connected by flexible couplings. Each alternate length of line shaft shall preferably have not more than two bearings, and the flexible couplings shall be placed outside of the bearings and close to them; and the intermediate length of shaft shall be supported by the flexible couplings. In case the pieces of line shaft become too long, an intermediate support between the bearings may be used to reduce the deflection of the shaft.

309. For shafts supporting their own weight only, the unsupported length of the shaft shall not exceed $d = 100\sqrt[3]{d^2}$,

in which L = the length of the shaft between bearings;
d = the diameter of the shaft; both in inches.

If a bridge is operated by man-power no shaft transmitting motion shall be less than 2 inches in diameter. This rule shall also apply to lever and link systems for operating bridge and rail locks and similar parts. It shall not apply to indicators, or limit switches.

310. Couplings.—Shafts liable to be thrown out of line by deflections of the structure shall be connected by jaw couplings, designed to center the shafts connected. Each half coupling shall be keyed to the end of the shaft. The jaws shall be accurately fitted together, but with enough clearance to permit a slight angular motion without bending the shafts. Other couplings may be jaw, muff, or flanged couplings. If flanged couplings are used, the bolt heads and nuts shall be seated in recesses or protected by flanges.

All couplings shall be strong enough to develop the full strength of the shafts which they connect, and shall be keyed to the shafts.

311. Longitudinal thrust.—If a shaft, trunnion, or axle receives a longitudinal force there shall be a thrust bearing to prevent longitudinal movement. Wheels and similar parts shall be securely fastened to prevent longitudinal movement, by set-screws through the hub or by clamps around the shaft. The hubs of bevel gears shall bear against the shaft bearings; and thrust collars, with oil grooves for good lubrication, shall be provided.

312. Collars.—Collars shall be used where necessary to hold shafts from moving longitudinally. Each collar shall have at least two set-screws, placed at an angle of 120 degrees from each other.

313. Keys.—All pieces that transmit torsion to shafts, or axles, shall be secured by standard keys, proportioned for the torsional stress, and seated in close-fitting keyways in both the shafts and hubs. If two keys are used they shall be placed at 120 degrees from each other. All trunnions passing through structural steel parts

shall be fastened to the structural steel, to make sure that they turn in the proper bearings.

314. Set-screws.—Set-screws shall not be used for transmitting torsion to shafts or axles. They may be used for holding collars, keys and light parts in place.

315. Steps.—Vertical shafts shall preferably turn on flat discs or collars. The discs or collars shall be bronze or hardened steel except that for unimportant or slow-moving shafts they may be cast iron, or may be omitted when unnecessary.

316. Gears.—Gear wheels shall be designed on the assumption that one tooth transmits the whole load. The minimum face of any gear shall be one and one-half times the circular pitch. The face of bevel gears shall not exceed three times the circular pitch, or one-third of the slant height of the pitch cone. No pinion shall have less than 12 teeth, and whenever feasible at least 15 teeth shall be used.

No pinion with 20-degree involute stub teeth shall have less than 14 teeth.

All gears preferably shall have involute teeth shaped to an accepted interchangeable standard.

317. Hubs of gears.—If practicable, the length of gear-wheel hubs shall be $1\frac{1}{4}$ times the face of the gear or $1\frac{1}{4}$ times the diameter of the shaft, whichever is the greater.

For gears with $14\frac{1}{2}$ or 20-degree involute teeth, with standard addendum, the minimum diameter of the hubs shall be $0.085D + 3.5P$; and for gears with 20-degree stub teeth it shall be $0.09D + 3.7P$,

in which D = the pitch diameter, and
P = the circular pitch of the gear, both in inches.

318. Worm gearing.—Worm gearing shall be designed with the worm and wheel of metals of different hardness or crystalline structure. If the worm is of rolled structural or cast steel, the worm wheel may be of cast iron or semi-steel, provided that the strength of the teeth is made as specified in Art. 712 for cast iron. If stronger teeth are required, also for worms running at high speeds, the worm-wheel rim shall be of phosphor-bronze of Grade D, as specified in Arts. 803 to 808; and the strength of the teeth shall be made as specified in Art. 712 for phosphor-bronze. If worms and wheels are used to transmit motion to the end and center wedge rocking shafts, and in similar cases where the load should not cause reverse motion, the angle of the worm threads shall be about 6 degrees with a plane normal to the axis of the worm; and the worm wheel shall not have less than 20 teeth.

If worm gearing is used to transmit power, the angle of the worm threads shall not be less than 20 degrees with a plane normal to the axis of the worm; and the worm wheel shall not have less than 24 teeth. Hindley worms and wheels shall not be used, as their adjustment is too delicate for use in movable-bridge machinery. The end thrust of the worm shall be taken by phosphor-bronze collars, discs, or some other effective device.

The worms and wheels shall be enclosed in a cast-iron casing and the worm shall be below the wheel and run in a bath of oil. Solid greases shall not be used for lubricating worms and wheels. Castor oil or a heavy mineral valve oil should be used.

319. Screws.—All screws that transmit motion shall have either Acme, square, or buttress threads. No steel screw shall run in an unlined steel nut. If the design of the nut is such that cast iron does not provide the necessary strength, rolled, forged, or cast steel nuts may be used; but they shall be lined with bronze bushings, securely fastened to the body of the nuts, with the threads cut in the bushing.

320. Lubrication.—Provision shall be made for good lubrication of all journals, pivots, steps, and sliding surfaces. Lubricating devices shall be easily accessible. If not accessible, pipes may be run to some accessible point. Screw compression grease cups shall be provided for all bearings of vertical shafts. Oil grooves shall be provided in the bearings wherever necessary for the proper distribution of the lubricant. The edges of all oil and grease grooves shall be carefully rounded to a radius not less than one-half of the width of the groove; except that when the grooves are over $\frac{3}{8}$ in. wide the radius may be $\frac{3}{16}$ in.

321. Oil grooves in trunnions.—The grooves for large trunnion bearings shall be cut in the trunnions. They shall be straight and parallel with the axis of the trunnions. They shall be located, when possible, so that the entire bearing surface of the lining will be swept by the lubricant in each movement of opening or closing the bridge. The size shall be such that a $\frac{5}{16}$-in. rod may lie wholly within the groove. The grooves shall be accessible for cleaning with a rod. The edges and bottom corners of the grooves shall be well rounded.

322. Covers.—Dust covers shall be provided wherever necessary to protect lubricated surfaces.

323. Drainage holes.—At places where water is likely to collect there shall be drainage holes not less than 1 in. in diameter.

4. SHEAVES, WIRE ROPES, AND ATTACHMENTS

401. Diameters of sheaves.—The diameters of sheaves for various forms of wire ropes shall not be less than those given in the following table.

Construction of Ropes	Counterweight Ropes. Values of D	Operating Ropes. Values of D
6 strands of 19 wires each and 1 hemp center	$60 \times d$	$48 \times d$
6 strands of 37 wires each and 1 hemp center	$45 \times d$	$35 \times d$
6 strands of 61 wires each and 1 hemp center	$35 \times d$	$27 \times d$

In the table, d = the diameter of the rope and D = the pitch diameter of the sheaves, from center to center of the rope.

402. Sheaves and drums.—Sheaves and drums shall be designed for all stresses caused by the direct loads on the ropes and for the stresses caused by the operating machinery, due to accelerating the bridge and its counterweight, and to frictional resistances.

403. When built-up sheaves are used, the rims shall be connected to the webs by an ample number of rivets, or bolts, to carry all stresses caused by the loads upon the sheaves, by the operating machinery, and by frictional resistances. They shall also be strong enough to resist the tendency of the rim to elongate along its circumference, due to the lateral distortions which result from the radial pressure of the counterweight ropes.

404. The hubs of all sheaves and drums shall be of ample size and length. When structural-steel sheaves are used the hubs shall be designed and connected to the structural web so as to resist all stresses. The barrel of the hub shall be thick enough so that the elastic elongation of its inner circumference, due to arm stresses and

trunnion reactions, will not produce a gap between the inside of hub and the trunnion at the bottom of the trunnion. Trunnions shall be pressed into the hubs or the hubs shrunk on the trunnions.

405. Operating ropes.—Operating ropes shall have at least two full circumferential turns around the drum when the span is in fully opened or closed positions. The ropes shall be attached to the drums so as to avoid sharp bends in the wires. When necessary there shall be turnbuckles, or other devices, for taking up the slack of the ropes.

406. Counterweight ropes.—The deviation of counterweight ropes from the vertical shall be as small as practicable. The deviation from the plane through the center of the sheave groove shall not exceed 1 in 30.

407. Equalizers.—If equalizers are used, they shall be designed so that in all positions the stresses in the ropes attached to them will be equalized.

408. Wire; physical properties.—The wire from which ropes are made shall show the following physical properties:

1. *Tensile Strength*

Ultimate Tensile Strength	A Extra Strong Crucible Cast Steel, Pounds per Sq. In.	B Plow Steel, Pounds per Sq. In.	C Monitor Steel, Pounds per Sq. In.
Average	200,000	230,000	250,000
Minimum	180,000	210,000	230,000
Maximum	220,000	250,000	270,000

2. *Torsion*

The number of times a piece of wire 8 ins. long can be twisted around its longitudinal axis without rupture shall not be less than 1.9 divided by the diameter expressed in inches.

3. *Bending*

The number of times the wire can be bent 90 degrees alternately in opposite directions, over a jaw the radius of which is equal to twice the diameter of the wire, without fracture, shall not be less than 6. This test shall be made in a mechanical bender constructed so that the wire actually conforms to the radius of the jaws and is subjected to little tensile stress.

409. Wire ropes.—Wire ropes shall be laid up in the best manner. They shall be thoroughly impregnated with an effective lubricant during the process of manufacture. Intermediate splices shall not be used in wire ropes.

The ropes shall be made up with a hemp center and shall be of the construction shown in the table in Art. 413, for the various diameters of rope. They shall be of the commercial grades of steel known as:

A. Extra Strong Crucible Cast Steel.

B. Plow Steel.

C. Monitor Steel.

410. Sizes.—The diameter of operating ropes shall not be less than $\frac{1}{2}$ in.

The diameter of counterweight ropes shall not be less than $\frac{7}{8}$ in. nor more than $2\frac{1}{2}$ ins.

411. Sockets.—The sockets shall be stronger than the ropes to which they are attached. They shall be fastened to the ropes in a reliable manner, so that the rope will not slip in the socket. When test pieces of rope, with sockets attached, are tested to destruction, the ropes shall not pull out of the sockets.

412. Tests.—A number of test pieces of rope shall be prepared with sockets attached not over 7 ft. apart. The sockets shall be selected at random from those that are to be used in filling the order. The number of test pieces shall not be more than 10 per cent. of the total number of finished lengths of rope to be made, nor less than 2 from each original length of rope.

When the ropes are tested to destruction the ultimate strength shall not be less than the values given in the table in Art. 413.

413.—Dimensions and strength of wire ropes.

DIMENSIONS AND MINIMUM ULTIMATE STRENGTH OF WIRE ROPES

Dimensions and Construction				Minimum Ultimate Strength in Pounds		
Diameter of Rope, Ins.	Construction	Approx. Area of Metal Section, Sq. Ins. *a*	Approx. Average Diam. of Wire, In. *d*	A Extra Strong Crucible Cast Steel	B Plow Steel	C Monitor Steel
$\frac{1}{4}$	6 × 19	0.108	.032	18,000	19,000	20,000
$\frac{5}{8}$	6 × 19	0.166	.041	28,000	30,000	32,000
$\frac{3}{4}$	6 × 19	0.229	.050	38,000	42,000	46,000
$\frac{7}{8}$	6 × 19	0.314	.058	50,000	56,000	62,000
1	6 × 19	0.405	.065	66,000	73,000	80,000
$1\frac{1}{8}$	6 × 19	0.505	.074	83,000	90,000	100,000
$1\frac{1}{4}$	6 × 19	0.612	.082	102,000	110,000	120,000
$1\frac{3}{8}$	6 × 19	0.753	.090	123,000	135,000	148,000
$1\frac{1}{2}$	6 × 19	0.909	.098	142,000	158,000	178,000
$1\frac{5}{8}$	6 × 37	1.010	.074	160,000	180,000	200,000
$1\frac{3}{4}$	6 × 37	1.155	.079	185,000	205,000	225,000
$1\frac{7}{8}$	6 × 37	1.324	.085	212,000	235,000	255,000
2	6 × 37	1.520	.091	242,000	270,000	290,000
$2\frac{1}{8}$	6 × 37	1.695	.096	270,000	300,000	325,000
$2\frac{1}{4}$	6 × 61	1.912	.079	298,000	330,000	360,000
$2\frac{3}{8}$	6 × 61	2.084	.083	324,000	360,000	400,000
$2\frac{1}{2}$	6 × 61	2.370	.088	360,000	390,000	445,000

414. Rejection of rope.—If the strength of a rope falls below that specified, the entire length from which the test pieces were taken shall be replaced with a new length which shall fulfill the requirements.

415. Rejection of sockets.—If a socket should break during the tests, two others shall be selected and attached to another piece of rope, and the test repeated. This process shall be continued until the reliability of the sockets is established; in which case the lot shall be accepted. If, however, 10 per cent. or more of the sockets tested break at a load less than the specified minimum strength of the rope, the lot shall be rejected.

416. Facilities for testing.—The manufacturer shall provide proper facilities for making the tests, and shall make them at his own expense.

417. Length.—The length of each counterweight rope shall be measured while the portion under measurement is supported throughout its entire length in a straight line and under a tension of not less than 2 per cent. of its specified minimum tensile strength. A variation from the required length at the rate of not more than $\frac{1}{4}$ in. in each 100 ft. of length will be allowed.

418. Shipment.—All wire ropes shall be shipped on reels and not in coils.

5. POWER EQUIPMENT

I. General

501. General.—The kind of power, whether man, steam, gasoline, oil, or electric, depends upon local conditions; and unless determined by the engineer shall be left to the judgment of the contractor.

502. Man-power.—If the bridge is to be operated by man-power the number of men and the time required to operate it shall be calculated on the assumption that the force one man can exert on a capstan lever is 40 lbs., at a speed of 160 ft. per minute, developing about $\frac{1}{5}$ h.p.

For proportioning the machinery, the force exerted by each man shall be assumed as 125 lbs.; but not less than 150 lbs., applied at the end of the capstan lever, shall be assumed in any case.

503. Mechanical power.—When the bridge is operated by mechanical power the engine or motor shall be of ample capacity to move the bridge within the required time. (See Arts. 617 to 622.)

504. Brakes.—The bridge-operating machinery shall be equipped with brakes as follows:

A. Swing Bridges

1. If operated by man-power, no brakes shall be provided.
2. If operated by steam or gasoline engine power, a brake operated by man-power shall be provided. The brake shall preferably be installed on the first counter shaft from the engine, and shall be capable of producing a braking force equivalent to that from the normal torque of the engine.
3. If electric power is used, each main operating motor shall be provided with a full-torque solenoid brake.

B. Bascule Bridges

1. If operated by man-power, an emergency brake shall be provided.
2. If operated by steam or gasoline engine power, a brake operated by man-power shall be provided. The brake shall preferably be installed on the

first counter shaft from the engine, and shall be capable of producing a braking force equivalent to that from the normal torque of the engine. When required, an emergency brake shall also be provided, acting as close to the rack, or other operating device, as practicable; and operated from a source of power independent of the engine.

3. If electric power is used, each main operating motor shall be provided with a full-torque solenoid brake.

When required, an emergency brake shall also be provided, acting as close to the rack, or other operating device, as practicable; and operated from a source of power independent of the motor.

C. Vertical-lift Bridges

1. If operated by man-power, no brake shall be provided.
2. If operated by steam or gasoline engine power, a brake operated by man-power shall be provided. The brake shall preferably be installed on the first counter-shaft from the engine, and shall be capable of producing a braking force equivalent to that from the normal torque of the engine.
3. If electric power is used, each main operating motor shall be provided with a full-torque solenoid brake.

505. Whistle.—Every movable bridge shall be supplied with a suitable whistle or horn. The whistle or horn shall be audible at a distance of 1500 ft., with the wind blowing in any direction.

506. Gates.—All movable bridges carrying highway traffic, which leave an opening in the floor when opened for navigation, shall be provided with safety gates.

507. Operator's house.—If mechanical power is used to operate a movable bridge, a suitable house shall be provided for the operator. The house shall be of dimensions adequate for the purpose for which it is intended. It shall be placed in a position where the operator can observe the signals and see the approaching vessels and bridge traffic and shall have enough windows, of sufficient size, so that his view will not be obstructed.

508. Stairways, walks, and ladders.—If the operator's house is above or below the floor of the bridge, suitable metal stairs, platforms, and walks, with railings, shall be provided to give easy access to the house. Metal ladders shall be provided to give access to the machinery, trunnions, counterweights, lights, bridge seats, and similar parts.

509. Heating.—In locations where the climatic conditions require it, provision shall be made for heating the operator's house. If steam power is used, the house shall be heated by steam coils, or radiators, fed from the boiler. When other power is used, a coal, wood, oil, or gas stove, as directed by the Engineer, shall be provided.

II. STEAM EQUIPMENT

510. Engines.—If a steam engine is used, it shall be a two-cylinder, reversing engine, the piston speed of which shall not exceed 500 ft. per minute. It shall develop the desired power and speed with a steam pressure of 100 lbs. per square inch at the throttle. The engine shall be connected to the operating machinery by an approved friction clutch, arranged so that the moving and locking machinery can be operated alternately, or stopped, without stopping the engine.

511. Boilers.—There shall be one or two boilers, as specified by the Engineer. They shall be upright tubular boilers. They shall be designed for a steam pressure of 150 lbs. per square inch, and shall be adapted to the kind of fuel designated by the Engineer. Boilers shall be in accordance with the code of the American Society of Mechanical Engineers adopted March 19, 1924, and subsequent revisions thereto; and shall conform to civil regulations. They shall be encased in asbestos, covered with hammered, polished sheet steel.

512. Horse-power of boilers.—The boiler horse-power of each boiler shall be twice the horse-power of the engine.

513. Pipe covering.—Steam pipes shall be covered in a manner approved by the Engineer. Water pipes shall be protected from freezing, where necessary.

514. Engine room.—The engine room shall be provided with a steel water tank, of sufficient capacity, a duplex steam feed pump, and an injector for each boiler; with the necessary pipes and connections for feeding the boilers separately or together. Steam water lifters, with necessary strainers, flexible hose, and piping to lift the water from the river into the tank shall be provided, if the river water is suitable for use in the boilers. If the river water cannot be used, the Owner shall arrange for a suitable supply of water and furnish and install all piping required to deliver it at a convenient point in the engine room. A coal hoist, or chute, and a steel coal bin of sufficient capacity shall be provided. The engine room shall be provided with indicators to show the positions of the moving span and of the end-lifting and locking apparatus.

III. Internal Combustion Equipment

515. Engines.—Gasoline and other internal combustion engines shall be of substantial construction. The piston speed shall not exceed 1000 ft. per minute.

The engine shall have a reversing gear, provided with approved friction clutches, to be operated by a hand wheel, lever, or other approved device. The counter shaft connecting the engine with the operating machinery shall be fitted with disengaging clutches, arranged so that the moving and locking machinery can be operated alternately and in either direction without stopping the engine.

516. Starters.—Single-cylinder, low-speed engines of more than 15 h.p. shall be equipped with a compressed-air starting device, with necessary air pump, air tank, and piping. High-speed, multiple-cylinder, gasoline engines of more than 40 h.p. shall be equipped with a complete compressed-air, or electric, starting device.

517. Engine room.—If low-speed engines are used, the engine room shall be provided with a cooling-water tank of ample capacity. High-speed engines may be cooled by approved radiators, with fans. The fuel tank shall be located outside of the engine room and below the level of the cylinders.

It shall be protected from the sun.

The engine room shall be provided with indicators to show the position of the moving span and of the end-lifting and locking apparatus.

IV. Electrical Equipment

518. Design.—The electrical equipment shall be as simple as possible to assure satisfactory operation. All unnecessary auxiliary apparatus should be eliminated.

Unless otherwise specified by the Owner, the equipment for various types of bridges shall be that described in Arts. 519 to 537, inclusive.

A. Swing Bridges

519. Motors.—The main operating motors shall be of the standard, totally enclosed, crane type, and equipped with full-torque solenoid brakes and suitable control.

End and center mechanism, including wedges, rail lifts, and bridge and rail locks, shall preferably be operated independently of the main operating motors, by totally enclosed, high-torque motors, with reversing drum or magnetic control.

When desirable, such mechanism may be operated from the main motors through suitable clutches.

520. Interlocking.—The end mechanism shall be interlocked with the signals so that no current will be available until the signals have been set at danger. Current for the operating motors shall not be available for turning the bridge until the end mechanism has completed its function. When closing, the sequence of operations will be reversed. (See Art. 112.)

521. Limit switches.—Limit switches shall not be used for the turning movement, except when that movement is restricted. They shall be supplied for the end mechanism if it is designed so that an over-travel would cause damage.

522. Indicators.—Indicators showing the position of the moving span shall preferably be mechanical and in the form of a dial with a revolving pointer. When this form is not feasible, electric lamp signals may be used to indicate the open, nearly open, free, nearly closed, and closed positions of the span.

523. Navigation lights.—Navigation lights shall be in accordance with Government requirements. (See Art. 113.)

524. Lights.—A sufficient number of lights shall be supplied to illuminate properly the operator's house and machinery.

525. Switchboard.—A suitable switchboard shall be supplied, and separate circuits with proper switches and overload protection for each motor and group of lights.

The switchboard shall be provided with a voltmeter and an ammeter. A watt-hour meter will not be provided unless it is specified by the Owner.

526. Submarine cable.—Current shall be delivered to the movable span through a submarine cable and collector rings around the bridge center, whenever local conditions permit.

The Owner shall do any necessary dredging for the installation of the submarine cable.

B. Vertical-lift Bridges

527. Motors.—The main operating motors shall be of the standard totally enclosed, crane type, equipped with full-torque solenoid brakes and suitable control.

528. End mechanism shall be operated by totally enclosed high-torque motors, with reversing drum or magnetic control.

529. Interlocking.—The end mechanism shall be interlocked with the signals so that no current will be available until the signals have been set at danger. Current for the operating motors shall not be available for raising the bridge until the end mechanism has completed its function. When closing, the sequence of operations will be reversed. (See Art. 112.)

530. Limit switches.—Limit switches shall be provided for the two extreme positions of the span, also for the end mechanism, if it is designed so that an over-travel would cause damage.

531. Indicators.—Indicators showing the position of the moving span shall preferably be mechanical and in the form of a dial with a revolving pointer. When this form is not feasible, electric lamp signals may be used to indicate the open, nearly open, free, nearly closed, and closed positions of the span.

532. Navigation lights.—Navigation lights shall be in accordance with Government requirements. (See Art. 113.)

533. Lights.—A sufficient number of lights shall be supplied to illuminate properly the operator's house and machinery.

534. Switchboard.—A suitable switchboard shall be supplied, and separate circuits with proper switches and overload protection for each motor and group of lights.

The switchboard shall be provided with a voltmeter and an ammeter. A watt-hour meter will not be provided unless it is specified by the Owner.

535. Conductors.—When the motors are on the movable span, current shall be delivered by copper conductors, of ample size, with suitable collecting shoes.

C. Bascule Bridges

536. General.—Arts. 527, 528, 529, 530, 531, 532, 533 and 534 shall apply to bascule as well as to vertical-lift spans.

537. Conductors.—When the motors are on the movable span, current shall be delivered by suitable flexible cables. These cables shall be arranged so that they can easily be renewed.

D. General Specifications

538. Plans.—The Contractor shall furnish complete dimension and connection diagrams of all electrical equipment. Special apparatus shall be designated by the manufacturer's name and catalogue references.

539. Rules.—The electrical equipment shall conform to the Standards of the American Institute of Electrical Engineers, as given in the edition of 1922, or subsequent revisions thereof, unless modified herein.

540. Motors.—Motors shall preferably be of the standard crane or mill types, totally enclosed, and as moisture-resisting and splash-proof as practicable.

They shall be mounted so as to admit of easy access for maintenance, inspection, and repairs. They shall be securely supported and anchored. The anchor bolts shall closely fit their holes.

When the size of motor required cannot be obtained of the totally enclosed type, the windings shall be as moisture-resisting as practicable and the motors shall be installed so as to be fully protected from the weather.

When the motors and solenoid brakes must be installed on the moving leaves of bascule bridges they shall be capable of being operated in any position of the leaf.

541. Starting torque.—The maximum starting or running torque shall be equal to at least twice the normal running torque.

542. Speed.—If motors of a given output may be obtained with different armature speeds, preference shall be given to those running at the lower speeds, in order to secure motors of rugged construction and to simplify the speed-reduction gearing.

543. Temperature rise.—Each motor shall be capable of delivering its rated output continuously for fifteen minutes, without exceeding a temperature rise of 55° C. in any part.

544. Spare parts.—The Contractor shall not furnish spare parts unless they are definitely specified by the Owner in the inquiry and included in the contract.

545. Tests.—Copies of the manufacturer's certified tests of motors, solenoids, brakes, and other electrical mechanism will be furnished only when required by the Engineer.

546. Control.—If the full-load current is 300 amperes or less, the control shall be of the reversing drum type. For larger sizes, remote control, with magnetic switches and drum master-controller, shall be used.

There shall be a sufficient number of accelerating points to assure smooth and rapid operation. The resistors shall be of ample capacity to allow normal operation of the motors on any point. If a solenoid brake is used, the first point shall control it.

The control for the main operating motors shall be arranged so that the movable span can be allowed to drift without power applied or brakes set; or if direct-current motors are used, slow-speed control of the span may be obtained by shunting the armatures.

547. Switchboard.—The switchboard apparatus shall be mounted on a suitable panel located in the operator's house. The panel base shall be free from metallic veins and flaws.

The bottom of the switchboard shall clear the floor by at least 9 ins. There shall be a distance of at least 2½ ft. between the wall of the house and any of the parts on the back of the switchboard.

548. Switches and circuit breakers.—Where the power lines enter the operator's house, a suitable disconnecting switch shall be provided, so that current can be cut off at times of inspection and adjustment. If direct current is used, a 2-pole automatic air circuit breaker of suitable capacity shall be provided on the switchboard.

If alternating current is used, an oil circuit breaker, having one pole per line, shall be provided on the switchboard.

Circuits for lighting and motor control shall be provided with suitable knife switches, fused to the proper value so as to protect the apparatus connected thereto.

Magnetic switches or automatic circuit breakers, having one pole per line and equipped with overload and no-voltage protection, shall be placed in each motor circuit.

549. Push buttons.—Normally open push-button switches shall be mounted where they will be convenient to the operator for the purpose of cutting out the interlocking in emergencies.

550. Meters.—Meters, as specified in Arts. 525 and 534, shall be mounted on the switchboard.

551. Markers.—The appliances mounted on the switchboard shall be provided with durable metal plates designating their use.

552. Wiring.—Wires, switches, and other electrical parts shall be of ample size to allow the maximum output from the equipment without injurious heating.

553. Conduits.—Wires shall have approved rubber and braid insulation and shall be installed in standard sherardized or loricated metal conduits.

Bends in the conduits shall be used sparingly. Suitable fittings shall be used. Junction boxes may be used where ordinary fittings are not feasible.

Conduits and boxes shall have properly located drain holes, and shall be installed so that dirt will not accumulate around them.

554. Tags.—Wires shall be numbered and the number permanently marked on durable metal tags so that any wire may be traced from the switchboard to the motor or to the source of power.

6. LOADS AND STRESSES

601. Loads, stresses, and stress sheets.—The superstructure of movable bridges shall be proportioned for the loads specified herein.

When calculating the maximum live-load stresses, the load shall be considered as applied continuously and not in detached parts.

The stress sheet shall show the stresses in each member for each case of loading, together with the combinations which produce the greatest positive and negative stress in each member. A tensile stress shall be considered as positive and a compressive stress as negative.

Impact stresses shall be shown separately in connection with the live-load stresses whenever the addition of impact is required by the specifications for the bridge superstructure. The increase in dead load stresses specified in Art. 608 shall be shown separately in connection with the dead load stresses.

602. Swing bridges.—The maximum stresses in the main girders or trusses of swing bridges, continuous over three or four supports, shall be calculated for the following conditions:

Case I.—Dead load: Bridge open, or closed with no end reactions.

Case II.—Dead load: Bridge closed, with its ends lifted to cause positive end reactions equivalent to one and one-half times the maximum live-load negative reactions, without impact.

Case III.—Live load: Bridge closed, but with no dead-load truss reactions. One arm loaded and considered as a simple span.

Case IV.—Unbroken live load: Bridge closed and considered as a continuous girder.

Case V.—Unbroken live load: Bridge closed and considered as a continuous girder, but with the live load placed so as not to cause negative reactions.

The following combinations of these cases shall be used for determining the maximum stresses:

Case I alone.

Case I with *Case III.*

Case I with *Case V.*

Case II with *Case IV.*

603. Bascule bridges.—The maximum stresses in the movable span and in the towers and counterweight trusses of bascule bridges shall be calculated for the following conditions:

Case I.—Dead load: Bridge closed.

Case II.—Dead load: Bridge open and in any position.

Case III.—Live load: Bridge closed.

The following combinations of these cases shall be used for determining the maximum stresses:

Case II alone.

Case I with *Case III.*

604. Vertical-lift bridges.—The maximum stresses in the movable span and in the towers of vertical-lift bridges shall be calculated for the following conditions:

Case I.—Dead load: Bridge closed or open.

Case II.—Live load: Bridge closed.

The maximum stresses shall be determined by combining *Case I* with *Case II.*

605.—Wind and lateral loads.—The wind load on the structure shall be a moving load of 30 lbs. per square foot of area, as seen in elevation; of the floor system, considered as full area, the area of one truss, and one-half of the area of all remaining

trusses; but not less than 300 lbs. per linear foot at the loaded chord, and 150 lbs. per linear foot at the unloaded chord.

The lateral force due to the moving live load, and the wind pressure against it, shall be assumed as follows:

1. *Railway bridges:*
   To provide for the effect of the sway of engines and train, a moving lateral load equal to 5 per cent. of the specified live load on one track, but not more than 400 lbs. per linear foot, applied at the base of rail.
   A moving wind force against the train of 300 lbs. per linear foot on one track, applied 7 ft. above the base of rail.
2. *Highway bridges also carrying electric railway traffic:*
   300 lbs. per linear foot, applied in the plane of the bridge floor.
3. *Highway bridges:*
   200 lbs. per linear foot, applied in the plane of the bridge floor.

606. Whenever a wind pressure of 50 lbs. per square foot of the unloaded structure, applied as specified in Art. 605, causes greater stresses in any member than the loads specified in Art. 605, they shall be used.

607. When proportioning the members and determining the stability of the movable spans and the towers of bascule and vertical-lift bridges when the movable spans are open, the wind pressure, acting in any direction horizontally, shall be assumed as 30 lbs. per square foot, and applied as specified in Art. 605.

608. Operating stresses.—Dead-load stresses in structural members that vary with the movements of the span, as in bascule bridges, shall be increased 20 per cent. to include dynamic effect. This dynamic allowance shall not be combined with the live-load stresses.

609. Combined stresses.—Members subject to both axial and bending stresses shall be proportioned so that the combined fiber stresses will not exceed the specified unit stresses. When members are continuous over panel points, three-fourths of the bending stress, computed as for simple beams, shall be added to the axial stress.

610. If stresses in either tension or compression are produced by a combination of either longitudinal or lateral and wind forces, with dead-load, live-load, impact, and centrifugal force, the unit stresses may be increased 25 per cent. above those allowed for combined dead, live, impact, and centrifugal stresses; but in no case shall the section be less than that required for the combined dead, live, impact, and centrifugal stresses.

611. Alternate stresses.—Members that are subject to alternate stresses of tension and compression, due to combinations of dead, live, impact, and centrifugal stresses, but not including operating stresses, shall be proportioned for the kind of stress requiring the larger sections.

If the alternate stresses occur in succession during one passage of the unbroken live load each shall be increased by 50 per cent. of the smaller.

The connections of such members shall be proportioned for the sum of the net alternate stresses before they are increased.

612. Combined alternate stresses.—If members are subject to alternate stresses due to combinations of either longitudinal or lateral and wind forces, with dead-load, live-load, impact, and centrifugal force, but not including operating stresses, they shall be proportioned for the kind of stress requiring the larger section, using unit stresses 25 per cent. larger than those allowed for combined dead, live, impact, and centrifugal stresses; but in no case shall the section be less than that which is required by Art. 611.

The connections of such members shall be proportioned for the larger of the alternate stresses, using the unit stresses allowed for combined dead, live, impact, and centrifugal stresses; but in no case shall the strength of the connections be less than those which are required by Art. 611.

613. Combined operating stresses.—If the increased operating stresses in any member require larger sections than those determined from dead, live, impact, and centrifugal stresses, the sections shall be increased so that the normal unit stresses are not exceeded.

If the increased operating stresses, combined with wind stresses, require larger sections than the combinations specified in Arts. 610 and 612, the sections shall be increased so that the unit stresses will not exceed 125 per cent. of the normal unit stresses.

614. End and center reactions.—The end-lifting machinery of swing bridges shall be proportioned for an uplift equal to the positive reaction specified in Art. 602, *Case II.*

The end-lifting apparatus shall be adjusted so that, at normal temperature, an uplift will be produced equal to 110 per cent. of the maximum negative end reaction from live load, for single-track railway bridges; and equal to the maximum negative end reaction from live load for double-track railway and highway bridges.

The end and center bearings shall preferably be wedges.

The end and center wedges shall be proportioned for the maximum positive reactions, including impact.

615. Balance wheels.—On center-bearing swing spans, the transverse balance wheels and their axles, bearings, and brackets shall be designed for an overturning wind pressure of 15 lbs. per square foot of surface of each truss and the floor system as seen in elevation.

616. Rollers.—The rollers of rim-bearing, or combined rim-and-center-bearing swing bridges, shall be proportioned for the dead-load stresses when the bridge is swinging and for the dead-load, live-load, and impact stresses when the bridge is closed. The load on each roller shall be calculated on the supposition that the entire concentrated load on each loaded point of the drum is carried by a number of rollers not greater than twice the depth of the drum divided by the mean diameter of the roller.

617. Power for operation.—For determining the power required to operate the bridge, the various forces shall be assumed and applied as follows:

A. In the normal time of opening:

1. Swing and vertical-lift bridges against frictional resistances and accelerating forces.
2. Bascule bridges against frictional resistances, accelerating forces, and a wind pressure of 2½ lbs. per square foot of the span and its floor as seen in plan. This force shall be assumed as applied normally to the floor of the span throughout the movement of the span when opening or closing.

B. In one and one-half times the normal time of opening:

1. Swing bridges against a wind blowing at a velocity of 50 miles per hour upon each truss and one floor system, as seen in elevation, assumed as applied to both arms and normally to the span throughout the movement of the span when opening or closing; in addition to the forces specified in A, 1, above. (If V is the wind velocity in miles per hour and P is the corresponding pressure per square foot of surface normal to the direction of the wind: $P = 0.004V^2$.)

2. Vertical-lift and bascule railway bridges, in locations where ice forms, with an ice load of 2½ lbs. per square foot on 85 per cent. of the area of a quadrilateral of which the width is the distance between the centers of the trusses and the length is that of the floor of the movable span; in addition to the forces specified in A,1 or A, 2.

In highway bridges the full area of the quadrilateral, defined above, shall be considered as loaded with ice.

C. In twice the normal time of opening:

Bascule railway bridges, against frictional resistances, accelerating forces, and a wind pressure of 10 lbs. per square foot of area upon any vertical projection of the moving span and its floor; and in locations where ice forms, with an ice load of 2½ lbs. per square foot on 85 per cent. of the area of a quadrilateral of which the width is the distance between the centers of the trusses and the length is that of the floor of the movable span.

In highway bridges the full area of the quadrilateral, defined above, shall be considered as loaded with ice.

618. Holding span.—On unequal-arm swing bridges and bascule bridges the machinery shall be proportioned to hold the span in any position against a wind pressure of 15 lbs. per square foot, applied on any vertical projection of the moving span. When proportioning the machinery for this condition the unit stresses specified herein may be increased one-fourth.

619. Impact and excess loads.—The machinery shall be designed for motive forces determined as follows:

TORQUE OF PRIME MOVER EXPRESSED IN PER CENT. OF NORMAL TORQUE OR CALCULATED RESISTANCE

Character of Prime Mover	Engine-house Machinery, Including Transmission Machinery or Speed Reducers	Remainder of Operating Train	End-lifting and Locking Apparatus. Per Cent. of Calculated Resistance
Direct-current, crane type motor	200	200	100
Alternating-current motor	200	200	100
One-cylinder, 4-cycle gasoline engine, about 200 to 300 r.p.m.	225	150	100
Two-cylinder, 4-cycle gasoline engine, about 250 to 350 r.p.m.	175	125	100
Three-cylinder, 4-cycle gasoline engine, about 250 to 400 r.p.m.	150	100	100
Marine or truck type, 4-cylinder, 4-cycle gasoline engine, about 1000 to 1400 r.p.m.	125	100	100
Steam engine	125	100	100

620. Machinery resistances.—When calcuating the resistances to be overcome by the machinery the following coefficients shall be used:

	Starting	Running
Trunnion friction:		
a. Less than one complete rotation	0.15	0.10
b. One or more complete rotations	0.12	0.08
Center discs	0.15	0.12
Wedges:		
a. Against driving, each contact surface	0.15	0.15
b. Against slipping out when under load, each contact surface	0.05	0.00
Rolling friction of bridges having rollers with flanges, or built-up segmental girders	0.009	0.006
Rolling friction of rollers without flanges:		
a. In contact with one surface only	$\frac{0.004}{\sqrt{r}}$	$\frac{0.004}{\sqrt{r}}$
b. In contact with two surfaces	$\frac{0.008}{\sqrt{r}}$	$\frac{0.008}{\sqrt{r}}$

In *a* and *b*, r is the radius of the roller in inches.

Resistance of wire ropes bent around sheaves, or drums:

$$t = c\frac{d}{D}T,$$

in which T = the tension in the part of the rope approaching the sheave;
d = the diameter of the rope;
D = the pitch diameter of the rope upon the sheave;
c = a coefficient which equals 0.45 when the ratio $\frac{D}{d}$ is 30, or less, and which diminishes by 0.003 for each unit by which the ratio $\frac{D}{d}$ exceeds 30, and
t = the additional tension in the part of the rope leaving the sheave, which is required to overcome the internal frictional resistance of the rope against movement around the sheave, or drum.

When designing the machinery for holding the span against the 15 lbs. wind pressure, and for computing braking effect, one-half of the above coefficients shall be used.

Sliding friction between plane surfaces intermittently lubricated (such as guides for vertical-lift spans), 0.10.

621. Machinery losses.—When computing the machinery losses between the operating rack, the operating rope, the operating strut, or similar points and the motor, the following coefficients shall be used:

	Values of f
Horizontal shafts and journals	0.05
Intermediate vertical shafts and journals	0.06
Vertical main-pinion shafts and journals	0.10
Collar thrust bearings	0.10
Foot-steps for vertical shafts	0.10

Efficiency of gearing exclusive of bearings:

	Per Cent.
Spur gears with cut teeth	97
Spur gears with cast teeth	95
Bevel gears with cut teeth	95
Bevel gears with cast teeth	92
High-grade silent power-transmission chains	97
Well-made pin-connected power-transmission chains	95

Efficiency of worm gearing, excluding thrust bearings,

$$E = \frac{L}{L + 2f\pi R},$$

in which L = the lead of the worm and R its pitch radius, both in inches.

For cut worms and wheels, $f = 0.10$.

622. Brakes.—Brakes, other than solenoid brakes, shall be designed so that the braking effect shall not be greater than one and one-half times the normal accelerating effect of the motor.

When the brakes act through the machinery the unit stresses in the machinery shall not exceed one and one-half times those caused by the normal torque of the prime mover.

Parts that receive stress only from brakes shall be designed for normal unit stresses.

7. UNIT STRESSES

701. Structural steel.—Structural-steel parts, except as otherwise specified herein, shall be proportioned for unit stresses in accordance with the Specifications of the American Society of Civil Engineers. (See Art. 102.)

702. Structural-steel machinery parts.—For parts in tension during one operation of the span the tensile stress shall not exceed 16,000 lbs. per square inch. For parts in compression during one operation of the span the compressive stress shall not exceed $16{,}000 - 70\frac{l}{r}$ in pounds per square inch,

in which l = the effective length of the member and r its least radius of gyration.

For parts subject to reversal of stress during one operation of the span, or subject to vibration, the tensile stress shall not exceed 10,000 lbs. per square inch, and the compressive stress shall not exceed $10{,}000 - 45\frac{l}{r}$ in pounds per square inch.

(*Note.*—Structural-steel machinery supports shall be designed so as to reduce the effect of vibration and will frequently require stiffening at right angles to the planes of the principal stresses.)

703. Cast pedestals and wedges.—Steel and iron castings used for bridge pedestals, wedges, wedge seats, trunnion pedestals, and similar parts, shall be proportioned so that the maximum stresses, in pounds per square inch, shall not exceed the following:

	Cast Steel, Lbs. per Sq. In.	Cast Iron, Lbs. per Sq. In.
Direct compression and compression fiber stress due to bending	$16{,}000 - 70\frac{l}{r}$	$12{,}000 - 50\frac{l}{r}$
Direct tension	16,000	3,000
Tension fiber stress due to bending	16,000	4,000

704. Levers and bell cranks.—Levers and bell cranks shall be proportioned so that the fiber stresses, in pounds per square inch, shall not exceed the following:

Cast or forged steel in tension or compression	10,000
Cast iron in tension	3,000
Cast iron in compression	8,000

When the load is applied eccentrically, as in the case of an overhanging crank, the same unit stresses shall be used, but an equivalent bending moment from the following formula shall be used as the basis of design:

$$M_E = \tfrac{3}{8}M_B + \tfrac{5}{8}\sqrt{M_B{}^2 + M_T{}^2},$$

in which M_B = the simple bending moment;
M_T = the torsional moment;
M_E = the equivalent bending moment.

705. Bearings.—Bearings shall be proportioned so that the fiber stresses, in pounds per square inch, due to the resultant of all forces acting upon them shall not exceed the following:

Cast steel, tension or compression	10,000
Cast iron, tension	3,000
Cast iron, compression	8,000

706. Pins, bolts, and rivets.—The fiber stresses, in pounds per square inch, on pins, bolts, and rivets used in connection with the operating machinery shall not exceed the following:

Tension:	
Pins and bolts	10,000
Rivets	5,000
Bending:	
Pins and finished bolts	15,000
Bolts and rivets	10,000
Shear:	
Pins, finished bolts, and power-driven rivets	7,500
Bolts and hand-driven rivets	6,000
Bearing:	
Pins, finished bolts, and power-driven rivets	15,000
Bolts and hand-driven rivets	12,000

707. Pins.—When pins, not forming a part of the operating machinery, are used in the superstructure and do not serve as mechanical bearings, the fiber stresses, in pounds per square inch, shall not exceed:

Shear	12,000
Bending	24,000
Bearing	24,000

708. Trunnions.—The maximum fiber stresses, in pounds per square inch, in trunnions and similar mechanical joints shall not exceed:

Annealed carbon-steel forgings, Class C	15,000
Heat-treated carbon-steel forgings, Class G	25,000

709. Shafts.—The maximum torsional shear in rolled or Class C forged-steel shafts shall not exceed 8000 lbs. per square inch; and the maximum bending fiber stress shall not exceed 16,000 lbs. per square inch.

Shafts subject to combined torsional and bending stresses shall be proportioned by the following formulae:

$$T_E = \sqrt{(K_m M)^2 + (K_t T)^2}$$

$$\frac{16 T_E}{\pi d^3} \leqslant 8000,$$

in which M = the bending moment;
T = the torsional moment, and
T_E = the equivalent torsional moment, all in inch-pounds.
K_m and K_t = impact and fatigue factors, to be taken as follows:

Manner of Loading	Values for K_m	Values for K_t
Gradually applied load on shafts rotating at 10 r.p.m., or less	1.0	1.0
Gradually applied load on shafts rotating at more than 10 r.p.m.	1.5	1.0
Suddenly applied load	1.5	1.5

710. Shafts shall also be proportioned so that the angular deflections shall not exceed the following limits:

A. For ordinary service, as in the operating trains of movable bridges, one degree in a length equivalent to twenty times the diameter of the shaft.

B. For special cases where more positive action is desirable, as in shafts driving end wedges, 0.075 degree per linear foot of shaft.

When calculating the angular deflections the torsional modulus of elasticity of rolled or forged steel shall be taken as 11,600,000.

711. Shafts with keyways.—When standard keyways are cut in shafts at points of maximum stress the calculated diameters shall be increased as follows:

Calculated Diameters of Shafts, In.	Add to Diameter when 1 Keyway is Used	Add to Diameter when 2 Keyways are Used
1 to 3 in.	8 per cent.	6 per cent.
Over 3 in.	6 per cent.	4 per cent.

Most shafts proportioned for torsional stiffness require no increase in diameter on account of keyways; but the diameter of such shafts shall not be less than that determined from the limiting unit stresses and increased as specified for keyways.

712. Teeth of spur gears.—The entire load shall be assumed to be carried by one tooth. The permissible tooth load shall be determined from the formula:

$$W = Spfy.$$

In this formula:

W = the permissible tooth load, in pounds;
S = the permissible fiber stress, in pounds per square inch;
p = the circular pitch, in inches;
f = the face of gear, in inches;
y = a factor depending upon the number and form of the teeth.

The values of S shall be determined from the formula:

$$S = S_B\left\{1 - \tfrac{1}{4200}\sqrt{6200V - V^2}\right\},$$

in which V = the velocity at the pitch circle of a gear, in feet per minute;
S = the permissible fiber stress on a gear tooth when running at a velocity of V in feet per minute at the pitch circle.

S_B shall have the following values in pounds per square inch:

	Values of S_B
Class C forged-steel gears	22,000
Cast-steel gears	20,000
Phosphor-bronze gears	10,000
Cast-iron gears	8,000

The values of y shall be determined as follows:

Form of Gear Teeth	Values of y
For 20-degree involute stub teeth (Nuttall system)[1]	$0.178 - \frac{1.033}{N}$
For 20-degree involute teeth	$0.154 - \frac{0.912}{N}$
For 14½-degree involute teeth	$0.124 - \frac{0.684}{N}$
For radial-flank teeth	$0.075 - \frac{0.276}{N}$

In these formulae N is the number of teeth on the gear.

713. Bevel gears.—Bevel gears shall be designed in the same manner as spur gears with the following modifications:

$$W_1 = Spfy_1\frac{d}{D},$$

in which W_1 = the permissible tooth load at the outside pitch circle, in pounds;
d = the pitch diameter at the inside end of the teeth;
D = the pitch diameter at the outside end of the teeth.

The value used for y_1 shall be that corresponding to the formative number of teeth, N.

[1] Use no pinion with less than 14 teeth.

The value of N shall be determined from the formula:

$$N = n\sqrt{1 + \left\{\frac{n}{n_1}\right\}^2},$$

in which n = the number of teeth on the pinion;
n_1 = the number of teeth on the mating gear.

Bevel gears shall be designed so that $\frac{d}{D}$ shall not be less than $\frac{2}{3}$.

714. Bearings.—The pressures, in pounds per square inch, on bearings when moving at slow or intermittent speeds shall not exceed the following:

Center discs, on full projected area	3500
Trunnions, steel on phosphor-bronze	2000

Axles of balance wheels that receive their maximum stresses from overturning wind pressure:

Steel axles on cast-iron or lined cast-steel bearings	1500

Main-pinion shaft bearings and other slow-moving shafts:

Steel on phosphor-bronze	1000
Steel on babbitt	800

The pressures, in pounds per square inch, on bearings moving at ordinary speeds, shall not exceed the following

Steel shafts on phosphor-bronze	750
Steel shafts on babbitt	600
Steel shafts on cast iron	600
Cast iron on cast iron	400

For crank pins and similar joints with alternating pressures the limiting values given above for ordinary speeds may be doubled.

715. Heating and seizing. –In order to prevent heating and seizing at high speeds, the bearing pressures, in pounds per square inch, shall not exceed the following:

	Values of pressures in lbs. per sq. in. for ordinary speeds.	Maximum values in lbs. per sq. in. for slow speeds.
Collar bearings:		
Steel on steel	$\frac{20{,}000}{Nd}$	60
Steel or cast iron on cast iron	$\frac{60{,}000}{Nd}$	600
Steel on phosphor-bronze	$\frac{80{,}000}{Nd}$	750
Pivot bearings, steel on cast iron or phosphor-bronze	$\frac{100{,}000}{Nd}$	1500
Journals	$\frac{300{,}000}{Nd}$	As given above for various materials.

"Acme" or square threads transmitting motion:	Values of pressure in lbs. per sq. in. for ordinary speeds when Nd = 140 or more.	Maximum pressure in lbs. per sq. in. for slow speeds when Nd is less than 140.
Steel screw running in cast-iron or "semi-steel" nut	$\frac{262{,}500}{Nd}$	1875
Steel screw running in phosphor-bronze nut	$\frac{350{,}000}{Nd}$	2500

In these formulæ, N = number of revolutions per minute;
d = diameter of pivot or journal or the mean diameter of collar or screw in inches.

716. Rollers.—On practically solid rollers, pressures shall not exceed the following:

	In Motion	At Rest
Cast iron	$300dL$	$450dL$
Steel castings	$500dL$	$600dL$
Hard steel	$1000dL$	$1500dL$
Tempered steel	$1500dL$	$2000dL$

in which d = diameter and
L = length of roller, both in inches.

When rollers and bearings are of different materials, the lower limiting loads shall be used.

717. Balance wheels.—Balance wheels for center-bearing swing spans that receive their maximum stresses from overturning wind pressure shall be proportioned so that the load on the wheels will not exceed:

	In Motion
Steel castings	$1200dL$
Iron castings	$800dL$

718. Rollers for rim-bearing bridges.—Rollers for rim-bearing swing spans shall be proportioned so that the following loads on the rollers will not be exceeded:

	In Motion	At Rest
Steel castings	$400dL$	$600dL$
Iron castings	$250dL$	$400dL$

719. Tracks for rolling-lift bridges.—The stress per linear inch of line contact between the tracks on the rolling segments and the track-supporting girders of rolling lift bridges shall not exceed

$675\sqrt[4]{d^3}$,[1] when ordinary forged or cast steel with carbon of about 0.40 per cent. is used. In this expression, d = the diameter of the rolling segment in inches.

720. Wedges.—Wedges and their seats shall be proportioned so that the bearing pressure in pounds per square inch will not exceed:

Wedges during motion	1,500
Wedges driven and carrying dead or live load, or both	16,000

[1] Tentative, pending the completion of tests to determine the laws of line contact in this type of construction.

721. Wire ropes.—The direct tension in counterweight and operating wire ropes shall not exceed one-fifth of the minimum specified ultimate strength, given in Art. 413, less the equivalent tension due to bending as specified in Art. 722.

722. Bending stresses in wire ropes.—When a wire rope is bent over a sheave or drum, the bending stress and equivalent tension on the rope shall be calculated as follows:

Let: E_R = the modulus of elasticity of the wire rope as found from the formula

$$E_R = \frac{PL}{a\Delta},$$

in which P = the change in stress for which a change in elongation is measured;
L = the length of rope for which Δ is measured;
Δ = the change in elongation due to the addition of a stress P on a length L;
a = the sum of areas of all the wires in the rope.

Let d = the diameter of the wires in the rope;
D = the diameter of the sheave to the center of the rope;
S = the unit stress in the wires making up the rope;
T = the equivalent total tension stress in the rope due to bending it around the sheave.

Then:

$$S = E_R \frac{d}{D},$$

$$T = Sa.$$

The values of a and d are given in Art. 413.

For the types of rope shown in Art. 413 the maximum value of E_R shall be taken as 12,000,000.

8. MATERIALS

801. Materials in general.—Unless otherwise specified herein, all materials shall be made in accordance with the Specifications of the American Society for Testing Materials, and the current revisions thereof.

	Serial Designation of the American Society for Testing Materials
Structural steel	A 7–24
Carbon-steel forgings	A 18–21, Class C
Heat-treated carbon-steel forgings	A 18–21, Class G
Steel castings	A 27–24, Class B, Medium
Iron castings, except counterweights	A 48–18, Medium
Boiler steel	A 70–24
Cold-finished steel	A 54–24

[*Note.*—Copies of these specifications, in pamphlet form, may be obtained from the American Society for Testing Materials, 1315 Spruce Street, Philadelphia, Pennsylvania.]

802. Steel for center discs.—Center discs of ordinary size, which are forged, finished, and tempered without case-hardening, shall be made from homogeneous steel of the following quality:

Carbon	0.35 to 0.45; 0.40 desired
Phosphorus	0.04 maximum
Sulphur	0.05 maximum
Manganese	0.50 maximum

Large discs which are forged, finished, and case-hardened on the bearing surface shall be made from homogeneous steel of the following quality:

Carbon	0.10 to 0.20; 0.15 desired
Phosphorus	0.04 maximum
Sulphur	0.05 maximum
Manganese	0.50 maximum

803. Phosphor-bronze.—Phosphor-bronze shall be a homogeneous alloy of copper and tin of crystalline structure. It shall be made from new metals, except that clean scrap of known composition may be used. It shall not contain sulphur. The phosphorus shall be introduced in the form of phosphor-tin or phosphor-copper. Castings shall be sound, clean and free from blow holes, porous places, cracks and other defects.

804. There shall be four grades:

Grade A, for use under pressures exceeding 2000 lbs. per square inch, as for center-bearing discs of draw bridges.

Grade B, for use in low-speed bearings under pressures of 2000 lbs. per square inch, or less, such as trunnions and journals of bascule and lift bridges and for the end thrust washers of the cone rollers of roller-bearing turntable centers.

Grade C, for use in ordinary machinery bearings.

Grade D, for use in gears, worm-wheel rims, nuts transmitting motion, and similar parts that are subject to stresses other than compression.

805. The chemical and physical qualities desired are given in the following table:

	A	B	C	D
Copper.....Per cent.	79 to 81	82 to 84	79 to 81	87 to 89
Tin..... " "	19 to 21	16 to 18	9 to 12	8 to 11
Lead..... " "	.....	.....	8 to 11	.....
Zinc..... " "	.....	.....	.....	1 to 3
Phosphorus..... " "	1.00 max.	1.00 max.	0.25 max.	0.25 max.
Other elements..... " "	0.50 max.	0.50 max.	0.50 max.	0.50 max.
Yield point in compression, pounds per square inch.....	24,000 min.	18,000 min.	12,000 min.	.....
Permanent set under 100,000 lbs. per square inch, compression in inches.....	0.12 max.	0.25 max.	.....	.....
Tensile strength, minimum in pounds per square inch.....	.....	.....	.....	30,000
Elongation in 2 in., per cent.....	.....	.....	.....	13 min.

806. The yield point in compression shall be determined as the load which produces a permanent set of 0.001 in. in the compression-test specimen described in Art. 807.

807. Compression-test specimens shall be cylinders 1 sq. in. in cross-sectional area and 1 in. high.

808. Tension-test specimens for Grade D bronze shall be turned to the dimensions of the standard ½-in. by 2-in. test specimen of the American Society for Testing Materials. They should be cast slightly larger than the finished specimen. The central part should be $\frac{5}{8}$ in. in diameter and the ends $\frac{13}{16}$ in. in diameter.

The specimens should not be cast attached to the castings they represent. They should be subjected to the same heat-treatment, if any, as the castings they represent. They should be cast in green sand with moisture below 12 per cent, or in dry sand.

809. Babbitt metal.—Babbitt metal shall be a homogeneous alloy. It shall be made from new metal, except that clean scrap of known compositions may be used. Its composition shall be as follows:

Tin	2 parts by weight
Zinc	1 part by weight
Antimony	5 per cent. of the weight of tin and zinc

9. WORKMANSHIP

901. General.—All workmanship shall be equal to that of the best practice in modern machine shops. As the parts of the operating machinery of movable bridges are generally exposed to the weather, the finish shall be confined to bearing, rotating, and sliding surfaces, and wherever required to produce accurate fits and precise dimensions.

902. Castings.—All castings shall be cleaned and all fins and other irregularities removed so that the castings will have clean, smooth surfaces, suitable for this class of work. All castings that are attached to structural steelwork, other castings, or the masonry, shall have their contact surfaces finished. All unfinished edges of bases, ribs, and similar parts shall be neatly cast with suitable rounded corners, and all inside angles shall have ample fillets.

903. Planing girders.—The edges of the web plates of swing-bridge drum girders shall be planed and the flange angles set exactly flush with the planed edges, or if preferred the bottoms of the web plates and the outstanding lower flange angles may be planed after the girders are riveted.

In built track and segmental girders of rolling-lift bridges the edges of the webs and side plates and the backs of the outstanding flange angles shall be planed, so that a full bearing on the tread plates will be secured.

904. Tread plates.—In rolling-lift bridges the top and bottom surfaces of the tread plates shall be planed.

905. Rack and track.—Track segments shall be planed on the top and bottom and at the joints. Surfaces on which conical rollers bear shall be planed to the exact bevel required. A line shall be scribed on the bearing surface to which the outer ends of the rollers may be set.

The toothed rack segments shall be accurately fitted and the segments selected so as to produce the correct pitch of the teeth across each joint.

The pitch line shall be scribed on the upper ends of the teeth. On swing bridges, when the rack is cast separately from the track, the rack and track segments shall be accurately fitted together in the shop and properly match-marked before being taken apart for shipment.

The contact surfaces of the rack and track shall both be accurately planed.

906. Balance wheels.—The balance wheels of center-bearing swing bridges, unless of chilled cast iron, shall be turned on their treads and the circumferential edges rounded.

The hubs shall be bored and faced.

907. Rollers.—Rollers for rim-bearing turntables and similar purposes shall be turned on their circumference to accurate bevels and also on the faces of the rim; the circumferential corners of the rim shall be rounded.

The hubs shall be accurately bored and the ends faced.

908. Centers and pivot stands.—The pivot, oil box, and pivot stand shall be accurately finished, particular care being taken to make the bearing surface parallel to and at right angles with the vertical axis of the center.

909. Center discs.—The lower, or steel, disc, shall preferably be made from soft open-hearth steel, forged and turned to closely approximate dimensions and then case-hardened and the concave top surface accurately ground and polished. If more economical, the steel disc may be made from medium open-hearth steel, forged and turned to closely approximate dimensions and then hardened and tempered and the concave top surface accurately ground and polished.

The upper, or phosphor-bronze disc, shall be accurately finished and the lower convex surface polished.

The upper disc shall be perforated at its center and the hole threaded for the attachment of a pipe for lubricating the center.

910. Oil grooves in center discs.—If the discs are not more than 36 ins. in diameter, radial straight oil grooves shall be cut on the lower, convex bearing surface of the upper, or phosphor-bronze disc, extending from the central oil hole to the circumference of the disc. If the discs are more than 36 ins. in diameter, three, or more radial oil grooves shall extend from the central oil hole to the circumference of the disc. Each radial oil groove shall have two straight branches extending to the circumference of the disc. The radial grooves and their branches shall be arranged so that the circumferential distances between adjacent oil grooves shall not exceed 15 ins. The oil grooves shall not be less than $\frac{3}{16}$ in. in both width and depth, and the corners at the bearing surface shall be well rounded to a radius not less than two-thirds of the width of the groove. The corners at the bottom of the grooves shall be filleted, all sharp corners being avoided.

911. Main-pinion shaft.—The main-pinion shaft bearings shall be designed with enough shims to provide for underrun or overrun in the diameter of the rack as fitted up in the shop; and when feasible the bearings shall be shipped with the necessary shims in position.

912. Bearings.—The bearing surfaces and the joints between base and cap of bearings shall be finished. The holes in the base and cap shall be drilled.

913. Oil grooves in bearings.—An oil hole shall be drilled in each bearing cap, communicating with a circumferential oil groove on the inside of the cap. The contact corners of the base and cap shall be well rounded to within about $\frac{3}{8}$ in. of the ends of cap and bearing.

Ordinary oil grooves, with well-rounded corners, on the inside of cap and bearing, may be substituted when required by the Engineer.

914. Shafts.—All shafts, trunnions, and axles that are turned down for different diameters shall have suitable fillets at the shoulders.

915. Keys and keyways.—Keys shall be planed and keyways machine-cut. The finish of the keys and keyways shall be such as to give the key a driving fit on the sides. Tapered keys shall bear on the top, bottom and sides; and parallel-faced keys on the sides only.

916. Journals.—Journals, trunnions, and rubbing surfaces shall be polished after they are machined.

917. Oil grooves in trunnions.—The oil grooves in trunnions and similar large journals shall be machine-cut. Small inequalities may be removed by chipping and filing. The edges and bottom corners of the grooves shall be well-rounded. The radius of the rounded edge shall not be less than one-half of the width of the grooves, with a maximum of $\frac{1}{4}$ in.

918. Gears.—All gears moving with a pitch circumferential velocity of over 300 ft. per minute shall have machine-cut teeth. Gears with uncut cast teeth shall have their external circumferential faces turned when necessary, to remove any overrun in diameters.

The pitch circle shall be scribed on all gears.

919. Shrouded pinions.—At the juncture of the shrouds and teeth of shrouded pinions, cleaning, chipping or other means shall be used to insure the proper meshing of the pinion and gear or rack teeth.

920. Sheaves.—The grooves in sheaves shall be turned. The shape of the grooves shall conform as closely as feasible to the rope section, so that, while the ropes will run freely in the grooves, the sides of the grooves will prevent the wire ropes from flattening under static loads, as when supporting counterweights.

The hubs shall be accurately bored and the ends faced.

921. Hubs.—Hubs of wheels, gears, couplings, etc., shall be bored to close fits on the shafts or axles. If the hub acts as a collar, the end next to the bearing shall be faced. Care shall be taken to bore the hubs of all pinions and gears concentrically with their pitch circles.

922. Worms and wheels.—The threads of worms and the teeth of worm wheels shall be machine-cut.

923. Air buffers.—The workmanship on air buffers shall be so accurate that the weight of the cylinder and its attachment will be sustained by the confined air for five minutes, with a piston travel of not more than that which occurs during the closure of the bridge. During the test the valves must be closed and the buffers balanced so that the whole weight will be carried by the piston rod.

924. Bolts.—Unfinished bolts shall usually have a clearance of $\frac{1}{16}$ in. in the bolt holes. All finished bolts shall fit the drilled or reamed holes with the tolerances allowed in good machine work.

925. Oil and grease cups.—Adequate oil or grease cups, of appropriate size, shall be provided for all important bearings. Grease cups are preferred for vertical bearings.

926. Protection for shipment.—All finished bearing surfaces of the machinery shall be coated with white lead and tallow before shipment. The bearing surfaces of trunnions, and similar heavy axles and shafts, shall be protected by wooden lagging securely attached. The bearings of other shafts and similar parts shall be covered with burlap or by lagging. All electrical apparatus shall be thoroughly protected by crates or boxes. Electric motors that are not built to operate fully exposed to the weather shall be well protected for shipment.

10. ERECTION

1001. Protection of parts.—Parts that are protected from the weather in the finished structure shall also be protected during erection by housing or similar means. This applies particularly to electrical apparatus.

1002. Wire ropes.—Wire ropes shall be kept free from dirt, cinders, and sand. During erection, the ropes shall not be pulled through dirt nor bent into sharp angles and kinks.

1003. Camber.—When swing bridges are in process of erection, care shall be taken to set the camber blocking to the necessary heights, so that the span will be assembled to the proper camber curves when the trusses are in the unstressed condition. When the camber blocking is struck and the span swung, the rivets in all the main truss connections shall have been driven or the joints securely pinned and bolted, so that no slip will occur in the connections.

1004. Erection of machinery.—The alignment and adjustment of the machinery shall be done by skilled mechanics.

Special attention shall be given to the accurate alignment of trunnion and other large bearings.

1005. Lubrication.—All rotating and sliding parts shall be thoroughly cleaned and well lubricated during erection. All oil and grease cups shall be filled before the operating machinery is tested.

Counterweight and operating ropes shall be given one coat of American Steel and Wire Co.'s Shield Filler, or an equivalent lubricant. The Shield Filler should be heated and applied when the atmospheric temperature is above 40° F.

1006. Testing.—When completed, the operating machinery shall be tested by several complete cycles of operation. All bearings shall be inspected to detect heating of parts, and any defects found in the adjustment of the machinery shall be corrected.

1007. Counterweights.—Before a concrete counterweight is built, careful field tests shall be made with the materials that are to be used, and the results sent at once to the engineer responsible for the shop drawings, to serve as the basis of any necessary recalculations of the balancing of the bridge. The size of the test blocks and the time allowed for drying them before they are weighed shall be chosen so that the blocks will represent the final weight of the concrete in the finished counterweight as closely as possible.

During construction the materials shall be weighed and sample blocks made at frequent intervals in order to make sure that the actual counterweight will be of the correct weight. If the counterweights are adjustable, the adjustment shall be made during the tests of the operating machinery and so that the bridge will operate smoothly. The final adjustment shall be made in calm weather and when there is no snow or ice on the moving span.

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APPENDIX B

SHEAVES LOADED BY ROPES UNDER TENSION

1. Theory.—Figure BA, is a diagram of a rope under tension wrapped around a sheave.

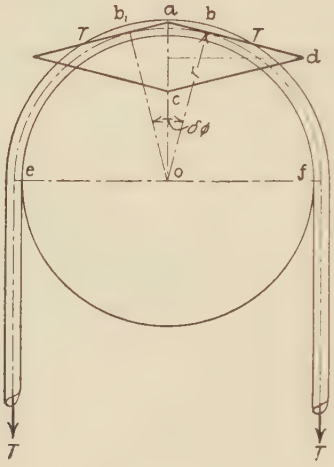


FIG. BA.

Let T = the tension in the rope;

r = the radius of the sheave,

and

p = the intensity of the normal pressure of the rope against the groove of the sheave.

Find the value of p in terms of T and r.

Let b_1b be a very small length of rope,

and

$\delta\phi$ be the angle subtended at the center by b_1b.

The forces acting on b_1b are the two tangents T and T and the resultant normal pressure, ac, of the rope upon the sheave. This is $pr\delta\phi$, very nearly.

From the triangle of forces, acd,

$$\frac{pr\delta\phi}{2} = T \sin \frac{\delta\phi}{2}, \text{ or } p = \frac{T \times 2 \sin \frac{\delta\phi}{2}}{r\delta\phi},$$

very nearly.

At the limit, $\delta\phi$ diminishes, and the expression becomes

$$p = \frac{T}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The length of an element of the circumference is $rd\phi$.
The pressure on this element is $prd\phi$.
The vertical component of this pressure is

$$dP_v = pr \cos \phi d\phi.$$

Integrate this between the limits of ϕ_2 and ϕ_1, for an arc measured in either direction from the vertical through O, and remember that $pr = T$.

$$P_v = pr \int_{\phi_1}^{\phi_2} \cos \phi d\phi = T \Big[\sin \phi \Big]_{\phi_1}^{\phi_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the same manner the horizontal component becomes

$$P_h = pr \int_{\phi_1}^{\phi_2} \sin \phi d\phi = T \Big[- \cos \phi \Big]_{\phi_1}^{\phi_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Assume that the rope is wrapped 180° around the sheave, and find the vertical reaction at the trunnion and the compression within the sheave as a whole.

$$\frac{R_T}{2} = T \Big[\sin \phi \Big]_0^{\frac{\pi}{2}} = T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This value of $\frac{R_T}{2}$ is for one quadrant; hence,

$$R_T = 2T.$$

Similarly,

$$H_T = T \Big[- \cos \phi \Big]_0^{\frac{\pi}{2}} = -T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

There are two equal and opposite forces applied to the sheave in the horizontal direction, each of the numerical value of T.

Equations (1), (2), (3), (4), and (5) give the external forces on a sheave carrying a rope under a tension T.

2. Example.—Assume a counterweight-rope sheave with 12 arms, as shown in Fig. BB. Assume a unit tension in the rope and find the vertical and horizontal components of the pressure applied by the tension in the rope at each apex-joint at the ends of the arms. Vertical components:

Joint	ϕ_2°	ϕ_1°	sin ϕ_2°	$-$ sin ϕ_1°	= vertical component
5–6– 7	15	0	0.2588	0.0000	$0.2588 \times 2 = 0.5176$
3–4– 5	45	15	0.7071	0.2588	0.4483
1–2– 3	75	45	0.9659	0.7071	0.2588
23–1–24	90	75	1.0000	0.9659	0.0341
Total on each side of the vertical.....					1.0000

Horizontal components:

Joint	$\phi_2°$	$\phi_1°$	$-\cos\phi_2°$	$+\cos\phi_1°$ =	horizontal component
5–6– 7	15	0	0.9659	1.0000	0.0341
3–4– 5	45	15	0.7071	0.9659	0.2588
1–2– 3	75	45	0.2588	0.7071	0.4483
23–1–24	90	75	0.0000	0.2588	0.2588
Total on each side of the vertical.....					1.0000

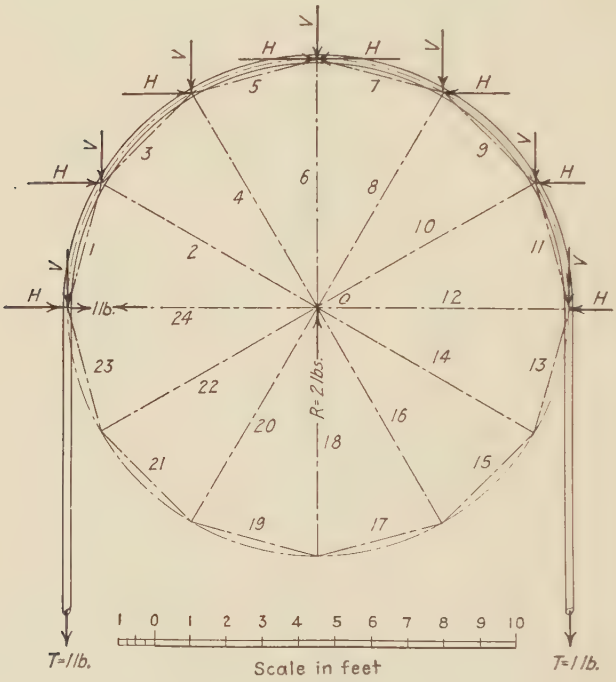


FIG. BB.

The vertical and horizontal components of the pressures applied at the joints have the same values, respectively, at joints symmetrically located about the vertical plane through the center of the sheave.

3. Stresses in the arms and rim of a counterweight sheave.—In Fig. BB, join the apices at the circumference of the sheave by lines and consider the polygon with twelve sides thus formed. If the sectional areas of the rim and arms are determined by some approximate method, the actual stresses can be found by an application of the theory of elastic structures with redundant members. New sectional areas are then found and the stresses corrected by repeating the process as many times as may be found necessary.

The stress in a redundant member may be found from the relation:

$$S = -\frac{\sum \frac{Pul}{EA}}{\sum \frac{u^2 l}{EA}} \quad \ldots \ldots \ldots \ldots \quad (6)$$

Refer to Vol. I, Chapter VII, Eq. (25).

In any ordinary sheave, E will be constant for all parts; hence,

$$S = -\frac{\sum \frac{Pul}{A}}{\sum \frac{u^2 l}{A}} \quad \ldots \ldots \ldots \ldots \quad (7)$$

Apply this method to the large counterweight sheave shown in Fig. BC. The vertical and horizontal components at each apex-joint, from the pressure of the rope against the rim, due to a tension of unity in the rope are given in Art. 2.

Assume that arm 24 is removed, and find the stresses in all other arms and the rim-chords, due to a rope tension of unity.

Assume that arm 24 is removed and replaced by a tension of unity, and find the values of u for all arms and the rim-chords.

Apply Eq. (7) to each member and sum the results. This process is conveniently performed in the manner given in Table BA.

The value of l for the rim was taken as the length of the chord measured on the center of gravity of the rim. For the arms, l was taken as the length from the center of gravity of the rim to the inside of the hole in the trunnion. The area of the arms used was an average area of the arm, the part of the hub adjoining the arm, and a wedge-shaped portion of the trunnion opposite the arm; for all of these parts will be distorted by stresses in the arms.

This sheave was designed to carry 16 plow-steel wire ropes $2\frac{1}{4}$ ins. in diameter, working under a tension of 60,000 lbs. in each rope, a total of 960,000 lbs. on each side of the sheave. The total stresses given in Column 11 were calculated by multiplying each value of S in Column 10 by 960,000. All calculations were made by a slide rule.

Column 12 gives the areas of the rim sections midway between the arms, and those of the arms near the hub and near the rim, adjacent to the curves connecting the webs of the arms with each other and with those of the rim. Column 13 shows the direct unit stresses, being Column 11 divided by Column 12. Column 14 gives the calculated secondary stresses. Those in the rim are so small that they do not affect the design. The secondary stresses given for the arms are those at their ends, and they diminish to the points of tangency of the web of the arms and the curves joining the arms, and those joining the arms and the rim-webs. The greatest secondary stresses occur in the horizontal arms, 12 and 24, in which the primary stresses are much smaller than those in arms 2 to 10. The calculations made in this case indicate that secondary stresses may safely be ignored when designing sheaves of this type.

The chords of the rim are all in direct compression. As the rim is curved between the arms, each segment also is subjected to negative bending moments. The forces are the direct stresses at the apices, and the lever arms are the ordinates between

TABLE

1 Member	2 l	3 A	4 P	5 u	6 $\frac{ul}{A}$	7 $\frac{Pul}{A}$	8 $\frac{u^2l}{A}$
1	41.79	102.4	−.4824	−1.9320	−0.7884	+0.3803	+ 1.5232
3	41.79	102.4	−.4824	−1.9320	−0.7884	+0.3803	+ 1.5232
5	41.79	102.4	−.4824	−1.9320	−0.7884	+0.3803	+ 1.5232
7	41.79	102.4	−.4824	−1.9320	−0.7884	+0.3803	+ 1.5232
9	41.79	102.4	−.4824	−1.9320	−0.7884	+0.3803	+ 1.5232
11	41.79	102.4	−.4824	−1.9320	−0.7884	+0.3803	+ 1.5232
Total...	.....	.....	.....	.....	.....	+2.2818	+ 9.1392
13	41.79	102.4	−.5176	−1.9320	−0.7884	+0.4081	+ 1.5232
15	41.79	102.4	−.5176	−1.9320	−0.7884	+0.4081	+ 1.5232
17	41.79	102.4	−.5176	−1.9320	−0.7884	+0.4081	+ 1.5232
19	41.79	102.4	−.5176	−1.9320	−0.7884	+0.4081	+ 1.5232
21	41.79	102.4	−.5176	−1.9320	−0.7884	+0.4081	+ 1.5232
23	41.79	102.4	−.5176	−1.9320	−0.7884	+0.4081	+ 1.5232
Total...	.....	.....	.....	.....	.....	+2.4486	+ 9.1392
2	77.25	76.2	−.2679	+1.0000	+1.0137	−0.2716	+ 1.0137
4	77.25	76.2	−.2679	+1.0000	+1.0137	−0.2716	+ 1.0137
6	77.25	76.2	−.2679	+1.0000	+1.0137	−0.2716	+ 1.0137
8	77.25	76.2	−.2679	+1.0000	+1.0137	−0.2716	+ 1.0137
10	77.25	76.2	−.2679	+1.0000	+1.0137	−0.2716	+ 1.0137
Total...	.....	.....	.....	.....	.....	−1.3580	+ 5.0685
14	77.25	76.2	+.2679	+1.0000	+1.0137	+0.2716	+ 1.0137
16	77.25	76.2	+.2679	+1.0000	+1.0137	+0.2716	+ 1.0137
18	77.25	76.2	+.2679	+1.0000	+1.0137	+0.2716	+1.0137
20	77.25	76.2	+.2679	+1.0000	+1.0137	+0.2716	+ 1.0137
22	77.25	76.2	+.2679	+1.0000	+1.0137	+0.2716	+ 1.0137
Total...	.....	.....	.....	.....	.....	+1.3580	+ 5.0685
12	77.25	76.2	0	+1.0000	+1.0137	0	+ 1.0137
24	77.25	76.2	0	+1.0000	+1.0137	0	+ 1.0137
Total...	.....	.....	.....	.....	.....	.....	+ 2.0274
Totals...	.....	.....	.....	.....	.....	+4.7304	+30.4428

$$S_{24} = -\frac{4.7304}{30.4428} = -0.1554.$$

BA

9 $S_{24}u$	10 S	11 Total Stress	12 Area of Section for Stresses	13 Direct Unit Stresses	14 Secondary Unit Stresses	
+.3002	−.1822	−174,900	102.4	−1725	Very small	
+.3002	−.1822	−174,900	102.4	−1725	Very small	
+.3002	−.1822	−174,900	102.4	−1725	Very small	
+.3002	−.1822	−174,900	102.4	−1725	Very small	
+.3002	−.1822	−174,900	102.4	−1725	Very small	
+.3002	−.1822	−174,900	102.4	−1725	Very small	
+.3002	−.2174	−208,700	102.4	−2058	Very small	
+.3002	−.2174	−208,700	102.4	−2058	Very small	
+.3002	−.2174	−208,700	102.4	−2058	Very small	
+.3002	−.2174	−208,700	102.4	−2058	Very small	
+.3002	−.2174	−208,700	102.4	−2058	Very small	
+.3002	−.2174	−208,700	102.4	−2058	Very small	
					At Rim	At Hub
−.1554	−.4233	−406,300	67.88 Hub 60.88 Rim	−5085 −6675	∓ 570	∓ 640
−.1554	−.4233	−406,300	67.88 Hub 60.88 Rim	−5085 −6675	± 120	± 50
−.1554	−.4233	−406,300	67.88 Hub 60.88 Rim	−5085 −6675	0	0
−.1554	−.4233	−406,300	67.88 Hub 60.88 Rim	−5085 −6675	± 120	± 50
−.1554	−.4233	−406,300	67.88 Hub 60.88 Rim	−5085 −6675	∓ 570	∓ 640
−.1554	+.1125	+108,000	67.88 Hub 60.88 Rim	+1591 +1774	∓ 525	∓ 650
−.1554	+.1125	+108,000	67.88 Hub 60.88 Rim	+1591 +1774	± 120	± 50
−.1554	+.1125	+108,000	67.88 Hub 60.88 Rim	+1591 +1774	0	0
−.1554	+.1125	+108,000	67.88 Hub 60.88 Rim	+1591 +1774	± 120	± 50
−.1554	+.1125	+108,000	67.88 Hub 60.88 Rim	+1591 +1774	∓ 525	∓ 650
−.1554	−.1554	−149,150	67.88 Hub 60.88 Rim	−2197 −2450	∓1485	∓1225
−.1554	−.1554	−149,150	67.88 Hub 60.88 Rim	−2197 −2450	∓1485	∓1225

For each member $S = P + S_{24}u$, from which Column 10 was calculated.

the chords and the arcs joining the apices. Each rim segment above the center of the sheave, further, is subjected to positive bending moments due to the normal pressures of the ropes. No appreciable error will be introduced if these forces are

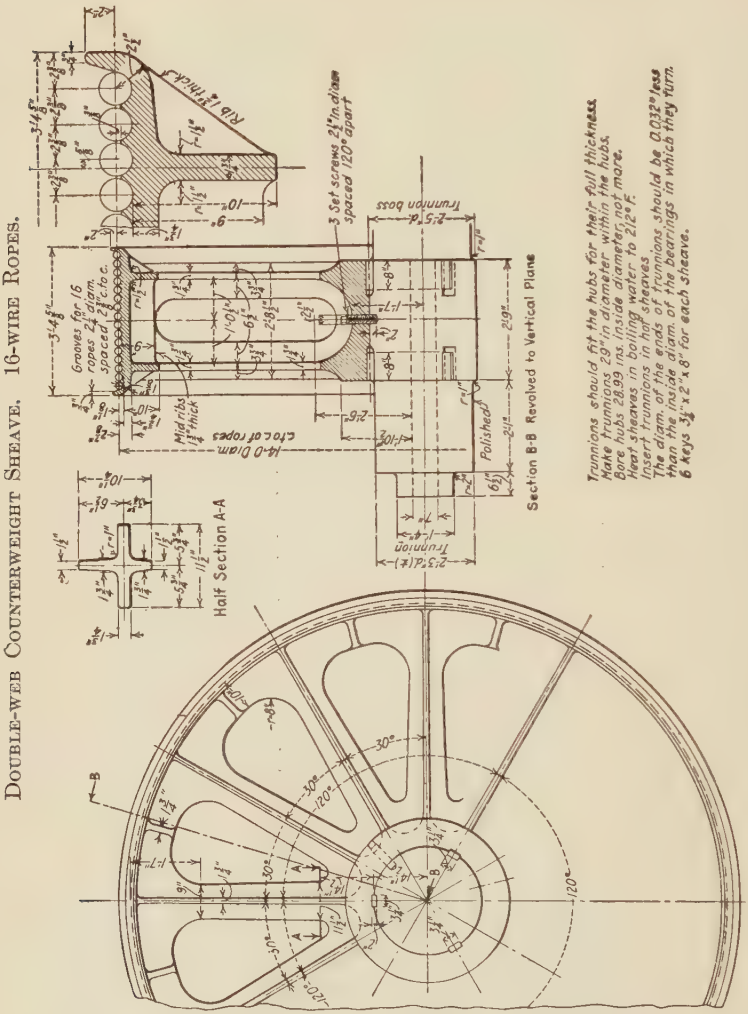

Double-web Counterweight Sheave. 16-wire Ropes.
Grooves for 16 ropes 2¼ diam. spaced 2¾ c. to c.
Rib 1¾ thick
3 Set screws 2½ in diam spaced 120° apart
Trunnion boss
Midribs 1¾ thick
c to c of ropes
Polished
Section B-B Revolved to Vertical Plane
Trunnions should fit the hubs for their full thickness
Make trunnions .29" in diameter within the hubs.
Bore hubs 28.99 ins. inside diameter, not more.
Heat sheaves in boiling water to 212° F.
Insert trunnions in hot sheaves
The diam. of the ends of trunnions should be .0.032" less than the inside diam. of the bearings in which they turn.
6 Keys ⅞ x 2 x 8" for each sheave.
Half Section A-A
Estimated weight of one cast steel sheave 42,100 lbs.
" " " one forged steel trunnion 44,000 lbs.
" " " six tool steel keys 90 lbs.

Fig. BC.

considered to be normal to the chords joining the apices and not normal to the arcs subtended by the chords.

The barrel of the rim of a sheave, of the type shown in Fig. BC, requires special attention. The part between the arm-webs and two consecutive arms acts somewhat like a flat plate partially anchored on its sides and ends, and subjected to a nearly uniform load from the radial rope pressures. Also, any sectional element of the

rim, as shown in Fig. BC, at BB, is similar to a beam supported by the webs and with overhanging cantilever ends, all loaded by the normal rope pressures.

The hub should be investigated for elastic distortions tending to enlarge the diameter of the hole bored for the trunnion. The stresses in the horizontal arms, and those above them, all tend to compress the wall of the hub and, owing to lateral distortions (Poisson's Ratio), this results in increasing the circumference of the hub. The arms below the horizontal are in tension, and tend directly to increase the circumference, but the lateral distortions here tend to reduce it. Further, the trunnion is compressed by its load, tending to add to the trouble.

In practice, the neglect of these distortions in the wall of the hub, together with ordinary pressed fits between the trunnion and the hub, have resulted in such a loose fit that there has been an actual gap between the bottom of the trunnion and the inside of the hub. This condition has resulted in the loosening of the keys and the gradual destruction of the key seats.

In cases of large sheaves with many ropes, it will usually be found that pressed fits cannot be depended upon to force an increased internal diameter of the hub great enough to overcome the enlargement due to the elastic distortions. When the trunnion is made too large, the metal is abraded during the pressing operation. In such cases the sheave should be uniformly heated in water, or by other effective means, and the trunnion inserted. The cooling of the sheave will then cause it to grip the trunnion with almost any desired force. In this way more than the theoretical adjustment in diameters may be made.

In some cases the hubs of large sheaves have been recessed so that the trunnion bore against the inside of the hub only at and near the ends. This has probably been done to reduce the calculated bending moment on the trunnion and to reduce its diameter. This should never be done; the hub should fit the trunnion for its entire length. When the recess is used, the load on the sheave is carried on the two bearings near the ends of the hub. Any deflection of the trunnion causes its upper fibers to shorten and its lower ones to lengthen. This causes a relative movement at the joint between the inside of the hub and the trunnion, which develops a tendency of the sheave to move along the trunnion. This is avoided when the trunnion fits the full length of the hub. Even in the latter case, dowel bolts, or some other effective means, should be used to make sure that no longitudinal movement takes place.

When the design of a large cast sheave has been made with due attention to all theoretical requirements, it will usually be found that some of its parts are too light and too thin for successful foundry casting. The cooling stresses are large, and excellent judgment is required in determining the final sizes. While the hub must be of ample thickness to provide for stresses and distortions, no extra metal should be used, for, at best, it is a large reservoir for heat, and cools the last of all parts, with resulting internal cooling stresses.

All cast sheaves should be thoroughly annealed.

When multiple sheaves with only a few ropes on each are used, all of the difficulties mentioned in this article are greatly reduced. The trunnions become small, with a large reduction in work lost in friction; repairs are simplified; and sound castings are more readily made.

Reference should be made to Chapter XI for the design of sheaves of ordinary size and for only a few ropes on each.

4. Forced fits of hubs on trunnions or shafts.—The preceding article calls attention to the fact that the hubs of large sheaves must be forced upon their trunnions

in order to make sure that the elastic distortions of the hubs, due to the large stresses within them, will not loosen the hubs on the trunnions. There are many other cases in the design of machinery in which forced fits are necessary. It is desirable to discuss this subject further.

There are three classes of forced fits applicable to ordinary machinery, namely:

1. Drive fits.
2. Press fits.
3. Shrink fits.

The General Electric Co. has adopted allowances for various types of machinery.[1] An analysis of these allowances for cases common in movable-bridge machinery shows that the allowances used by the General Electric Co. may be closely expressed by simple formulae.

Let d = the diameter of a shaft,
and
a = the amount by which the diameter of the shaft should be increased to provide for forced fits, then:

1. For drive fits,

$$a = 0.0008\sqrt{d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

2. For press fits,

$$a = 0.0016\sqrt{d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

3. For shrink fits,

$$a = 0.0021\sqrt{d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

For many cases these allowances appear to be too large, for they cause large hoop-tensional and radial-compressive stresses in the hubs. In some cases, particularly for small shafts, these stresses seem to be too great for the best practice.[2]

Mr. C. F. MacGill concludes from his experience that the allowances for forced fits should be from 0.002 to 0.004 in. for steel shafts pressed into steel hubs, and from 0.003 to 0.005 in. for steel shafts pressed into cast-iron hubs; and that these allowances should not increase with the diameter of the shafts.

In some cases it is necessary to calculate the stresses in the hubs caused by pressing the shafts into them. In addition to the notations given:

Let E = Young's modulus;
T = the unit hoop-tension in the hub;
P = the radial unit compressive stress in the hub at its inside surface;
S = the combined maximum unit shearing stress at the inside of the hub;
r_i = the inside radius of the bore in the hub; and
r_0 = the outside radius of the hub.

[1] Kent's "Mechanical Engineers' Handbook," Tenth Edition, p. 1586.

[2] "Shrinkage and Forced Fits and the Stresses They Produce," by A. Lewis Jenkins, *Eng. News*, March 17, 1910, p. 310. "A Record of Pressed Fits," by C. F. MacGill, *Trans. A. S. M. E.*, Vol. 35, p. 819, 1913; also Discussion by Sanford A. Moss, p. 831. "Strength of Materials," by Arthur Morley, London, 1921, p. 341.

If the shaft and the hub are both of the same material, the following relations may be deduced:

$$S = T + P = \frac{Ea}{d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\frac{P}{T} = \frac{r_0^2 - r_i^2}{r_0^2 + r_i^2} = m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

If $\frac{P}{T} = m$, $P = Tm$, then

$$S = T(1 + m), \quad \text{or} \quad T = \frac{S}{1 + m}. \quad . \quad . \quad . \quad . \quad (13)$$

From these relations the maximum unit stresses in the hub may be calculated. They occur at the inside surface of the bore in the hub.

In cases in which the allowance is determined from calculations of the elastic distortion of the hubs due to stresses from the arms or web of large wheels, like that of a large counterweight sheave upon its trunnion, the unit stresses should be found. If they are too large, the thickness of the wall of the hub can be increased, thus diminishing the increase in its internal diameter caused by the elastic distortions due to the stresses from the arms or web.

APPENDIX C

TABLES OF MISCELLANEOUS DATA

1. Dynamics.
2. Properties of beams.
3. Mensuration.
4. Centroids.
5. Centers of gravity.
6. Elements of lines.
7. Elements of sections.
8. Polar elements of sections.
9. Elements of solids.
10. Polar elements of solids.
11. Miscellaneous data.

TABLE 1

Dynamics

Mass, acceleration, velocity.—The following simple relations will frequently be required in the design of machinery:

Let W = the weight of a body, in pounds;

M = the mass of the body;

g = the acceleration due to gravity, in feet per second per second. The value of g is usually taken as 32.2, but 32.16 is more closely the average value for the United States;

a = the acceleration, in feet per second per second, for any constant force;

f = the accelerating force, in pounds;

t = the time during which acceleration occurs, in seconds;

v = the velocity acquired in the time t;

s = the distance through which the mass moves, in feet.

Finite Relations	Differential Relations
$M = \frac{W}{g} = \frac{W}{32.2}$	
$f = Ma = \frac{Wa}{g}$	$f = M\frac{dv}{dt}$
$a = \frac{v}{t}$	$a = \frac{dv}{dt}$ or $\frac{d^2s}{dt^2}$
$ft = Mv$	$v = \frac{ds}{dt}$ or $ds = vdt$
$f = \frac{Mv}{t} = \frac{Wv}{gt}$	$vdv = ads$
$s = \frac{vt}{2}$	
$v = \frac{s}{t}$ (uniform)	
Work $= fs = \frac{Wv}{gt} \times \frac{vt}{2}$	Work $= \int (df \times ds)$
Kinetic energy $= \frac{Wv^2}{2g} = \frac{Wv^2}{64.4} = \frac{1}{2}Mv^2$	

Work.—Work is the overcoming of resistance through a certain distance. It is measured by the product of the resistance and the path through which it is overcome. Work is also measured by the product of the moving force and the distance through which the force acts in overcoming the resistance. The common unit of work is the foot-pound.

Power.—Power is the time-rate at which work is performed, as for example, foot-pounds per second. The common units of power are the horse-power (h.p.) and the kilowatt (kw.). Their relative values are:

1 h.p. = 550 ft.-lbs. per second;

1 h.p. = 33,000 ft.-lbs. per minute = 0.7457 kw.;

1 kw. = 737.56 ft.-lbs. per second;

1 kw. = 44,254 ft.-lbs. per minute = 1.341 h.p.;

1 h.p.-hour = 0.7457 kw.-hour;

1 kw.-hour = 1.341 h.p.-hours.

Man-power.—A man can exert a force of 40 lbs. on a capstan-lever and walk at a rate of 160 ft. per minute.

1 man-power = 6400 ft.-lbs. per minute = 0.1939 h.p. = 0.1446 kw.

A man can exert the same power on a suitable crank for a short time.

A man can exert a force of 15 lbs. on a crank at a rate of 220 ft. per minute for several hours.

1 man-power = 3300 ft.-lbs. per minute = 0.1 h.p. = 0.07457 kw.,
for continuous service during several hours.

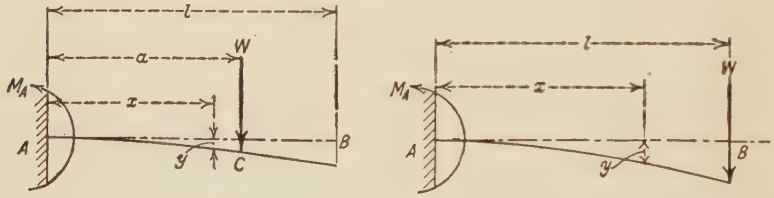
TABLE 2

MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION

W = the total load on the beam, excluding the weight of the beam;
I = the moment of inertia of the beam;
E = Young's modulus;
Tan θ = the slope of the elastic line;
y = the deflection of the beam at any point;
Δ = the maximum deflection;
f = the safe working extreme fiber stress;
c = the distance of the extreme fiber from the neutral axis.

TABLE 2

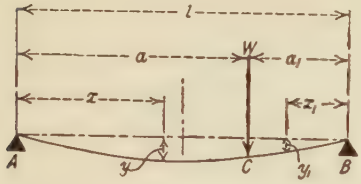
MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION

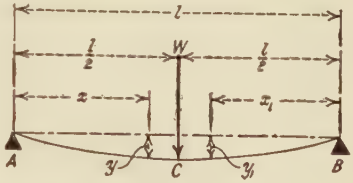


Reaction R_A	W	W
Moment M_A	$-Wa$	$-Wl$
Moment M_x	$-W(a-x)$	$-W(l-x)$
Tan θ	$-\frac{Wx}{2EI}(2a-x)$	$-\frac{Wx}{2EI}(2l-x)$
Tan θ at C	$-\frac{Wa^2}{2EI}$	At B, $-\frac{Wl^2}{2EI}$
Eq. of elastic line A to C	$y=-\frac{Wx^2}{6EI}(3a-x)$	A to B, $y=-\frac{Wx^2}{6EI}(3l-x)$
Eq. of elastic line C to B	$y=-\frac{Wa^2}{6EI}(3x-a)$	
Deflection of B	$\Delta=-\frac{Wa^2}{6EI}(3l-a)$	$\Delta=-\frac{Wl^3}{3EI}$
Deflection at B	$\Delta=-\frac{1}{6}\cdot\frac{f(3l-a)a}{Ec}$	$\Delta=-\frac{1}{3}\times\frac{fl^2}{Ec}$
Max. working load	$W=\frac{fI}{ca}$	$W=\frac{fI}{cl}$

TABLE 2—*Continued*

MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION

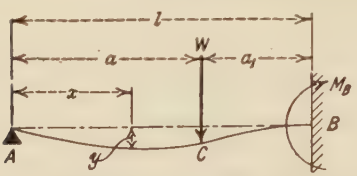


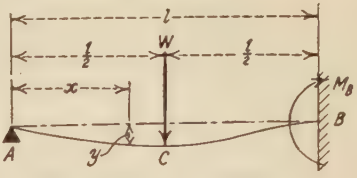


Reaction R_A	$\frac{Wa_1}{l}$	$\frac{W}{2}$
Reaction R_A	$\frac{Wa}{l}$	$\frac{W}{2}$
Moment at left of the load	$\frac{Wa_1}{l}x$	$\frac{Wx}{2}$
Moment at right of the load	$\frac{Wa}{l}x_1$	$\frac{Wx_1}{2}$
Max. moment at the load	$\frac{Wa_1a}{l}$	$\frac{Wl}{4}$
Tan θ left of the load	$\frac{Wa_1}{6EIl}(3x^2-l^2+a_1^2)$	$\frac{W}{16EI}(4x^2-l^2)$
Tan θ right of the load	$\frac{Wa}{6EIl}(3x_1^2-l^2+a^2)$	$\frac{W}{16EI}(4x_1^2-l^2)$
Max. tan θ is at $x=0$	$-\frac{Wa_1}{6EIl}(l^2-a_1^2)$	$-\frac{Wl^2}{16EI}$
Eq. of elastic line left of load, A to C	$y=\frac{Wa_1}{6EIl}[x^3-(l^2-a_1^2)x]$	$y=\frac{W}{48EI}(4x^3-3l^2x)$
Eq. of elastic line right of load, B to C	$y_1=\frac{Wa}{6EIl}[x_1^3-(l^2-a^2)x_1]$	$y_1=\frac{W}{48EI}(4x_1^3-3l^2x_1)$
Max. deflection at $x=a\sqrt{\frac{1}{3}+\frac{2}{3}\frac{a_1}{a}}$	$\Delta=-\frac{Wa^2a_1^2}{9EIl}\sqrt{\frac{1}{3}+\frac{2}{3}\frac{a_1}{a}}\left[\frac{a}{a_1}+2\right]$	At C, $\Delta=-\frac{Wl^3}{48EI}$
Deflection at C	$\Delta_c=-\frac{Wa^2a_1^2}{3EIl}$	At C, $\Delta=-\frac{1}{12}\times\frac{fl^2}{Ec}$
Max. working load	$W=\frac{lfI}{aa_1c}$	$W=4\times\frac{fI}{cl}$

TABLE 2—*Continued*

MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION

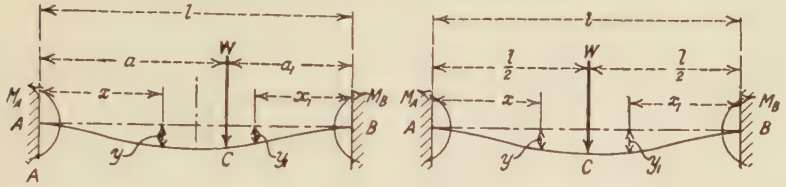




Reaction R_A	$\frac{Wa_1^2(2l+a)}{2l^3}$	$\frac{5}{16}W$
Reaction R_B	$W\left[1-\frac{a_1^2(2l+a)}{2l^3}\right]$	$\frac{11}{16}W$
Moment at left of the load	$\frac{Wa_1^2(2l+a)}{2l^3}x$	$\frac{5}{16}Wx$
Moment at right of the load	$W\left[\frac{a_1^2(2l+a)}{2l^3}x-x+a\right]$	$\frac{5}{16}Wx-W\left(x-\frac{l}{2}\right)$
Max. $+M$ is when $a=\frac{l}{2}(\sqrt{3}-1)$ $=0.366l$	$\frac{Wl}{4}(3\sqrt{3}-\frac{9}{2})$ $=0.1741Wl$	$\frac{5}{32}Wl$ $=0.15625Wl$
Max. $-M$ is when $a=l\sqrt{\frac{1}{3}}=0.577l$	$M_B=-\frac{1}{3}\sqrt{\frac{1}{3}}Wl=-0.192Wl$	$M_B=-\frac{3}{16}Wl=-0.1875Wl$
Tan θ left of the load	$\frac{Wa_1^2}{4EIl}\left[\frac{2l+a}{l^2}x^2-a\right]$	$\frac{W}{32EI}(5x^2-l^2)$
Tan θ right of the load	$\frac{W}{4EIl}\left[\left(\frac{2l+a}{l^2}x^2-a\right)a_1^2-2l(x-a)^2\right]$	$\frac{W}{32EI}[-11x^2+16lx-5l^2]$
Max. tan θ is at A when $a=\frac{1}{3}l$	$-\frac{Wl^2}{27EI}$	At A, $-\frac{Wl^2}{32EI}$
Eq. of elastic line left of load, A to C	$y=\frac{Wa_1^2}{4EIl}\left[\frac{2l+a}{3l^2}x^3-ax\right]$	$y=\frac{W}{96EI}(5x^3-3l^2x)$
Eq. of elastic line right of load, C to B	$y=\frac{W}{4EIl}\left\{\left[\frac{2l+a}{3l^2}x^3-ax\right]a_1^2-\frac{2l}{3}(x-a)^3\right\}$	$y=\frac{W}{96EI}[-11x^3+24lx^2-15l^2x+2l^3]$
When $a>l(\sqrt{2}-1)$ max. defl. is at $x=l\sqrt{\frac{a}{2l+a}}$	$\Delta=-\frac{Wa_1^2}{6EI}\sqrt{\frac{a}{2l+a}}$	
When $x=a=l(\sqrt{2}-1)$ max. deflection	$\Delta=-\frac{Wl^3}{101.35EI}$	At C, $\Delta=-\frac{7Wl^3}{768EI}$
Defl. at C when $a=l(\sqrt{2}-1)$ or $a=0.4142l$	$\Delta=-\frac{1}{17.485}\times\frac{fl^2}{Ec}$	At C, $\Delta=-\frac{7}{120}\times\frac{fl^2}{Ec}$
Max. working load	$W=5.2\times\frac{fI}{cl}$	$W=5.33\times\frac{fI}{cl}$

TABLE 2—*Continued*

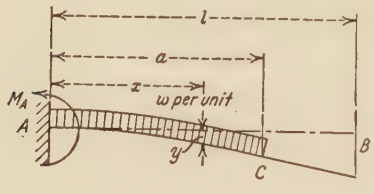
MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION

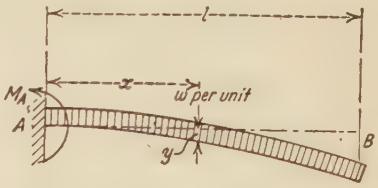


Reaction R_A	$\frac{Wa_1^2}{l^2}\left(1+\frac{2a}{l}\right)$	$\frac{W}{2}$
Reaction R_B	$\frac{Wa^2}{l^2}\left(1+\frac{2a_1}{l}\right)$	$\frac{W}{2}$
Moment at left of the load	$\frac{Wa_1^2}{l^2}\left[\left(1+\frac{2a}{l}\right)x-a\right]$	$\frac{W}{8}(4x-l)$
Moment at right of the load	$\frac{Wa^2}{l^2}\left[\left(1+\frac{2a_1}{l}\right)x_1-a_1\right]$	$\frac{W}{8}(4x_1-l)$
Max. $+M$ is when $a=\frac{l}{2}$	$\frac{Wl}{8}$	$\frac{Wl}{8}$
Max. $-M$ is when $a=\frac{l}{3}$	At A, $M=-\frac{4}{27}Wl$	$a=\frac{l}{2}$ At A and B, $M=-\frac{Wl}{8}$
Tan θ at left of the load	$\frac{Wa_1^2}{2EIl^2}\left[\left(1+\frac{2a}{l}\right)x^2-2ax\right]$	$\frac{W}{8EI}(2x^2-lx)$
Tan θ at right of the load	$\frac{Wa^2}{2EIl^2}\left[\left(1+\frac{2a_1}{l}\right)x_1^2-2a_1x_1\right]$	$\frac{W}{8EI}(2x_1^2-lx_1)$
Max. tan θ is at point of contraflexure when $a=\frac{l}{6}(\sqrt{13}-1)=0.434l$	$-\frac{10Wl^2}{619EI}$	At $x=\frac{l}{4}$, $-\frac{Wl^2}{64EI}$
Eq. of elastic line left of load	$y=\frac{Wa_1^2}{6EIl^2}\left[\left(1+\frac{2a}{l}\right)x^3-3ax^2\right]$	$y=\frac{W}{48EI}(4x^3-3lx^2)$
Eq. of elastic line right of load	$y_1=\frac{Wa^2}{6EIl^2}\left[\left(1+\frac{2a_1}{l}\right)x_1^3-3a_1x_1^2\right]$	$y_1=\frac{W}{48EI}(4x_1^3-3lx_1^2)$
Max. defl. is when $x=\frac{2al}{l+2a}$	$\Delta=-\frac{2Wa^3a_1^2}{3EI(l+2a)^2}$	At C, $\Delta=-\frac{Wl^3}{192EI}$
	$\Delta=\frac{2a^2}{3(l+2a)^2}\times\frac{fl^2}{Ec}$	At C, $\Delta=-\frac{1}{24}\times\frac{fl^2}{Ec}$
Max. working load	$W=6.75\times\frac{fI}{cl}$	$W=8\times\frac{fI}{cl}$

TABLE 2—*Continued*

MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION

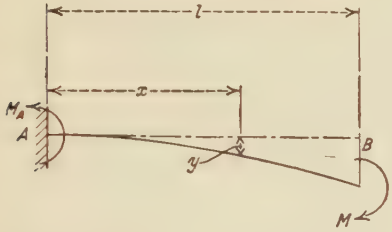


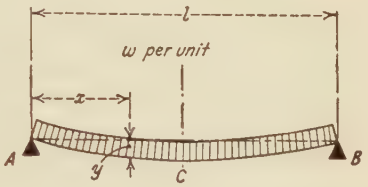


Reaction R_A	W	W
Moment M_A	$-\frac{Wa}{2}$	$-\frac{Wl}{2}$
Moment M_x	$-\frac{W}{2a}(a-x)^2$	$-\frac{W}{2l}(l-x)^2$
Max. moment	At A $\quad -\frac{Wa}{2}$	At A $\quad -\frac{Wl}{2}$
Tan θ	$-\frac{W}{6EIa}(x^3-3ax^2+3a^2x)$	$-\frac{W}{6EIl}(x^3-3lx^2+3l^2x)$
Max. tan θ	At C, $\quad -\frac{Wa^2}{6EI}$	At B, $\quad -\frac{Wl^2}{6EI}$
Eq. of elastic line, A to C	$y=-\frac{W}{24EIa}(x^4-4ax^3+6a^2x^2)$	A to B $\quad y=-\frac{W}{24EIl}(x^4-4lx^3+6l^2x^2)$
Eq. of elastic line, C to B	$y=-\frac{Wa^2}{24EI}(4x-a)$	
Max. deflection at B	$\Delta=-\frac{Wa^2}{24EI}(4l-a)$	$\Delta=-\frac{Wl^3}{8EI}$
Max. deflection at B	$\Delta=-\frac{1}{12}\times\frac{fa(4l-a)}{Ec}$	$\Delta=-\frac{1}{4}\times\frac{fl^2}{Ec}$
Max. working load	$W=2\times\frac{fI}{ca}$	$W=2\times\frac{fI}{cl}$

TABLE 2—*Continued*

MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION

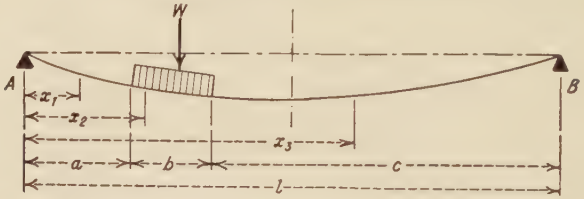




Reaction R_A	$-\frac{M}{l}$	$R_A = R_B$ $\frac{W}{2}$
Moment M_A	$-M$	0
Moment M_x	$-M$	$\frac{W}{2}\left(x-\frac{x^2}{l}\right)$
Max. moment	constant $=-M$	At C, $\frac{Wl}{8}$
Tan θ	$-\frac{Mx}{EI}$	$\frac{W}{24EI}\left(6x^2-4\frac{x^3}{l}-l^2\right)$
Max. tan θ	At B, $-\frac{Ml}{EI}$	At A and B, $\mp\frac{Wl^2}{24EI}$
Eq. of the elastic line	$y=-\frac{Mx^2}{2EI}$	$y=\frac{W}{24EI}\left(2x^3-\frac{x^4}{l}-l^2x\right)$
Max. deflection	At B, $\Delta=-\frac{Ml^2}{2EI}$	At C, $\Delta=-\frac{5Wl^3}{384EI}$
Max. deflection	At B, $\Delta=-\frac{1}{2}\times\frac{fl^2}{Ec}$	At C, $\Delta=-\frac{5}{48}\times\frac{fl^2}{Ec}$
Max. working load	$M=\frac{fI}{c}$	$W=8\times\frac{fI}{cl}$

TABLE 2—*Continued*

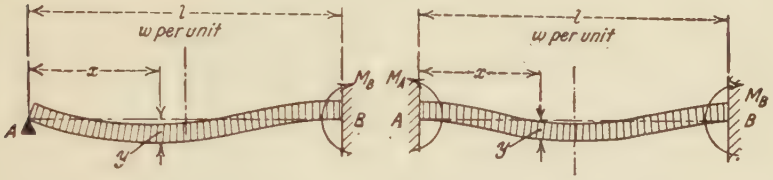
MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION



Reaction at A	$W\frac{2c+b}{2l}$
Reaction at B	$W\frac{2a+b}{2l}$
Moment at left of load	$W\frac{2c+b}{2l}x_1$
Moment in loaded length, b	$W\frac{2c+b}{2l}x_2-\frac{W}{2b}(x_2-a)^2$
Moment at right of load	$W\frac{2a+b}{2l}(l-x_3)$
Tan θ at left of load	$\frac{W}{2EI}\left(\frac{2c+b}{2l}\right)x_1^2+\frac{W}{24EIbl}[4l(a+b)^3-4a^3l-(a+b)^4-a^4-4bl^2(2a+b)]$
Tan θ, loaded length, b	$\frac{W}{2EI}\left(\frac{2c+b}{2l}\right)x_2^2-\frac{W}{24EIb}[x_2^3-3ax_2^2-3a^2x_2]+\frac{W}{24EIbl}[4l(a+b)^3-(a+b)^4-a^4-4bl^2(2a+b)]$
Tan θ at right of load	$\frac{W}{2EI}\left(\frac{2c+b}{2l}\right)x_3^2-\frac{W}{2EI}[x_3^2-(2a+b)x_3]+\frac{W}{24EIbl}[-(a+b)^4-a^4-4bl^2(2a+b)]$
Eq. of elastic line left of load	$y=\frac{W}{6EI}\left(\frac{2c+b}{2l}\right)x_1^3+\frac{W}{24EIbl}[4l(a+b)^3-4a^3l-(a+b)^4-a^4-4bl^2(2a+b)]x_1$
Eq. of elastic line in loaded length, b	$y=\frac{W}{6EI}\left(\frac{2c+b}{2l}\right)x_2^3-\frac{W}{24EIb}(x_2^4-4ax_2^3+6a^2x_2^2)+\frac{W}{24EIbl}[4l(a+b)^3-(a+b)^4-a^4-4bl^2(2a+b)]x_2-\frac{Wa^4}{24EIb}$
Eq. of elastic line right of load	$y=\frac{W}{6EI}\left(\frac{2c+b}{2l}\right)x_3^3-\frac{W}{12EI}[2x_3^3-3(2a+b)x_3^2]+\frac{W}{24EIbl}[-(a+b)^4-a^4-4bl^2(2a+b)]x_3-\frac{Wl^2}{6EI}(2a+b)-\frac{W}{24EIb}[-(a+b)^4-a^4-4bl^2(2a+b)]$

TABLE 2—*Continued*

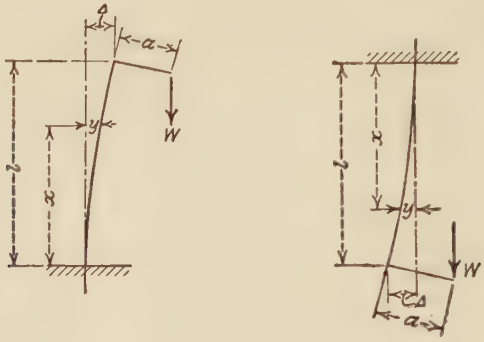
MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION



Reaction R_A	$\frac{3}{8}W$	$\frac{W}{2}$
Reaction R_B	$\frac{5}{8}W$	$\frac{W}{2}$
Moment	At B, $-\frac{Wl}{8}$	At A and B, $-\frac{Wl}{12}$
Moment M_x	$\frac{W}{8}\left(3x-\frac{4x^2}{l}\right)$	$\frac{W}{12}\left(6x-\frac{6x^2}{l}-l\right)$
Max. +moment	At $x=\frac{3}{8}l$, $\frac{9}{128}Wl$	At $x=\frac{l}{2}$, $\frac{Wl}{24}$
Max. −moment	At B, $-\frac{Wl}{8}$	At A and B, $-\frac{Wl}{12}$
Tan θ	$\frac{W}{48EI}\left(9x^2-\frac{8x^3}{l}-l^2\right)$	$\frac{W}{12EI}\left(3x^2-\frac{2x^3}{l}-lx\right)$
Max. tan θ	At A, $-\frac{Wl^2}{48EI}$	At point of contraflexure
Tan θ at point of contraflexure	At $x=\frac{3}{4}l$, $\frac{11Wl^2}{768EI}$	At $x=\frac{l}{2}(1\mp\frac{1}{3}\sqrt{3})=0.2113l$ $-\frac{Wl^2}{216EI}\sqrt{3}=-\frac{Wl^2}{124.7EI}$
Eq. of the elastic line	$y=\frac{W}{48EI}\left(3x^3-\frac{2x^4}{l}-l^2x\right)$	$y=\frac{W}{24EI}\left(2x^3-\frac{x^4}{l}-lx^2\right)$
Max. deflection	At $x=0.4215l$, $\Delta=-\frac{Wl^3}{185EI}$	At $x=\frac{l}{2}$, $\Delta=-\frac{Wl^3}{384EI}$
Max. deflection	At $x=0.4215l$, $\Delta=-\frac{fl^2}{12.807EI}$	At $x=\frac{l}{2}$, $\Delta=-\frac{1}{16}\times\frac{fl^2}{Ec}$
Max. working load	$W=8\times\frac{fI}{cl}$	$W=12\times\frac{fI}{cl}$

TABLE 2—*Continued*

MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION



Moment M_x	$Wa\frac{\cos \omega x}{\cos \omega l}$	$Wa\frac{\cosh \omega x}{\cosh \omega l}$
Moment M_{max}	At $x=0$ $\quad \frac{Wa}{\cos \omega l}$	At $x=l$ $\quad Wa$
Equation of the elastic line	$y=\frac{a}{\cos \omega l}(1-\cos \omega x)$	$y=\frac{a}{\cosh \omega l}(\cosh \omega x-1)$
Maximum deflection	$\Delta=a\left(\frac{1}{\cos \omega l}-1\right)$	$\Delta=a\left(1-\frac{1}{\cosh \omega l}\right)$
Maximum extreme fiber stresses	$f=\frac{W}{A}\left(1\mp\frac{aA}{S\cos \omega l}\right)$	$f=\frac{W}{A}\left(1\pm\frac{aA}{S}\right)$

In these expressions,

$\omega=\sqrt{\frac{W}{EI}}\cdot\frac{180^\circ}{\pi}$; cosh ω is the hyperbolic cos of ω,

A is the area of a right section of the column,

and

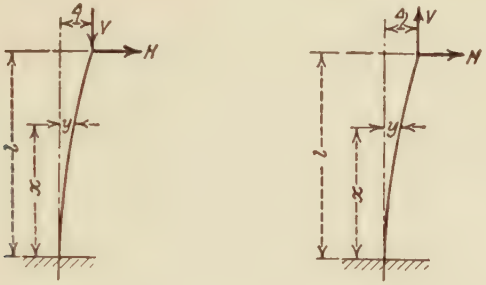
S is the section modulus of the column in inches.[3]

Approximate expressions may be formed by substituting the following for the exact values:

For cos ωl use $1-\frac{Wl^2}{2EI}$ and for cosh ωl use $1+\frac{Wl^2}{2EI}$.

TABLE 2—*Continued*

MOMENTS, SLOPES AND EQUATIONS OF THE ELASTIC LINES, AND DEFLECTIONS AND CAPACITIES OF BEAMS OF UNIFORM SECTION



Moment M_x	$\frac{H}{\omega}\frac{\sin \omega(l-x)}{\cos \omega l}$	$\frac{H}{\omega}\frac{\sinh \omega(l-x)}{\cosh \omega l}$
Moment M_{max}	At $x=0$, $\frac{H}{\omega}\tan \omega l$	At $x=0$, $\frac{H}{\omega}\tanh \omega l$
Equation of the elastic line	$y=\frac{H}{V}\left[-x+\frac{1}{\omega}\frac{\sin \omega l-\sin \omega(l-x)}{\cos \omega l}\right]$	$y=\frac{H}{V}\left[x-\frac{1}{\omega}\frac{\sinh \omega l-\sinh \omega(l-x)}{\cosh \omega l}\right]$
Max. deflection	$\Delta=\frac{H}{V}\left[\frac{\tan \omega l}{\omega}-l\right]$	$\Delta=\frac{H}{V}\left[l-\frac{\tanh \omega l}{\omega}\right]$
Max. extreme fiber stress	$f=-\frac{V}{A}\pm\frac{M_{max}}{S}$	$f=\frac{V}{A}\pm\frac{M_{max}}{S}$

In these expressions,

$\omega=\sqrt{\frac{V}{EI}}\cdot\frac{180^\circ}{\pi}$; sinh, cosh, and tanh are hyperbolic functions,

A is the area of a right section of the column,

and

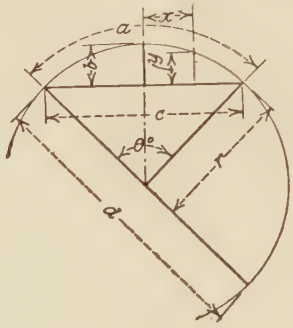
S is the section modulus of the column in inches.[3]

TABLE 3

MENSURATION

Properties of the Circle

Circumference of circle of diameter 1	$= \pi = 3.14159265$	
Circumference of circle	$= 2\pi r$	
Diameter of circle	= circumference	$\times 0.31831$
Diameter of circle of equal periphery as square	= side	$\times 1.27324$
Side of square of equal periphery as circle	= diameter	$\times 0.78540$
Diameter of circle circumscribed about square	= side	$\times 1.41421$
Side of square inscribed in circle	= diameter	$\times 0.70711$



Arc, $a = \dfrac{\pi r\theta^\circ}{180} = 0.017453r\theta^\circ.$

Angle, $\theta = \dfrac{180^\circ a}{\pi r} = 57.29578\dfrac{a}{r}.$

Radius, $r = \dfrac{4b^2 + c^2}{8b}.$ **Diameter,** $d = \dfrac{4b^2 + c^2}{4b}.$

Chord, $c = 2\sqrt{2br - b^2} = 2r \sin \dfrac{\theta^\circ}{2}.$

Rise, $b = r - \tfrac{1}{2}\sqrt{4r^2 - c^2} = \dfrac{c}{2} \tan \dfrac{\theta^\circ}{4} = 2r \sin^2 \dfrac{\theta^\circ}{4}.$

Rise, $b = r + y - \sqrt{r^2 - x^2},$

$y = b - r + \sqrt{r^2 - x^2}.$ $x = \sqrt{r^2 - (r + y - b)^2},$

$\pi = 3.14159265,$	$\log = 0.4971499$
$\dfrac{1}{\pi} = 0.3183099,$	$\log = \bar{1}.5028501$
$\pi^2 = 9.8696044,$	$\log = 0.9942997$
$\dfrac{1}{\pi^2} = 0.1013212,$	$\log = \bar{1}.0057003$
$\sqrt{\pi} = 1.7724539,$	$\log = 0.2485749$
$\sqrt{\dfrac{1}{\pi}} = 0.5641896,$	$\log = \bar{1}.7514251$
$\dfrac{\pi}{180} = 0.0174533,$	$\log = \bar{2}.2418774$
$\dfrac{180}{\pi} = 57.2957795,$	$\log = 1.7581226$

TABLE 3—*Continued*

AREA OF PLANE FIGURES

Triangle, base $\times$ $\frac{1}{2}$ perpendicular height,

$$\sqrt{s(s-a)(s-b)(s-c)},$$

s = $\frac{1}{2}$ sum of the three sides a, b, and c.

Trapezium, sum of area of the two triangles.

Trapezoid, $\frac{1}{2}$ sum of parallel sides $\times$ perpendicular height.

Parallelogram, base $\times$ perpendicular height.

Regular polygon, $\frac{1}{2}$ sum of sides $\times$ inside radius.

Circle, $\pi r^2 = 0.78540 \times \text{dia.}^2 = 0.07958 \times \text{circumference}^2$.

Sector of circle, $\frac{\pi r^2 \theta^\circ}{360} = 0.0087266 r^2 \theta^\circ$ = arc $\times$ $\frac{1}{2}$ radius.

Segment of circle, $\frac{r^2}{2}\left(\frac{\pi\theta^\circ}{180} - \sin\theta^\circ\right)$.

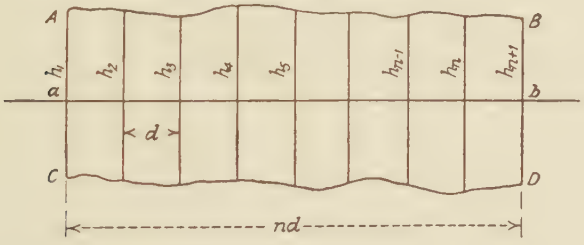
Circle of same area as square, diameter = side $\times$ 1.12838.

Square of same area as circle, side = diameter $\times$ 0.88623.

Ellipse, long diameter $\times$ short diameter $\times$ 0.78540.

Parabola, base $\times$ $\frac{2}{3}$ perpendicular height.

Irregular plane surface



Divide any plane surface A, B, C, D, along a line ab into an even number, n, of parallel and sufficiently small strips, d, whose ordinates are h_1, h_2, h_3, h_4, h_5 . . . , h_{n-1}, h_n, h_{n+1}, and considering contours between three ordinates as parabolic curves then for section $ABCD$,

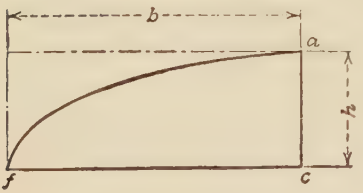
$$\text{Area} = \frac{d}{3}[h_1 + h_{n+1} + 4(h_2 + h_4 + h_6 \ldots h_n) + 2(h_3 + h_5 + h_7 \ldots h_{n-1})]$$

or, approximately, area = sum of ordinates $\times$ width, d.

TABLE 3—*Continued*

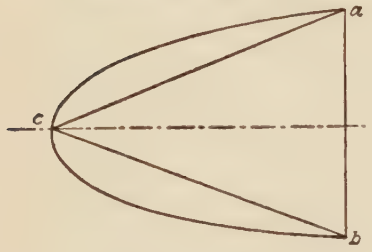
AREA OF PLANE FIGURES

Parabolic segment



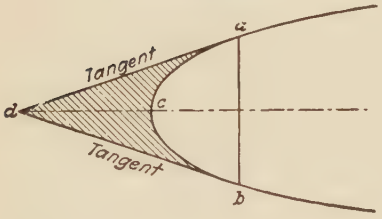
Area of $acf = \frac{2}{3}bh = \frac{2}{3}$ (area of circumscribing rectangle).

Parabolic segment



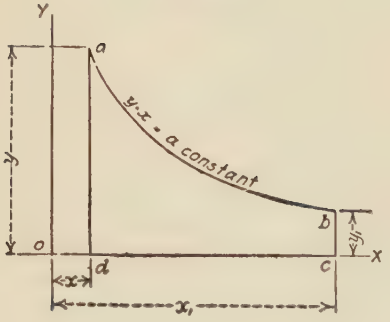
Area of $abc = \frac{1}{3}$ area of triangle abc.

Shaded figure



Area of $adbc = \frac{1}{3}$ area of triangle adb.

Hyperbola



Area $abcd = xy \log_e r$,

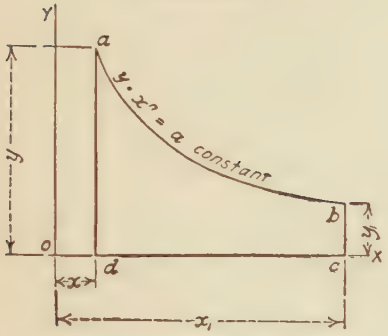
$$r = \frac{y}{y_1} \quad \text{or} \quad \frac{x_1}{x}$$

$\text{Log}_e = 2.302585 \times$ common log.

TABLE 3—*Continued*

AREA OF PLANE FIGURES

Hyperbola

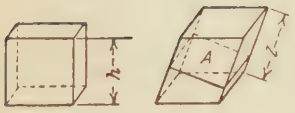


$$\text{Area } abcd = \frac{xy - x_1y_1}{n - 1}.$$

SURFACE AND VOLUME OF SOLIDS

S = Lateral or Convex Surface. V = Volume

Parallelopiped

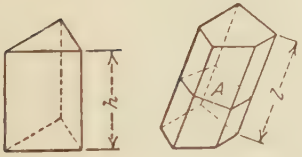


S = perimeter, P, perpendicular to sides × lat. length, l: $= Pl$

V = area of base, B, × perpendicular height, h: $= Bh$

V = area of section, A, perpendicular to sides × lat. length, l: $= Al$

Prism, right of oblique, regular or irregular

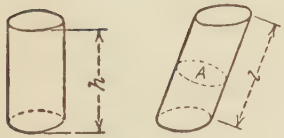


S = perimeter, P, perpendicular to sides × lat. length, l: $= Pl$

V = area of base, B, × perpendicular height, h: $= Bh$

V = area of section, A, perpendicular to sides × lat. length, l: $= Al$

Cylinder, right or oblique, circular or elliptic, etc.



S = perimeter of base, P, × perpendicular height, h: $= Ph$

S = perimeter, P_1, perpendicular to sides × lat. length, l: $= P_1l$

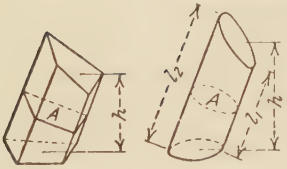
V = area of base, B, × perpendicular height, h: $= Bh$

V = area of section, A, perpendicular to sides × lat. length, l: $= Al$

TABLE 3—*Continued*

SURFACE AND VOLUME OF SOLIDS

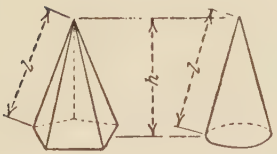
S = Lateral or Convex Surface. V = Volume



Frustum of any prism or cylinder

V = area of base, B, $\times$ perpendicular distance, h, from base to center of gravity of opposite face: $= Bh$

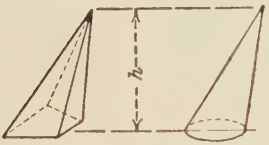
V for cylinder $= \frac{1}{2}A(l_1 + l_2)$.



Pyramid or cone, right and regular

S = perimeter of base, P, $\times \frac{1}{2}$ slant height, l: $= \frac{1}{2}Pl$

V = area of base, B, $\times \frac{1}{3}$ perpendicular height, h: $= \frac{1}{3}Bh$

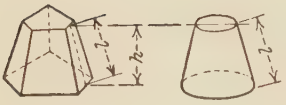


Pyramid or cone, right or oblique, regular or irregular

V = area of base, B, $\times \frac{1}{3}$ perpendicular height, h: $= \frac{1}{3}Bh$

$V = \frac{1}{3}$ volume of prism or cylinder of same base and perpendicular height.

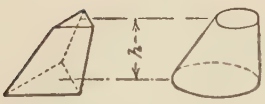
$V = \frac{1}{2}$ volume of hemisphere of same base and perpendicular height.



Frustum of pyramid or cone, right and regular, parallel ends

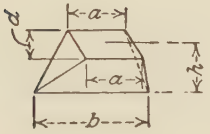
S = (sum of perimeter of base, P, and top, p) $\times \frac{1}{2}$ slant height, l: $= \frac{1}{2}l(P + p)$

V = (sum of areas of base, B, and top, b + square root of their products) $\times \frac{1}{3}$ perpendicular height, h: $= \frac{1}{3}h(B + b + \sqrt{Bb})$



Frustum of any pyramid or cone, parallel ends

V = (sum of areas of base, B, and top, b + square root of their products) $\times \frac{1}{3}$ perpendicular height, h: $= \frac{1}{3}h(B + b + \sqrt{Bb})$



Wedge, parallelogram face

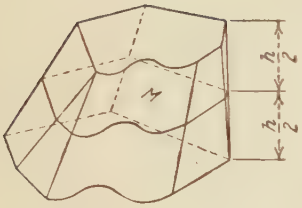
$V = \frac{1}{6}$ (sum of three edges, aba, $\times$ perpendicular height, h, $\times$ perpendicular width, d): $= \frac{1}{6}dh(2a + b)$

TABLE 3—*Continued*

SURFACE AND VOLUME OF SOLIDS

S = Lateral or Convex Surface. V = Volume

Prismatoid



$V = \frac{1}{6}$ perpendicular height, h (sum of areas of base, B, and top b, $+ 4 \times$ area of section, M, parallel to bases and midway between them):

$$= \tfrac{1}{6}h(B + b + 4M)$$

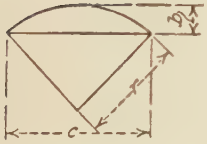
The prismatoid formula applies also to any of the foregoing solids with parallel bases, to pyramids, cones, spherical sections, and to many solids with irregular surfaces.

Sphere

$$S = 4\pi r^2 = \pi d^2 = 3.14159265d^2$$

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{1}{6}\pi d^3 = 0.52359878d^3$$

Spherical sector



$$S = \tfrac{1}{2}\pi r(4b + c)$$

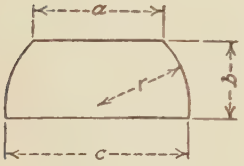
$$V = \tfrac{2}{3}\pi r^2 b$$

Spherical segment

$$S = 2\pi rb = \tfrac{1}{4}\pi(4b^2 + c^2)$$

$$V = \tfrac{1}{3}\pi b^2(3r - b) = \tfrac{1}{24}\pi b(3c^2 + 4b^2)$$

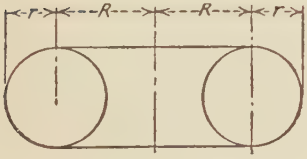
Spherical zone



$$S = 2\pi rb$$

$$V = \tfrac{1}{24}\pi b(3a^2 + 3c^2 + 4b^2)$$

Circular ring



$$S = 4\pi^2 Rr$$

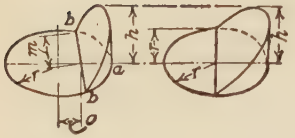
$$V = 2\pi^2 Rr^2$$

TABLE 3—*Continued*

SURFACE AND VOLUME OF SOLIDS

S = Lateral or Convex Surface. V = Volume

Ungula of right, regular cylinder



Base = segment, *bab*

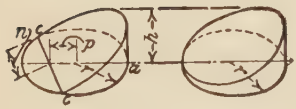
$$S = (2rm - o \times \text{arc, } bab)\frac{h}{r-o}$$

$$V = (\tfrac{2}{3}m^3 - o \times \text{area, } bab)\frac{h}{r-o}$$

Base = half circle

$$S = 2rh$$

$$V = \tfrac{2}{3}r^2h$$



Base = segment, *cac*

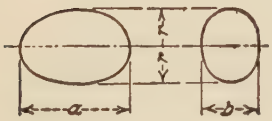
$$S = (2rn + p \times \text{arc, } cac)\frac{h}{r+p}$$

$$V = (\tfrac{2}{3}n^3 + p \times \text{area, } cac)\frac{h}{r+p}$$

Base = circle

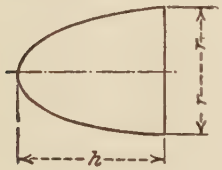
$$S = \pi rh$$

$$V = \tfrac{1}{2}\pi r^2h$$



Ellipsoid

$$V = \tfrac{1}{3}\pi rab$$

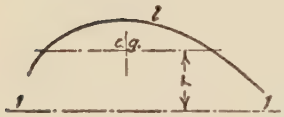


Paraboloid

$$V = \tfrac{1}{2}\pi r^2h$$

Ratio of corresponding volumes of a Cone, Paraboloid, Sphere, and Cylinder of equal height: $\frac{1}{3} : \frac{1}{2} : \frac{2}{3} : 1$.

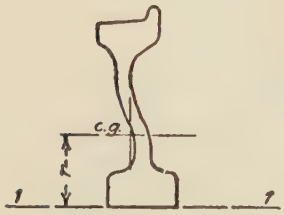
Bodies generated by partial or complete revolution



l = length of a curve
A = area of a plane
} rotating about an axis 1–1 on one side and in plane of axis.

r = distance of center of gravity of line or plane from axis 1–1, and for any angle of revolution, $a°$,

$\frac{2\pi ra°}{360}$ = length of arc described by center of gravity.



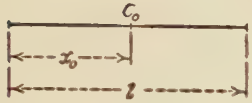
S = length of curve × length of arc about axis,
$= l\frac{2\pi ra°}{360}$. For complete revolution $S = 2\pi rl$.

V = area of plane × length of arc about axis,
$= A\frac{2\pi ra°}{360}$. For complete revolution $V = 2\pi rA$.

TABLE 4

CENTROIDS

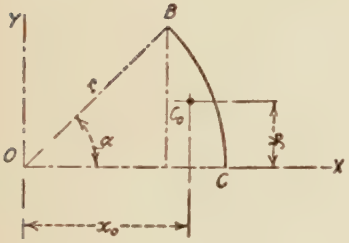
CENTROIDS OF LINES



Straight line

Centroid is at C_0, the middle point of the line.

$$x_0 = \frac{l}{2}.$$



Circular arc

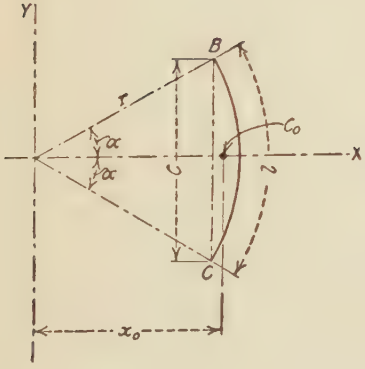
Centroid is at C_0 where

$$x_0 = \frac{r \sin \alpha}{\text{rad } \alpha},$$

$$y_0 = \frac{2r \sin^2 \frac{\alpha}{2}}{\text{rad } \alpha},$$

$$1 \text{ radian} = \frac{180^\circ}{\pi} = 57.295780^\circ,$$

$$1 \text{ degree} = 0.017453 \text{ radian}.$$



Circular arc

Centroid is at C_0 on the axis of symmetry and at a distance from the center of

$$x_0 = \frac{rc}{l} = \frac{r \sin \alpha}{\text{rad } \alpha},$$

$$y_0 = 0.$$

When the arc, BC, becomes a quadrant,

$$x_0 = \frac{2r \sin 45^\circ}{\pi} = \frac{2r\sqrt{2}}{\pi} = 0.900317r.$$

$$y_0 = 0.$$

When the arc, BC, becomes a semi-circumference,

$$x_0 = \frac{2r \sin 90^\circ}{\pi} = \frac{2r}{\pi} = 0.636620r.$$

When the arc is greater than 180, the expression

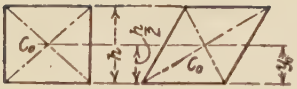
$$x_0 = \frac{rc}{l} = \frac{r \sin \alpha}{\text{rad } \alpha}$$

still holds good.

TABLE 4—*Continued*

CENTROIDS OF AREAS

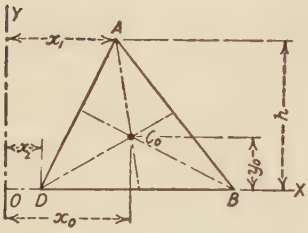
Parallelogram



The centroid is at the intersection of the diagonals.

$$y_0 = \frac{h}{2}.$$

Triangle

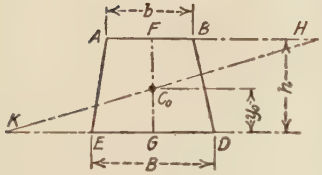


Graphical method.—The centroid is at the intersection of lines drawn from the apices to the middle of the opposite sides.

Analytical method.—If x_1, x_2, and x_3 are the distances of the vertices from any axis, the distance to the centroid is

$$x_0 = \frac{x_1 + x_2 + x_3}{3}. \quad y_0 = \frac{h}{3}$$

Trapezoid



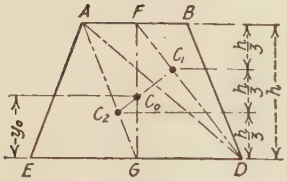
Graphical method.—The centroid is on the line, *FG*, joining the middle points of the parallel sides. Draw $BH = ED$ and $EK = AB$. The centroid is at C_0.

Analytical method.—The height of C_0 above the long base is

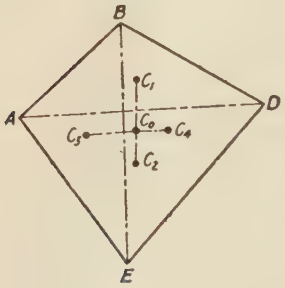
$$y_0 = \frac{h(2b + B)}{3(b + B)}.$$

The distance of C_0 below the short base is

$$h - y_0 = \frac{h(b + 2B)}{3(b + B)}.$$



Graphical method.—Draw *FG* joining the middle points of the bases. Draw *AD* and join the centroids of the triangles *ADB* and *ADE* by the line C_1C_2. The centroid of the trapezoid *ABDE* is at C_0.

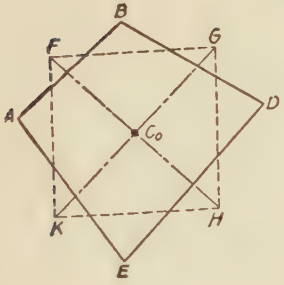


Quadrilateral

First graphical method.—Divide the quadrilateral *ABDE* into two triangles by the diagonal *AD*. Join the centroids of these triangles by the line c_1c_2. Similarly, draw *BE* and join the centroids by the line C_3C_4. The centroid of the quadrilateral *ABDE* is at the intersection C_0.

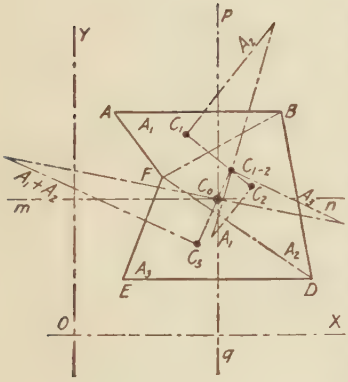
TABLE 4—*Continued*

CENTROIDS OF AREAS



Second graphical method.—Trisect each side and draw *FGHK* through the points thus found. The centroid of *ABDE* is at the intersection of the diagonals of the parallelogram *FGHK* at C_0.

Any plane figure limited by straight lines

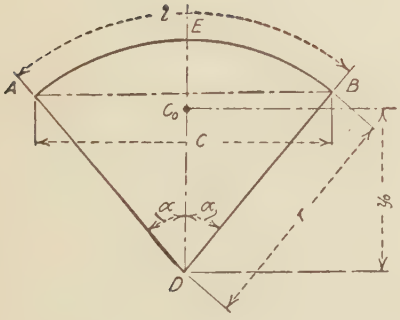


Graphical method.—Divide *ABDEF* into triangles and find the centroids C_1, C_2 and C_3 of each. Find the areas A_1, A_2, and A_3. Find the centroid of A_1 and A_2 at C_{1-2} as indicated. Find the centroid of $A_1 + A_2$ and A_3 as shown. The centroid of *ABDEF* is then at C_0.

Analytical method.—Choose any convenient coordinate axes of reference, as *OX* and *OY*. Sum the moments of the areas of the triangles, considered as concentrated at their centroids, about each axis successively, and divide the sums of the moments by the area of the whole figure. The quotients are the distances of the centroid from the respective axes. Draw lines *mn* and *pq* at these distances from the axes and parallel the axes. The centroid of the whole figure *ABDEF* is at their intersection C_0.

When there is a hole in the area its moment about each axis is negative.

Circular sector



The centroid of the circular sector *ABD* is on the axis of symmetry, *DE*, at a distance from *D* of

$$y_0 = \frac{2rc}{3l} = \frac{2r \sin \alpha}{3 \text{ rad } \alpha}.$$

For a quadrant

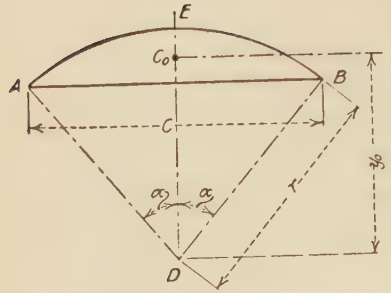
$$y_0 = \frac{4r\sqrt{2}}{3\pi} = 0.600216r.$$

The distance of the centroid from each bounding radius is $\frac{4r}{3\pi} = 0.424413r$.

For semi-circle

$$y_0 = \frac{4r}{3\pi} = 0.424413r.$$

TABLE 4—*Continued*
CENTROIDS OF AREAS



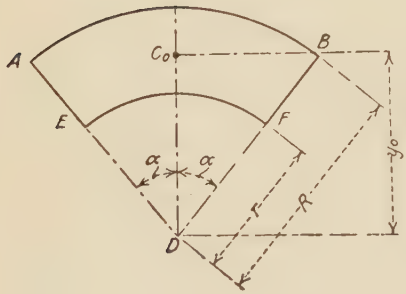
Circular segment

The centroid of the circular segment ABE is on the axis of symmetry, DE, at a distance from D of

$$y_0 = \frac{c^3}{12A} = \frac{2r^3 \sin^3 \alpha}{3A},$$

in which A is the area of the segment

$$= \frac{r^2}{2}(\text{rad } 2\alpha - \sin 2\alpha).$$



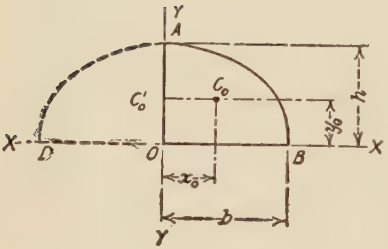
Sector of an annulus

The centroid of the sector of an annulus $ABFE$ is on the axis of symmetry at a distance from D of

$$y_0 = \frac{2(R^3 - r^3)\sin \alpha}{3(R^2 - r^2)\text{ rad } \alpha}$$

or

$$y_0 = 38.197187\frac{(R^3 - r^3)\sin \alpha}{(R^2 - r^2)\alpha^\circ}.$$



Elliptic quadrant

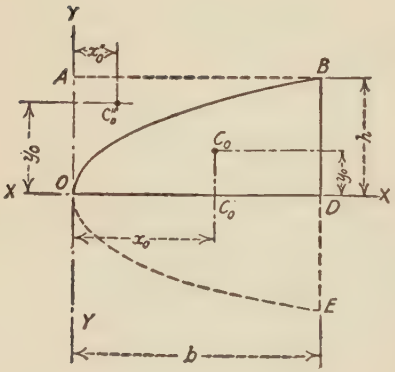
The centroid of the elliptic quadrant ABO is at C_0 where

$$x_0 = \frac{4b}{3\pi} \quad \text{and} \quad y_0 = \frac{4h}{3\pi}.$$

Semi-ellipse

The centroid of the semi-ellipse ABD is on the axis of symmetry and at a distance from O of

$$y_0 = \frac{4h}{3\pi}.$$



Parabolic half-segment

The centroid of the parabolic half-segment OBD is at C_0 where

$$x_0 = \frac{3b}{5} \quad \text{and} \quad y_0 = \frac{3h}{8}.$$

Parabolic segment

The centroid of the parabolic segment BOE is at C'_0 on the axis of symmetry and at a distance from O of

$$x_0 = \frac{3b}{5}.$$

Parabolic spandrel

The centroid of the parabolic spandrel OAB is at C''_0 where

$$x''_0 = \frac{3b}{10} \quad \text{and} \quad y''_0 = \frac{3h}{4}.$$

TABLE 5

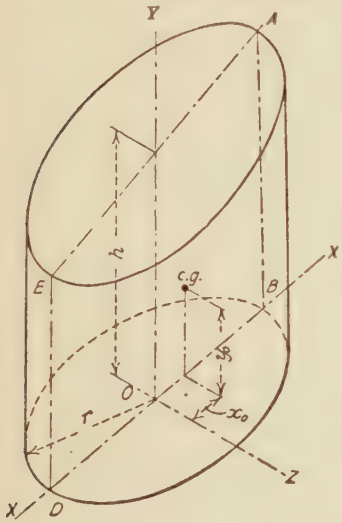
CENTERS OF GRAVITY

CENTERS OF GRAVITY OF SOLIDS

General.—The centers of gravity of many homogeneous bodies may be seen by inspection. The center of gravity will be at the geometrical center of the body, if it has one, as for example, the center of a sphere or a parallelopiped, or the center of a cylinder of revolution, thin, thick, or solid, with parallel bases.

If a cylinder or prism has parallel bases the centroids of which are known, the center of gravity is in a line joining the centroids of the bases. Generally, when a homogeneous body has an axis of symmetry, the center of gravity will lie on the axis.

When a body, having no geometrical center, can be divided into parts, each of which has a geometrical center, the center of gravity of the body may be found from its parts by taking moments about any convenient axes.



Oblique frustum of a right cylinder

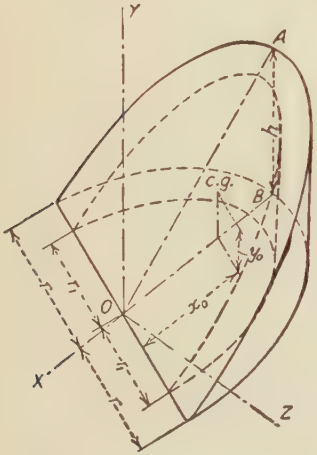
Let *ABDE* be the plane of symmetry. Axis *OY* joins the centroids of the bases. Let α be the angle of inclination of the oblique section with the base.

The distance from the base to the c. g. is

$$y_0 = \frac{h}{2} + \frac{r^2 \tan^2 \alpha}{8h}.$$

The distance from the axis of the cylinder to the c. g. is

$$x_0 = \frac{r^2 \tan \alpha}{4h}.$$



Right cylinder, oblique top cutting the base in a diameter

$$y_0 = \frac{3\pi h}{32}.$$

$$x_0 = \frac{3\pi r}{16}.$$

The centroid of the outer curved surface is at

$$y_0 = \frac{\pi h}{8}.$$

$$x_0 = \frac{\pi r}{4}.$$

Hollow right cylinder with oblique plane cutting the base in a diameter

The c. g. is at

$$y_0 = \frac{3\pi}{32}\left(\frac{r^4 - r_1^4}{r^3 - r_1^3}\right) \times \frac{h}{r}.$$

$$x_0 = \frac{3\pi}{16}\left(\frac{r^4 - r_1^4}{r^3 - r_1^3}\right).$$

TABLE 5—*Continued*

CENTERS OF GRAVITY OF SOLIDS

Cone or pyramid

The center of gravity lies on the line joining the centroid of the base with the vertex, and at a distance from the base of one-fourth of the altitude.

The centroid of the exterior surface, not including the base, lies on the line joining the vertex with the centroid of the perimeter of the base, and at a distance from the base of one-third of the length of that line.

Frustum of a circular cone

Let R be the radius of the larger base and r that of the smaller base, and let h be the altitude.

The center of gravity lies on the axis of the cone and at a distance from the larger base of

$$y_0 = \frac{h(R^2 + 2Rr + 3r^2)}{4(R^2 + Rr + r^2)}.$$

The centroid of the exterior surface, not including the base, is at a distance from the larger base of

$$y_0 = \frac{h(R + 2r)}{3(R + r)}.$$

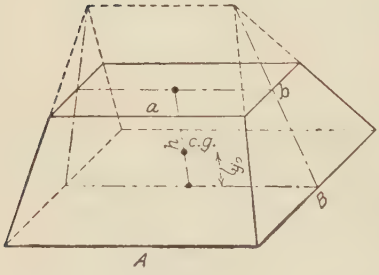
Frustum of a pyramid

When the pyramid has regular bases, let B and b be the lengths of the sides of the larger and smaller bases, respectively, and let A and a be the areas of the bases, and h the altitude. The center of gravity is at a distance from the larger base of

$$y_0 = \frac{h(A + 2\sqrt{Aa} + 3a)}{4(A + \sqrt{Aa} + a)}.$$

The centroid of the exterior surface, not including the bases, is at a distance from the larger base of

$$y_0 = \frac{h(B + 2b)}{3(B + b)}.$$



Obelisk and wedge

Let A and B and a and b be the lengths of the sides of the respective bases, then

$$y_0 = \frac{h(AB + Ab + aB + 3ab)}{2(2AB + Ab + aB + 2ab)}.$$

When b becomes zero, the center of gravity of the wedge is at a distance from the base of

$$y_0 = \frac{h(A + a)}{2(2A + a)}.$$

TABLE 5—*Continued*

CENTERS OF GRAVITY OF SOLIDS

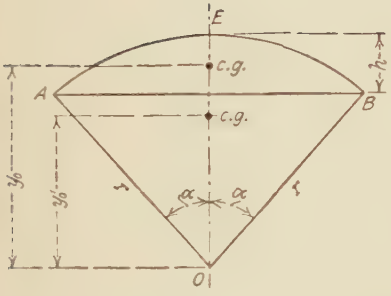
Hemisphere

$$y_0 = \tfrac{3}{8}r.$$

Hollow hemisphere

$$y_0 = \frac{3(R^4 - r^4)}{8(R^3 - r^3)}.$$

Spherical sector



or

$$y'_o = \tfrac{3}{8}(1 + \cos\alpha)r,$$

$$y'_o = \tfrac{3}{8}(2r - h).$$

Spherical segment

The distance from the center of the sphere to the center of gravity of the spherical segment, ABE is

$$y_0 = \frac{3(2r - h)^2}{4(3r - h)}.$$

Ellipsoid

Let the three coordinate axes be X, Y and Z; and let a, b and c be the lengths of the semi-axes. The center of gravity of one octant of the ellipsoid is at

$$x_0 = \tfrac{3}{8}a, \quad y_0 = \tfrac{3}{8}b \quad \text{and} \quad z_0 = \tfrac{3}{8}c.$$

Paraboloid of revolution

Assume a paraboloid formed by the revolution of a parabola about its axis. Let h be its height from apex to base. The distance from the base to the center of gravity is

$$y_0 = \tfrac{1}{3}h.$$

TABLE 6

ELEMENTS OF LINES

Line

$$A = d$$

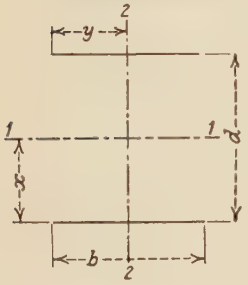
$$x = \frac{d}{2}$$

$$I_{1-1} = \frac{d^3}{12}$$

$$S_{1-1} = \frac{d^2}{6}$$

$$r_{1-1} = \frac{d}{\sqrt{12}} = 0.288675d$$

Two parallel lines



$$A = 2b$$

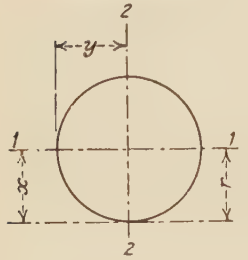
$$x = \frac{d}{2} \qquad y = \frac{b}{2}$$

$$I_{1-1} = \frac{bd^2}{2} \qquad I_{2-2} = \frac{b^3}{6}$$

$$S_{1-1} = bd \qquad S_{2-2} = \frac{b^2}{3}$$

$$r_{1-1} = \frac{d}{2} \qquad r_{2-2} = \frac{b}{\sqrt{12}} = 0.288675b$$

Circumference



$$A = \pi d$$

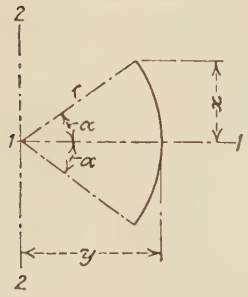
$$x = \frac{d}{2} \qquad y = \frac{d}{2}$$

$$I_{1-1} = \pi r^3$$

$$S_{1-1} = \pi r^2$$

$$r_{1-1} = r\sqrt{\tfrac{1}{2}} = 0.70711r$$

Circular arc



$$A = 2r\alpha$$

$$x = r\sin\alpha \qquad y = r$$

$$I_{1-1} = r^3\alpha\left[1 - \frac{\sin\alpha\cos\alpha}{\alpha}\right]$$

$$I_{2-2} = r^3\alpha\left[1 + \frac{\sin\alpha\cos\alpha}{\alpha}\right]$$

$$S_{1-1} = \frac{r^2\alpha}{\sin\alpha}\left[1 - \frac{\sin\alpha\cos\alpha}{\alpha}\right]$$

$$S_{2-2} = r^2\alpha\left[1 + \frac{\sin\alpha\cos\alpha}{\alpha}\right]$$

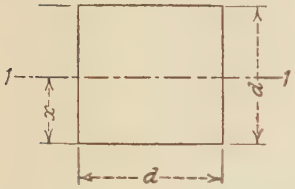
$$r_{1-1} = r\sqrt{\frac{1}{2}} \times \sqrt{1 - \frac{\sin\alpha\cos\alpha}{\alpha}}$$

$$r_{2-2} = r\sqrt{\frac{1}{2}} \times \sqrt{1 + \frac{\sin\alpha\cos\alpha}{\alpha}}$$

TABLE 7

ELEMENTS OF SECTIONS

Square
Axis of moments through center



$$A = d^2$$

$$x = \frac{d}{2}$$

$$I_{1-1} = \frac{d^4}{12}$$

$$S_{1-1} = \frac{d^3}{6}$$

$$r_{1-1} = \frac{d}{\sqrt{12}} = 0.288675d$$

Square
Axis of moments on base

$$A = d^2$$

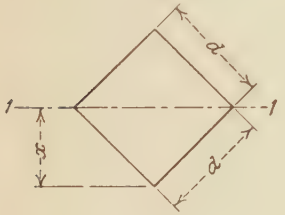
$$x = d$$

$$I_{1-1} = \frac{d^4}{3}$$

$$S_{1-1} = \frac{d^3}{3}$$

$$r_{1-1} = \frac{d}{\sqrt{3}} = 0.577350d$$

Square
Axis of moments on diagonal



$$A = d^2$$

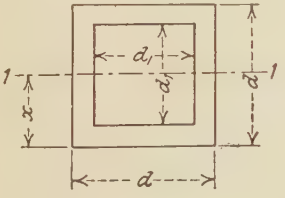
$$x = \frac{d}{\sqrt{2}} = 0.707107d$$

$$I_{1-1} = \frac{d^4}{12}$$

$$S_{1-1} = \frac{d^3}{6\sqrt{2}} = 0.117851d^3$$

$$r_{1-1} = \frac{d}{\sqrt{12}} = 0.288675d$$

Hollow square
Axis of moments through center



$$A = d^2 - d_1^2$$

$$x = \frac{d}{2}$$

$$I_{1-1} = \frac{d^4 - d_1^4}{12}$$

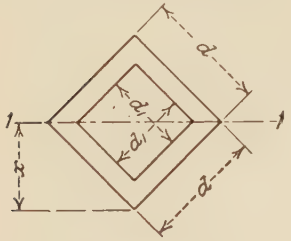
$$S_{1-1} = \frac{d^4 - d_1^4}{6d}$$

$$r_{1-1} = \sqrt{\frac{d^2 + d_1^2}{12}} = 0.288675\sqrt{d^2 + d_1^2}$$

TABLE 7—*Continued*

ELEMENTS OF SECTIONS

Hollow square
Axis of moments on diagonal



$$A = d^2 - d_1^2$$

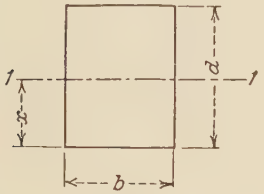
$$x = \frac{d}{\sqrt{2}}$$

$$I_{1-1} = \frac{d^4 - d_1^4}{12}$$

$$S_{1-1} = \frac{d^4 - d_1^4}{6d\sqrt{2}} = 0.117851\frac{d^4 - d_1^4}{d}$$

$$r_{1-1} = \sqrt{\frac{d^2 + d_1^2}{12}} = 0.288675\sqrt{d^2 + d_1^2}$$

Rectangle
Axis of moments through center



$$A = bd$$

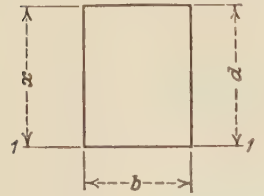
$$x = \frac{d}{2}$$

$$I_{1-1} = \frac{bd^3}{12}$$

$$S_{1-1} = \frac{bd^2}{6}$$

$$r_{1-1} = \frac{d}{\sqrt{12}} = 0.288675d$$

Rectangle
Axis of moments on base



$$A = bd$$

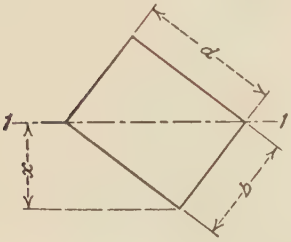
$$x = d$$

$$I_{1-1} = \frac{bd^3}{3}$$

$$S_{1-1} = \frac{bd^2}{3}$$

$$r_{1-1} = \frac{d}{\sqrt{3}} = 0.577350d$$

Rectangle
Axis of moments on diagonal



$$A = bd$$

$$x = \frac{bd}{\sqrt{b^2 + d^2}}$$

$$I_{1-1} = \frac{b^3d^3}{6(b^2 + d^2)}$$

$$S_{1-1} = \frac{b^2d^2}{6\sqrt{b^2 + d^2}}$$

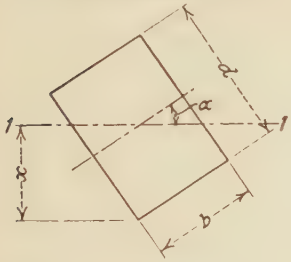
$$r_{1-1} = \frac{bd}{\sqrt{6(b^2 + d^2)}}$$

TABLE 7—*Continued*

ELEMENTS OF SECTIONS

Rectangle

Axis of moments on any line through center of gravity



$$A = bd$$

$$x = \frac{b \sin \alpha + d \cos \alpha}{2}$$

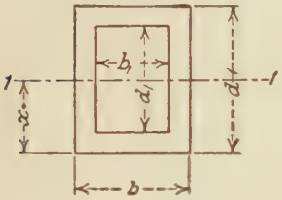
$$I_{1-1} = \frac{bd(b^2 \sin^2 \alpha + d^2 \cos^2 \alpha)}{12}$$

$$S_{1-1} = \frac{bd(b^2 \sin^2 \alpha + d^2 \cos^2 \alpha)}{6(b \sin \alpha + d \cos \alpha)}$$

$$r_{1-1} = \sqrt{\frac{b^2 \sin^2 \alpha + d^2 \cos^2 \alpha}{12}}$$

Hollow rectangle

Axis of moments through center



$$A = bd - b_1d_1$$

$$x = \frac{d}{2}$$

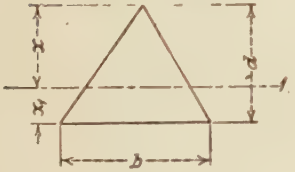
$$I_{1-1} = \frac{bd^3 - b_1d_1^3}{12}$$

$$S_{1-1} = \frac{bd^3 - b_1d_1^3}{6d}$$

$$r_{1-1} = \sqrt{\frac{bd^3 - b_1d_1^3}{12(bd - b_1d_1)}}$$

Triangle

Axis of moments through center of gravity



$$A = \frac{bd}{2}$$

$$x = \frac{2d}{3} \qquad x_1 = \frac{d}{3}$$

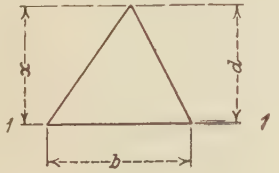
$$I_{1-1} = \frac{bd^3}{36}$$

$$S_{1-1} = \frac{bd^2}{24}$$

$$r_{1-1} = \frac{d}{\sqrt{18}} = 0.235702d$$

Triangle

Axis of moments on base



$$A = \frac{bd}{2}$$

$$x = d$$

$$I_{1-1} = \frac{bd^3}{12}$$

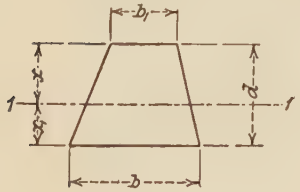
$$S_{1-1} = \frac{bd^2}{12}$$

$$r_{1-1} = \frac{d}{\sqrt{6}} = 0.408248d$$

TABLE 7—*Continued*

ELEMENTS OF SECTIONS

Trapezoid
Axis of moments through center of gravity



$$A = \frac{d(b + b_1)}{2}$$

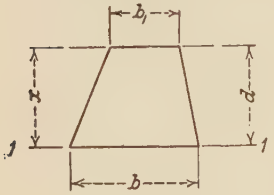
$$x = \frac{d(b_1 + 2b)}{3(b + b_1)} \qquad x_1 = \frac{d(b + 2b_1)}{3(b + b_1)}$$

$$I_{1-1} = \frac{d^3(b^2 + 4bb_1 + b_1^2)}{36(b + b_1)}$$

$$S_{1-1} = \frac{d^2(b^2 + 4bb_1 + b_1^2)}{12(b_1 + 2b)}$$

$$r_{1-1} = \frac{d}{6(b + b_1)}\sqrt{2(b^2 + 4bb_1 + b_1^2}$$

Trapezoid
Axis of moments on base



$$A = \frac{d(b + b_1)}{2}$$

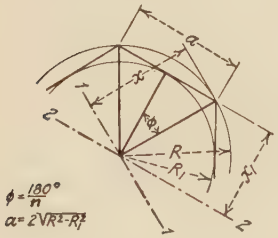
$$x = d$$

$$I_{1-1} = \frac{d^3(b + 3b_1)}{12}$$

$$S_{1-1} = \frac{d^2(b + 3b_1)}{12}$$

$$r_{1-1} = \frac{d}{\sqrt{6}}\sqrt{\frac{b + 3b_1}{b + b_1}}$$

Regular polygon
Axis of moments through center



n = number of sides

$A = \frac{1}{4}na^2 \cot \phi = \frac{1}{2}nR^2 \sin 2\phi = nR_1^2 \tan \phi$

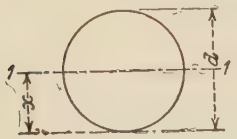
$$x = R = \frac{a}{2 \sin \phi} \qquad x_1 = R_1 = \frac{a}{2 \tan \phi}$$

$$I_{1-1} = \frac{A(6R^2 - a^2)}{24} = I_{2-2} = \frac{A(12R_1^2 + a^2)}{48}$$

$$S_{1-1} = \frac{A(6R^2 - a^2)}{24R} \qquad S_{2-2} = \frac{A(12R_1^2 + a^2)}{48R_1}$$

$$r_{1-1} = \sqrt{\frac{6R^2 - a^2}{24}} = r_{2-2} = \sqrt{\frac{12R_1^2 + a^2}{48}}$$

Circle
Axis of moments through center



$$A = \frac{\pi d^2}{4} = 0.785398d^2$$

$$x = \frac{d}{2}$$

$$I_{1-1} = \frac{\pi d^4}{64} = 0.049087d^4$$

$$S_{1-1} = \frac{\pi d^3}{32} = 0.098175d^3$$

$$r_{1-1} = \frac{d}{4}$$

TABLE 7—*Continued*

ELEMENTS OF SECTIONS

Annulus

Axis of moments through center

$$A = \frac{\pi(d^2 - d_1^2)}{4} = 0.785398(d^2 - d_1^2)$$

$$x = \frac{d}{2}$$

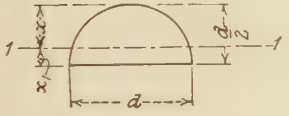
$$I_{1-1} = \frac{\pi(d^4 - d_1^4)}{64} = 0.049087(d^4 - d_1^4)$$

$$S_{1-1} = \frac{\pi(d^4 - d_1^4)}{32d} = 0.098175\frac{d^4 - d_1^4}{d}$$

$$r_{1-1} = \frac{\sqrt{d^2 + d_1^2}}{4}$$

Semicircle

Axis of moments through center of gravity



$$A = \frac{\pi d^2}{8} = 0.392699d^2$$

$$x = \frac{d(3\pi - 4)}{6\pi} = 0.287793d$$

$$x_1 = \frac{2d}{3\pi} = 0.212207d$$

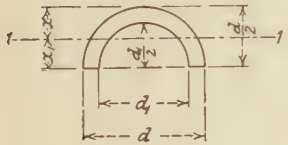
$$I_{1-1} = \frac{d^4(9\pi^2 - 64)}{1152\pi} = 0.006860d^4$$

$$S_{1-1} = \frac{d^3(9\pi^2 - 64)}{192(3\pi - 4)} = 0.023836d^3$$

$$r_{1-1} = \frac{d\sqrt{9\pi^2 - 64}}{12\pi} = 0.132168d$$

Semi-annulus

Axis of moments through center of gravity



$$A = \frac{\pi(d^2 - d_1^2)}{8} = 0.392699(d^2 - d_1^2)$$

$$x = \frac{2(d^3 - d_1^3)}{3\pi(d^2 - d_1^2)} \qquad x_1 = \frac{3\pi d(d^2 - d_1^2) - 4(d^3 - d_1^3)}{6\pi(d^2 - d_1^2)}$$

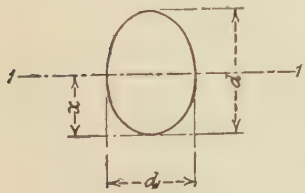
$$I_{1-1} = \frac{9\pi^2(d^4 - d_1^4)(d^2 - d_1^2) - 64(d^3 - d_1^3)^2}{1152\pi(d^2 - d_1^2)}$$

$$S_{1-1} = \frac{I}{x} \text{ if } x > x_1 \quad \text{or} \quad \frac{I}{x_1} \text{ if } x_1 > x$$

$$r_{1-1} = \frac{1}{12\pi}\sqrt{\frac{9\pi^2(d^4 - d_1^4)(d^2 - d_1^2) - 64(d^3 - d_1^3)^2}{(d^2 - d_1^2)^2}}$$

Ellipse

Axis of moments through center



$$A = \frac{\pi d d_1}{4} = 0.785398 d d_1$$

$$x = \frac{d}{2}$$

$$I_{1-1} = \frac{\pi d^3 d_1}{64} = 0.049087 d^3 d_1$$

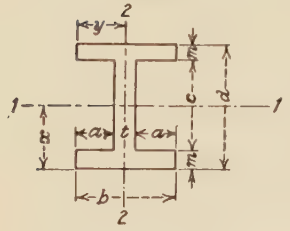
$$S_{1-1} = \frac{\pi d^2 d_1}{32} = 0.098175 d^2 d_1$$

$$r_{1-1} = \frac{d}{4}$$

TABLE 7—*Continued*

ELEMENTS OF SECTIONS

Beam



$$A = bd - 2ac$$

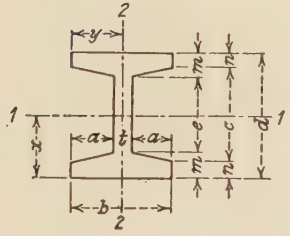
$$x = \frac{d}{2} \qquad y = \frac{b}{2}$$

$$I_{1-1} = \frac{bd^3 - 2ac^3}{12} \qquad I_{2-2} = \frac{(d-c)b^3 + ct^3}{12}$$

$$S_{1-1} = \frac{1}{6}\left[bd^2 - \frac{2ac^3}{d}\right] \qquad S_{2-2} = \frac{1}{6}\left[(d-c)b^2 + \frac{ct^3}{b}\right]$$

$$r_{1-1} = \sqrt{\frac{bd^3 - 2ac^3}{12(bd - 2ac)}} \qquad r_{2-2} = \sqrt{\frac{(d-c)b^3 + ct^3}{12(bd - 2ac)}}$$

Beam



$$A = dt + 2a(m+n)$$

$$x = \frac{d}{2} \qquad y = \frac{b}{2}$$

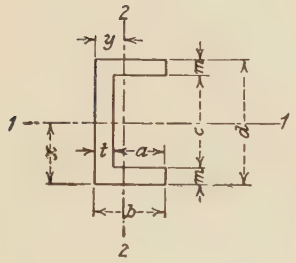
$$I_{1-1} = \frac{bd^3 - \frac{a}{4(m-n)}(c^4 - e^4)}{12}$$

$$I_{2-2} = \frac{2nb^3 + et^3 + \frac{m-n}{4a}(b^4 - t^4)}{12}$$

$$S_{1-1} = \frac{2I_{1-1}}{d} \qquad S_{2-2} = \frac{2I_{2-2}}{b}$$

$$r_{1-1} = \sqrt{\frac{I_{1-1}}{A}} \qquad r_{2-2} = \sqrt{\frac{I_{2-2}}{A}}$$

Channel



$$A = bd - ac \qquad = ct + 2mb$$

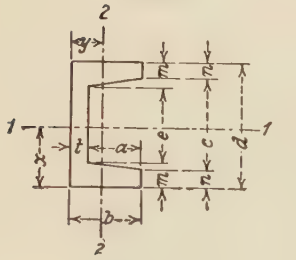
$$x = \frac{d}{2} \qquad y = \frac{ct^2 + 2mb^2}{2(ct + 2mb)}$$

$$I_{1-1} = \frac{bd^3 - ac^3}{12} \qquad I_{2-2} = \frac{ct^3 + 2mb^3}{3} - Ay^2$$

$$S_{1-1} = \frac{1}{6}\left[bd^2 - \frac{ac^3}{d}\right] \qquad S_{2-2} = \frac{I_{2-2}}{b - y}$$

$$r_{1-1} = \sqrt{\frac{bd^3 - ac^3}{12(bd - ac)}} \qquad r_{2-2} = \sqrt{\frac{I_{2-2}}{A}}$$

Channel



$$A = dt + a(m+n)$$

$$x = \frac{d}{2}$$

$$y = \frac{b^2 n + \frac{ct^2}{2} + \frac{a(m-n)}{3}(b + 2t)}{A}$$

$$I_{1-1} = \frac{bd^3 - \frac{a}{8(m-n)}(c^4 - e^4)}{12}$$

$$I_{2-2} = \frac{2nb^3 + et^3 + \frac{m-n}{2a}(b^4 - t^4)}{3} - Ay^2$$

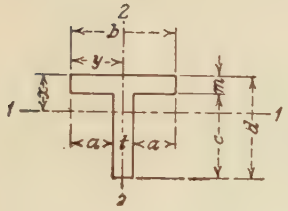
$$S_{1-1} = \frac{2I_{1-1}}{d} \qquad S_{2-2} = \frac{2I_{2-2}}{b - y}$$

$$r_{1-1} = \sqrt{\frac{I_{1-1}}{A}} \qquad r_{2-2} = \sqrt{\frac{I_{2-2}}{A}}$$

TABLE 7—*Continued*

ELEMENTS OF SECTIONS

Tee



$$A = td + 2am \qquad = mb + ct$$

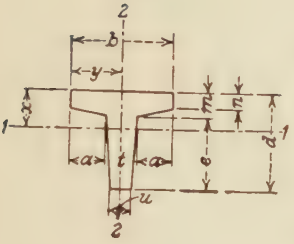
$$x = \frac{td^2 + 2am^2}{2(td + 2am)} \qquad y = \frac{b}{2}$$

$$I_{1-1} = \frac{td^3 + 2am^3}{3} - Ax^2 \qquad I_{2-2} = \frac{mb^3 + ct^3}{12}$$

$$S_{1-1} = \frac{I_{1-1}}{d - x} \qquad S_{2-2} = \frac{mb^3 + ct^3}{6b}$$

$$r_{1-1} = \sqrt{\frac{I_{1-1}}{A}} \qquad r_{2-2} = \sqrt{\frac{mb^3 + ct^3}{12(mb + ct)}}$$

Tee



$$A = \frac{e(t + u)}{2} + mt + a(m + n)$$

$$x = \frac{6an^2 + 2a(m-n)(m+2n) + 3td^2 - e(t-u)(3d-e)}{6A}$$

$$y = \frac{b}{2}$$

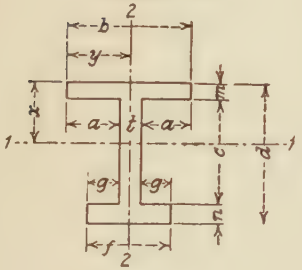
$$I_{1-1} = \frac{e^3(3u+t) + 4bm^3 - 2a(m-n)^3}{12} - A(x-m)^2$$

$$I_{2-2} = \frac{nb^3 + (m-n)t^3 + eu^3}{12} + \frac{a(m-n)[2a^2 + (2a+3t)^2]}{36} + \frac{e(t-u)[(t-u)^2 + 2(t+2u)^2]}{144}$$

$$S_{1-1} = \frac{I_{1-1}}{d - x} \qquad S_{2-2} = \frac{2I_{2-2}}{b}$$

$$r_{1-1} = \sqrt{\frac{I_{1-1}}{A}} \qquad r_{2-2} = \sqrt{\frac{I_{2-2}}{A}}$$

Flanged tee



$$A = fd - 2g(c + m) - 2am \qquad = mb + ct + nf$$

$$x = \frac{fd^2 - 2g(c+m)^2 + 2am^2}{2[fd - 2g(c+m) + 2am]} \qquad y = \frac{b}{2}$$

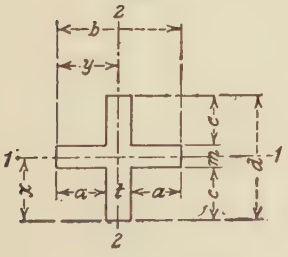
$$I_{1-1} = \frac{fd^3 - 2g(c+m)^3 + 2am^3}{3} - Ax^2$$

$$I_{2-2} = \frac{mb^3 + ct^3 + nf^3}{12}$$

$$S_{1-1} = \frac{I_{1-1}}{d - x} \qquad S_{2-2} = \frac{mb^3 + ct^3 + nf^3}{6b}$$

$$r_{1-1} = \sqrt{\frac{I_{1-1}}{A}} \qquad r_{2-2} = \sqrt{\frac{I_{2-2}}{A}}$$

Star



$$A = td + 2am \qquad = mb + 2ct$$

$$x = \frac{d}{2} \qquad y = \frac{b}{2}$$

$$I_{1-1} = \frac{td^3 + 2am^3}{12} \qquad I_{2-2} = \frac{mb^3 + 2ct^3}{12}$$

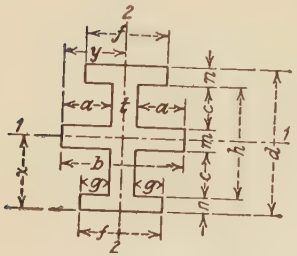
$$S_{1-1} = \frac{td^3 + 2am^3}{6d} \qquad S_{2-2} = \frac{mb^3 + 2ct^3}{6b}$$

$$r_{1-1} = \sqrt{\frac{td^3 + 2am^3}{12(td + 2am)}} \qquad r_{2-2} = \sqrt{\frac{mb^3 + 2ct^3}{12(mb + 2ct)}}$$

TABLE 7—*Continued*

ELEMENTS OF SECTIONS

Flanged star



$$A = fd - 2gh + 2am = mb + 2ct + 2nf$$

$$x = \frac{d}{2} \qquad y = \frac{b}{2}$$

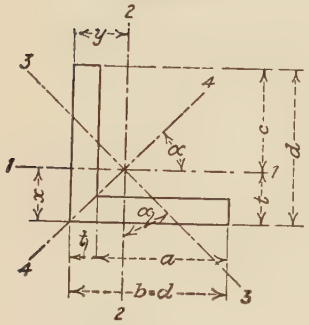
$$I_{1-1} = \frac{fd^3 - 2gh^3 + 2am^3}{12}$$

$$I_{2-2} = \frac{mb^3 + 2ct^3 + 2nf^3}{12}$$

$$S_{1-1} = \frac{2I_{1-1}}{d} \qquad S_{2-2} = \frac{2I_{2-2}}{b}$$

$$r_{1-1} = \sqrt{\frac{I_{1-1}}{A}} \qquad r_{2-2} = \sqrt{\frac{I_{2-2}}{A}}$$

Equal angle



$$A = t(b + c)$$

$$x = \frac{b^2 + ct}{2b + c} \qquad y = \frac{b^2 + ct}{2b + c} = x$$

$$\alpha = 45°$$

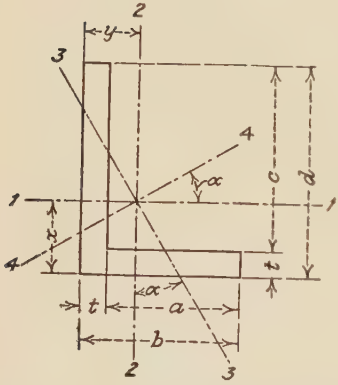
$$I_{1-1} = \frac{t(b - x)^3 + bx^3 - a(x - t)^3}{3}$$

$$I_{2-2} = I_{1-1}$$

$$I_{3-3} = \frac{\left\{ \begin{array}{c} ct^3 + c^3t + 3ct(b - 4x + 2t)^2 \\ + t^4 + 6t^2(2x - t)^2 \end{array} \right\}}{12}$$

$$I_{4-4} = \frac{ct^3 + c^3t + 3ctb^2 + t^4}{12}$$

Unequal angle



$$A = t(b + c)$$

$$x = \frac{t(b + 2c) + c^2}{2(b + c)}$$

$$y = \frac{t(2a + d) + a^2}{2(a + d)}$$

$$\text{Tan } 2\alpha = \frac{\left\{ \begin{array}{c} t[2y - t)d(d - 2x) \\ + a(2x - t)(b + t - 2y)] \end{array} \right\}}{2(I_{1-1} - I_{2-2})}$$

$$I_{1-1} = \frac{t(d - x)^3 + bx^3 - a(x - t)^3}{3}$$

$$I_{2-2} = \frac{t(b - y)^3 + dy^3 - c(y - t)^3}{3}$$

$$I_{3-3} = \frac{I_{2-2}\cos^2\alpha - I_{1-1}\sin^2\alpha}{\cos 2\alpha}$$

$$I_{4-4} = \frac{I_{1-1}\cos^2\alpha - I_{2-2}\sin^2\alpha}{\cos 2\alpha}$$

MOMENTS OF INERTIA OF AREAS—GENERAL CASE

When the moment of inertia of an area about any axis is known, that about any parallel axis in the same plane is readily found.

Let I_1 be the known moment of inertia about an axis 1–1,

A the area,

and

d the distance from axis 1–1 to a parallel axis X–X;

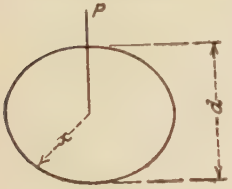
then

$$I_x = Ad^2 + I_1$$

TABLE 8

POLAR ELEMENTS OF SECTIONS

Circumference



$$A = \pi d$$

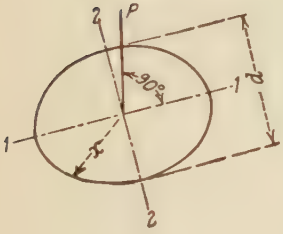
$$x = \frac{d}{2}$$

$$J_p = \frac{\pi d^3}{4}$$

$$S_p = \frac{\pi d^2}{2}$$

$$r_p = \frac{d}{2}$$

Circle



$$A = \frac{\pi d^2}{4}$$

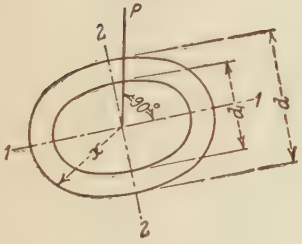
$$x = \frac{d}{2}$$

$$J_p = \frac{\pi d^4}{32}$$

$$S_p = \frac{\pi d^3}{16}$$

$$r_p = d\sqrt{\tfrac{1}{8}} = 0.353553d$$

Annulus



$$A = \frac{\pi}{4}(d^2 - d_1{}^2)$$

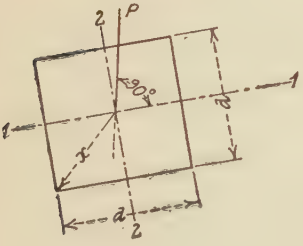
$$x = \frac{d}{2}$$

$$J_p = \frac{\pi}{32}(d^4 - d_1{}^4)$$

$$S_p = \frac{\pi}{16}\frac{d^4 - d_1{}^4}{d} = 0.19635\frac{d^4 - d_1{}^4}{d}$$

$$r_p = \sqrt{\tfrac{1}{8}(d^2 + d_1{}^2)} = 0.353553\sqrt{d^2 + d_1{}^2}$$

Square



$$A = d^2$$

$$x = d\sqrt{\tfrac{1}{2}} = 0.70711d$$

$$J_p = \frac{d^4}{6}$$

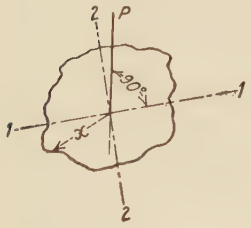
$$S_p = \frac{d^3}{3\sqrt{2}} = 0.235702d^3$$

$$r_p = d\sqrt{\tfrac{1}{6}} = 0.408248d$$

TABLE 8—*Continued*

POLAR ELEMENTS OF SECTIONS

Any area



A = area

x = distance from pole to extreme fiber

$J_p = I_{1-1} + I_{2-2}$

$S_p = \frac{J_p}{x}$

$r_p = \sqrt{\frac{J_p}{A}}$

POLAR MOMENTS OF INERTIA—GENERAL CASE

When the moments of inertia of any area are known about two rectangular axes in the plane of the area, the polar moment of inertia about an axis passing through the intersection of these axes and perpendicular to the plane of the area is readily found.

Let I_1 and I_2 be the known moments of inertia about two rectangular axes 1-1 and 2-2,

and

A the area;

then

$$J_p = I_1 + I_2$$

TABLE 9

ELEMENTS OF SOLIDS—RECTANGULAR AXES

Let w = the weight of the solid per unit of volume;
g = the weight of a unit of mass = 32.2 lbs. (Std. = 32.16);
I_m = the moment of inertia in mass units;
r = the radius of gyration.

Solid	I Axis	I_m	r
Slender rod of length l and sectional area A	Tranverse at the middle of length	$\frac{w}{g}A\frac{l^3}{12}$	$\frac{l}{\sqrt{12}} = 0.288675l$
Thin, rectangular plate of width b, length l, and thickness t	Transverse at the middle of length	$\frac{w}{g}\frac{tbl^3}{12}$	$\frac{l}{\sqrt{12}} = 0.288675l$
Thin, triangular plate with base b, altitude h, and thickness t	Parallel to the base through the center of gravity	$\frac{w}{g}\frac{tbh^3}{36}$	$\frac{h}{\sqrt{18}} = 0.235702h$
Thin, circular plate of radius r and thickness t	Diameter	$\frac{w}{g}\frac{\pi tr^4}{4}$	$\frac{r}{2} = 0.5r$
Thin, elliptical plate with axes $2a$ and $2b$ and thickness t	a axis	$\frac{w}{g}\frac{\pi tab^3}{4}$	$\frac{b}{2} = 0.5b$
Thin, elliptical plate with axes $2a$ and $2b$ and thickness t	b axis	$\frac{w}{g}\frac{\pi ta^3b}{4}$	$\frac{a}{2} = 0.5a$
Rectangular prism with dimensions a, b, and l	Axis through c.g. and normal to the face whose dimensions are a and l	$\frac{w}{g}abl\frac{a^2+l^2}{12}$	$\sqrt{\frac{a^2+l^2}{12}}$
Right circular cylinder of radius r and length l	Axis through c.g. and perpendicular to the axis of the cylinder	$\frac{w}{g}\pi r^2 l\left[\frac{r^2}{4}+\frac{l^2}{12}\right]$	$\sqrt{\frac{r^2}{4}+\frac{l^2}{12}}$
Hollow right circular cylinder with radii R and r and length l	Axis through c.g. and perpendicular to the axis of the cylinder	$\frac{w}{g}\pi l(R^2-r^2)\left[\frac{R^2+r^2}{4}+\frac{l^2}{12}\right]$	$\sqrt{\frac{R^2+r^2}{4}+\frac{l^2}{12}}$
Elliptical cylinder with axes $2a$ and $2b$ and length l	$2a$ axis through c.g.	$\frac{w}{g}\pi abl\left[\frac{b^2}{4}+\frac{l^2}{12}\right]$	$\sqrt{\frac{b^2}{4}+\frac{l^2}{12}}$

TABLE 9—*Continued*

ELEMENTS OF SOLIDS—RECTANGULAR AXES

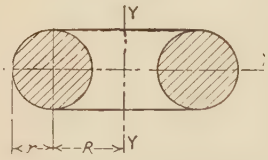
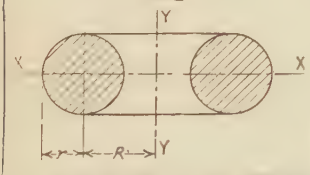
Solid	I Axis	I_m	r
Sphere of radius r	Diameter	$\frac{w}{g}\frac{8}{15}\pi r^5$	$r\sqrt{\frac{2}{5}} = 0.632456r$
Thin, spherical shell of radius r and thickness t	Diameter	$\frac{w}{g}\frac{8}{3}\pi r^4 t$	$r\sqrt{\frac{2}{3}} = 0.816497r$
Thick, spherical shell of radii R and r	Diameter	$\frac{w}{g}\frac{8}{15}\pi(R^5-r^5)$	$\sqrt{\frac{2}{5}\frac{R^5-r^5}{R^3-r^3}}$
Right rectangular pyramid with sides of base a and b and altitude h	Axis through c. g. and parallel to the base and side a	$\frac{w}{g}\frac{abh}{3}\frac{1}{20}\left[\frac{3}{4}h^2+b^2\right]$	$\sqrt{\frac{1}{20}\left[\frac{3}{4}h^2+b^2\right]}$
Right circular cone with radius of base r and altitude h	Axis through c. g. and parallel to the base	$\frac{w}{g}\frac{\pi r^2 h}{3}\frac{3}{20}\left[r^2+\frac{h^2}{4}\right]$	$\sqrt{\frac{3}{20}\left[r^2+\frac{h^2}{4}\right]}$
Ring 	Axis XX	$\frac{w}{g}2\pi^2 R r^2\left[\frac{R^2}{2}+\frac{5r^2}{8}\right]$	$\sqrt{\frac{R^2}{2}+\frac{5r^2}{8}}$

TABLE 10

ELEMENTS OF SOLIDS—POLAR AXES

Solid	I Axis	I_m	r
Rectangular plate of width a, length l, and thickness t	Through the center and normal to the surface.	$\frac{w}{g}alt\frac{a^2+l^2}{12}$	$\sqrt{\frac{a^2+l^2}{12}}$ $=0.288675\sqrt{a^2+l^2}$
Circular plate of radius r and thickness t	Through the center and normal to the surface	$\frac{w}{g}\frac{\pi tr^4}{2}$	$r\sqrt{\frac{1}{2}}$ $=0.70711r$
Elliptical plate with axes $2a$ and $2b$ and thickness t	Through c.g. and normal to the plate	$\frac{w}{g}\frac{\pi abt(a^2+b^2)}{4}$	$\frac{1}{2}\sqrt{a^2+b^2}$
Right parallelopiped with sides a and b length l	Axis of the parallelopiped	$\frac{w}{g}abl\frac{a^2+b^2}{12}$	$\sqrt{\frac{a^2+b^2}{12}}$ $=0.288675\sqrt{a^2+b^2}$
Right circular cylinder of radius r and length l	Axis of the cylinder	$\frac{w}{g}\frac{\pi lr^4}{2}$	$r\sqrt{\frac{1}{2}}$ $=0.70711r$
Hollow, right circular cylinder with radii R and r and length l	Axis of the cylinder	$\frac{w}{g}\pi l\frac{R^4-r^4}{2}$	$\sqrt{\frac{R^2+r^2}{2}}$
Right rectangular pyramid with sides of base a and b and altitude h	Axis of the pyramid	$\frac{w}{g}\frac{abh}{3}\frac{a^2+b^2}{20}$	$\sqrt{\frac{a^2+b^2}{20}}$
Right circular cone with radius of base r and altitude h	Axis of the cone	$\frac{w}{g}\frac{\pi hr^4}{10}$	$r\sqrt{\frac{3}{10}}$ $=0.547723r$
Frustum of a right circular cone with bases of radii R and r and altitude h	Axis of the frustum	$\frac{w}{g}\frac{\pi h}{10}\frac{R^5-r^5}{R-r}$	$\sqrt{\frac{3}{10}\frac{R^5-r^5}{R^3-r^3}}$
Ellipsoid having axes $2a$, $2b$, and $2c$	Axis $2c$	$\frac{w}{g}\pi abc\frac{4(a^2+b^2)}{15}$	$\sqrt{\frac{a^2+b^2}{5}}$
Ellipsoid of revolution with polar axis of $2c$ and equatorial axis of $2a$	Axis $2c$	$\frac{w}{g}\frac{8}{15}\pi a^4c$	$a\sqrt{\frac{2}{5}}$ $=0.63246a$
Paraboloid of revolution with radius of base r and altitude h	Axis of the paraboloid	$\frac{w}{g}\frac{\pi r^4h}{6}$	$r\sqrt{\frac{1}{3}}$ $=0.57735r$
Ring 	Axis YY	$\frac{w}{g}2\pi^2Rr^2(R^2+\frac{3}{4}r^2)$	$\sqrt{R^2+\frac{3}{4}r^2}$

The mass moments of inertia of the solids enumerated in the tables are all given about axes passing through their centers of gravity. The moment of inertia about any axis parallel to that passing through the center of gravity is readily found.

Let I_{cm} be the mass moment of inertia about an axis through the center of gravity,
V the volume,
d the distance to the required axis, as X,

and $\frac{w}{g}$ the density;

then $$I_{xm} = \frac{w}{g}Vd^2 + I_{cm}$$

TABLE 11

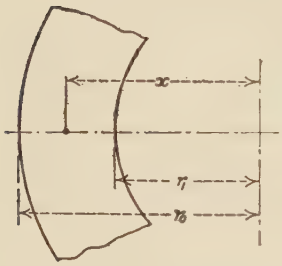
MISCELLANEOUS DATA

1. Approximate length of curves

Let l = the span length;
h = the middle ordinate;
$\frac{h}{l} = n$,
and
L = the length of the curve.

For a parabola:

$$L = l[1 + \tfrac{8}{3}n^2 - \tfrac{32}{5}n^4 + \tfrac{256}{7}n^6 - \tfrac{2560}{9}n^8 + \ldots]$$



This expression also is closely approximate for other curves, as the catenary and circle, if h is small with respect to l.

2. Center of pressure on an annulus

Assume a uniform unit pressure on the area of an annulus. The distance from the center to the center of pressure is:

$$x = \frac{2}{3}\left(\frac{r_0^3 - r_1^3}{r_0^2 - r_1^2}\right).$$

3. Maximum pressure for cylindrical contact

Assume that the outside of a cylinder is pressed against the inside of a second close-fitting cylinder.

Let d = the diameter of the cylinders;
l = the length of contact;
P = the force applied to the inside cylinder;
p = the unit pressure on the projected area of contact
$= \frac{P}{dl}$;
p_{max} = the maximum unit pressure between the cylinders at the end of the diameter in the direction of the force P.

Then:

$$p_{max} = \frac{4}{\pi}p \text{ or about } 1.27p.$$

This is the case of a bolt or rivet in a hole or of a journal in a tight-fitting bearing.

4. Flat plates uniformly loaded

Let w = the pressure on a flat plate per unit of area;
t = the thickness of the plate;
λ = Poisson's ratio, taken at $\frac{1}{4}$ for cast iron and $\frac{1}{3}$ for steel;
T = the maximum unit tensile stress in the plate;
r = the radius of a circular plate;
r_0 = the radius of a loaded portion at the center of a circular plate;

TABLE 11—*Continued*

a = the semi-major axis of an elliptical plate;
b = the semi-minor axis of an elliptical plate;
$2l$ = the length of a rectangular plate,
and
$2m$ = the width of a rectangular plate.
The loads should be taken in pounds and all dimensions in inches.

VALUES OF T

Shape of Plate	Loading	Supports of Edges of Plate	VALUES OF T	
			Cast Iron	Steel
Circular	Uniform	Supported	$\frac{3}{2}(1-\lambda)\left(\frac{r}{t}\right)^2 w$	$\frac{3}{2}(1-\lambda)\left(\frac{r}{t}\right)^2 w$
Circular	Uniform	Supported	$\frac{9}{8}\left(\frac{r}{t}\right)^2 w$	$\left(\frac{r}{t}\right)^2 w$
Circular	Uniform	Fixed	$\frac{3}{4}\left(\frac{r}{t}\right)^2 w$	$\frac{2}{3}\left(\frac{r}{t}\right)^2 w$
Circular	Uniform on circle at center of radius r_0	Supported	$\frac{3}{4}\left(\frac{3}{2}+3\log_e\frac{r}{r_0}\right)\left(\frac{r_0}{t}\right)^2 w$	$\frac{2}{3}\left(\frac{3}{2}+3\log_e\frac{r}{r_0}\right)\left(\frac{r_0}{t}\right)^2 w$
Elliptical	Uniform	Supported	$\frac{9}{4}\frac{a^2b^2}{(a^2+b^2)t^2}w$	$2\frac{a^2b^2}{(a^2+b^2)t^2}w$
Elliptical	Uniform	Fixed	$\frac{3}{2}\frac{a^2b^2}{(a^2+b^2)t^2}w$	$\frac{4}{3}\frac{a^2b^2}{(a^2+b^2)t^2}w$
Rectangular	Uniform	Supported	.....	$\frac{9}{4}\frac{l^2m^2}{(l^2+m^2)t^2}w$
Rectangular	Uniform	Fixed	.....	$\frac{3}{2}\frac{l^2m^2}{(l^2+m^2)t^2}w$
Square	Uniform	Supported	.....	$\frac{9}{8}\left(\frac{l}{t}\right)^2 w$
Square	Uniform	Fixed	.....	$\frac{3}{4}\left(\frac{l}{t}\right)^2 w$

For the development of these expressions and a discussion of their application and limitations, see "Mechanics of Materials," by Mansfield Merriman, John Wiley and Sons, Inc., New York.

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